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Thermal instability of rotating Jeffrey nanofluids in porous media with variable gravity

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Abstract

It is investigated how changes in gravity affect the thermal instability rotating Jeffrey nanofluids in porous media. Along with the Galerkin method and normal mode approach, the Darcy model is used. The distinct variable gravity parameters taken in this paper are: $h(z) = z^2 - 2z$, $h(z) = -z^2$, $h(z) = -z$ and $h(z) = z$ and their effects on the Jeffrey parameter, Taylor number, moderated diffusivity ratio, porosity of porous media, Lewis number and nanoparticle Rayleigh number on stationary convection have been scrutinized and graphically shown. Our finding demonstrates that varying gravity parameter $h(z) = z^2 - 2z$ has more stabilising impact on stationary convection. We have also discovered the necessary condition for overstability in the instance of oscillatory convection for this problem.

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1. Introduction

Choi [1] devised the term "nanofluid", which was defined as a liquid having a dispersion of submicronic solid particles (nanoparticles). Convective transport in nanofluids was an issue that Buongiorno [2] examined. He advanced Choi [1] work by including mathematical terms. Different uses for nanofluid were introduced by Tzeng *et al.* [3], Kim *et al.* [4], Routbort *et al.* [5] and Donzelli *et al.* [6].

The theoretical and experimental findings in Chandrasekhar [7] are based on the Newtonian fluid's capacity to convect steadily in the absence of a porous medium while subject to rotation and a magnetic field. Papamarkos *et al.* [8] described a method based upon octagonally symmetric design and IIR digital filterations. A Study of Non-Newtonian Nanofluids like Rivlin-Ericksen, Maxwellian and Modified Darcy-Maxwell Model for S.C. is employed by Rana *et al.* [9], Chand [10] and Singh *et al.* [11], respectively. Linearised stability theory was used by Lapwood [12] to investigate convective flow in a porous material. Nield *et al.* [13] introduced convection in porous media. Convection with internal heating in a porous material saturated by a nanofluid was examined by Nield *et al.* [14]. The results reveal that the inclusion of nanofluid particles increases the system's instability. Later, Nield *et al.* [15] provided brief introduction to the book Nield *et al.* [13]. Tzou [16, 17] investigated how

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natural convection affected nanofluids' thermal instability. Several researchers Nield *et al.* [18, 19, 20], Sheu [21] and Chand *et al.* [22, 23, 24] used the Buongiorno [2] model to investigate nanofluids' thermal instability in porous media. Ramanuja *et al.* [25] also used porous medium in their problem.

The development of objects in an astrophysical plasma environment is caused by thermal instability, which is studied by Kaothekar [26] for partly ionised thermal plasma. This plasma has a relation to astrophysical condensations. Chand *et al.* [27] studied T.I. effect on Oldroydian nanofluid by considering realistic boundary conditions. Nield *et al.* [20], Sharma [28], Yadav *et al.* [29], Chand *et al.* [30, 31], Govender [32] and Chand *et al.* [33] examined the of nanofluid's thermal instability in rotation. Some of them examined rotation's interactions with suspended particles, many non-newtonian fluids, couple-stress rotation's interactions with porous media, and rotation's interactions with itself. They discovered that a system's thermal instability depends heavily on rotation. Yadav *et al.* [34] employed magneto-convection in rotatory layer of nanofluid and electro-thermo-convection in a horizontal layer of rotating nanofluid is examined by Chand *et al.* [35]. Additionally, several of them created rotation-based industrial applications, including those found in nuclear reactors, power plants, the petroleum sector, geophysics etc. Nanofluid oscillating convection in a porous media was explored by Chand *et al.* [36]. Gautam *et al.* [37] established the concepts of free-free, rigid-free, and rigid-rigid boundary conditions for the electrohydrodynamic T.I. of a Jeffrey nanofluid layer saturating a porous medium and concluded that the rotation parameter stabilises the system for bottom and top-heavy layouts. A porous-medium-saturated Jeffrey nanofluid flow was studied by Rana [38] for the effects of rotation. For both bottom and top-heavy arrangements and provided evidence that the rotation parameter stabilises the system. Sharma *et al.* [39] studies the electrohydro dynamics convection in dielectric rotating Oldroydian nanofluid in porous medium.

The idealisation of uniform gravity used in theoretical research, while appropriate for lab applications, is seldom warranted for large-scale convection events happening in the Earth's atmosphere, ocean, or mantle. Gravity must thus be viewed as a changeable quantity that changes with distance from a surface or other reference point. Pradhan *et al.* [40] investigated the thermal instability of a fluid layer in a changeable gravitational field and discovered that boosting the gravitational field vertically accelerates the commencement of convection. A porous media with an internal heat source and an inclined temperature gradient was studied by Alex *et al.* [41] to see how changing gravity affected thermal instability. Straughan [42] used both linear theory and nonlinear energy theory to analyse the issue for the case of stiff boundaries in a spatially changing gravitational field and discovered that the nonlinear conclusions were remarkably similar to the linear ones. Chand *et al.* [43] looked into how changing gravity would affect a layer of nanofluid in a porous medium and found that the gravity parameter had a big impact on fluid stability. Theoretically and visually, Chand [10] investigates thermal instability of Maxwell non-Newtonian fluid with varying gravity. Using a higher order Galerkin method, Yadav [44] investigated the joint effects of variable gravity fields and throughflow on the beginning of convective motion in a porous medium layer. The results showed that both the throughflow and gravity variation parameters serve to delay the motion's onset. Mahajan *et al.* [45] analyses the effects of several fundamental temperature and concentration gradients on a layer of reactive fluid in a varied gravity field utilising both linear and non-linear analysis. Surya *et al.* [46] examine the thermal instability of a horizontal layer of liquid heated from below that is contained between thermally conductive porous walls under the influence of a fluctuating gravitational field. As limiting examples of the permeability parameters of the borders, the impact of the gravity variation growing vertically upward for various particular situations of the boundary conditions is derived and graphically depicted. In a layer of porous media, the effects of rotation and varying gravitational strengths on the beginning of heat convection were computed by Yadav [47]. The findings demonstrate how the gravity variation parameter and the rotation parameter both delay convection's arrival. With increased rotation and gravity variation parameters, the measurement of the convection cells diminishes. Shekhar *et al.* [48] investigates numerically how varying gravity affects rotational convection in a porous material that is poorly packed. The linear, parabolic, cubic, and exponential functions are taken into account for variations in gravitational force. While the Darcy number increases convection cell size, convection cell size falls when the variable gravity parameter and rotation parameter are increased. Additionally, it has been found that the system is more stable for exponential gravity functions than for cubic gravity functions. Chand *et al.* [49] investigated the impact of variable gravity on the thermal instability of rotating nanofluids in porous media and discovered that, in the presence of rotation and also for nanoparticle Rayleigh numbers, decreasing the gravity parameter has a stabilising effect while increasing it has a destabilising effect.

By taking into account its numerous applications in various fields like geophysics, astrophysics, food processing, oil reservoir modeling, building of thermal insulations and nuclear reactors etc. This brief review of the literature leads one to believe that such a problem was nonexistent; hence, the current problem of thermal instability of rotating Jeffrey nanofluids in porous media with variable gravity was chosen.

2. Mathematical Formulation

Here, we examine a rotating horizontal Jeffrey nanofluid layer heated from below in a porous medium with medium permeability k_1 and porosity ε and angular velocity $\Omega(0, 0, \Omega)$ bordered by plane $z = 0$ and $z = d$, working upward under the influence of variavle gravity. Furthermore, it is assumed from Nield *et al.* [18] and Chand *et al.* [49] that there is variable gravity along z -direction *i.e.* $\boldsymbol{g} = (1+\delta h(z))g$, where $\delta h(z)$ is the variable gravity parameter. When the top boundary layer is at $z = d$, the temperature T and volumetric fraction φ of nanoparticles are assumed to be T_1 and φ_1, respectively, with $T_0 > T_1$ and $\varphi_0 > \varphi_1$.

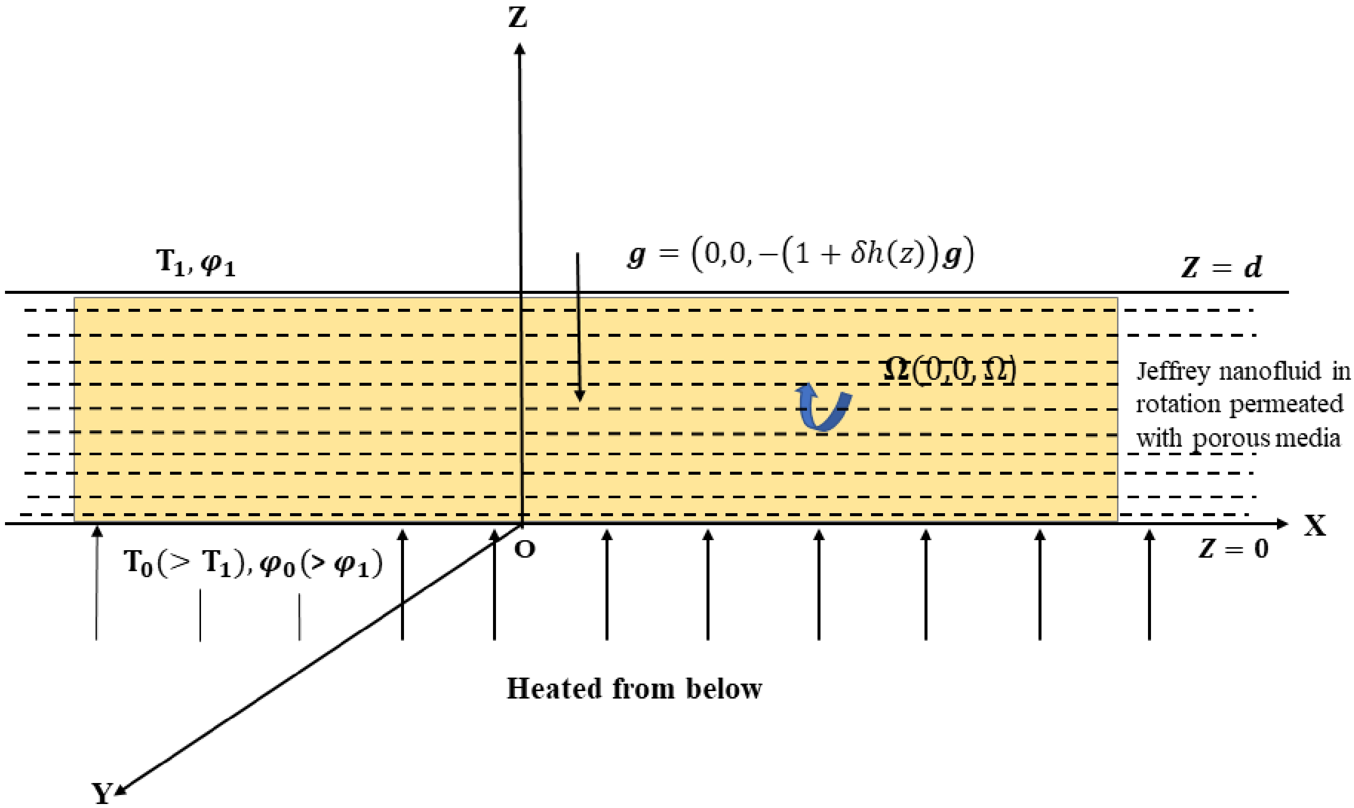

Figure 1: Physical Configuration

For the sake of simplicity, Oberbeck-Boussinesq approximation is used and Darcy's law is taken to be true by Nield *et al.* [18] and Chand *et al.* [49]. Thus from Buongiorno [2], Chandrasekhar [7], Nield *et al.* [18] and Chand *et al.* [33, 49] the pertinent governing equations for the study of spinning Jeffrey nanofluid in porous medium are

$$\nabla.\boldsymbol{q} = 0 \tag{1}$$

$$0 = -\nabla p + \left(\varphi\rho_p + (1-\varphi)\left\{\rho_0\left(1-\alpha\left(T-T_1\right)\right)\right\}\right)\boldsymbol{g} - \frac{\mu}{k_1(1+\lambda)}\boldsymbol{q} + \frac{2\rho_0}{\varepsilon}(\boldsymbol{q}\times\Omega) \tag{2}$$

For nanoparticle, the continuity equation is given by (Buongiorno [2])

$$\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}\boldsymbol{q}.\nabla\varphi = D_B\nabla^2\varphi + \frac{D_T}{T_1}\nabla^2 T \tag{3}$$

For the nanofluid, the equation of thermal energy is given by (Buongiorno [2] and Chand *et al.* [49])

$$(\rho c)_m\frac{\partial T}{\partial t} + (\rho c)_f\,\boldsymbol{q}.\nabla T = k_m\nabla^2 T + \varepsilon\,(\rho c)_p\left[D_B\nabla\varphi.\nabla T + \frac{D_T}{T_1}\nabla T.\nabla T\right] \tag{4}$$

where $\boldsymbol{q}$ is the fluid velocity, p is the pressure, ρ_0 is nanofluid density at $z = 0$, ρ_P is nanoparticles density, φ is the volume fraction of the nanoparticles, T is temperature, T_1 is the reference temperature, α is thermal expansion coefficient, $\boldsymbol{g}$ is gravitational acceleration and k_1 is medium fluid permeability, μ is coefficient of viscosity, ε is the porosity of the porous media, $\lambda = \frac{\lambda_1}{\lambda_2}$ the Jeffrey parameter (which is the ratio of stress-relaxation-time parameter, λ_1 to strain-retardation-time parameter, λ_2), the fluid's heat capacity in porous medium is $(\rho c)_m$, $(\rho c)_P$ stands for heat capacity of nanoparticles, $(\rho c)_f$ stands for fluid's heat capacity, k_m is the fluid's thermal conductivity, the Brownian diffusion coefficient is D_B and D_T is nanoparticles' the thermophoretic diffusion coefficient (Chand *et al.* [49]).

We presumed nanoparticles' temperature and volumetric fraction as constant. Thus, boundary conditions (Chandrasekhar [7] and Nield *et al.* [18]) are

$$\left\{\begin{array}{llll} w = 0, & T = T_0, & \varphi = \varphi_0 & at \quad z = 0 \\ w = 0, & T = T_1, & \varphi = \varphi_1 & at \quad z = d \end{array}\right. \tag{5}$$

On introducing non-dimensional variables as (Chandrasekhar [7])

$$(x^*, y^*, z^*) = \frac{(x,y,z)}{d}, \quad \boldsymbol{q}^* = \boldsymbol{q}\frac{d}{\kappa_m}, \quad t^* = \frac{t\kappa_m}{\sigma d^2}$$

$$p^* = \frac{pk_1}{\mu\kappa_m}, \quad \varphi^* = \frac{\varphi-\varphi_0}{\varphi_1-\varphi_0}, \quad T^* = \frac{T-T_1}{T_0-T_1}$$

where $\kappa_m = \frac{k_m}{(\rho c)_f}$, $\quad\sigma = \frac{(\rho c)_m}{(\rho c)_f}$ are fluid's thermal diffusivity and thermal capacity ratio, respectively. Relaxing the star $(*)$ for simplification. The reduced non-dimensional form of equations 1,2,3,4 are:

$$\nabla.\boldsymbol{q} = 0 \tag{6}$$

$$0 = -\nabla p - \frac{1}{1+\lambda}\boldsymbol{q} - R_m(1+\delta h(z))\hat{k} + R_D(1+\delta h(z))T\hat{k} - R_n(1+\delta h(z))\varphi\hat{k} + \sqrt{T_a}\left(\boldsymbol{q}\times\hat{k}\right) \tag{7}$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}\boldsymbol{q}.\nabla\varphi = \frac{1}{L_n}\nabla^2\varphi + \frac{N_A}{L_n}\nabla^2 T \tag{8}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{q}.\nabla T = \nabla^2 T + \frac{N_B}{L_n}\nabla\varphi.\nabla T + \frac{N_A N_B}{L_n}\nabla T.\nabla T \tag{9}$$

where dimensionless parameters are
$R_m = \frac{(\rho_P\varphi_0+\rho_0(1-\varphi_0))gk_1 d}{\mu\kappa_m}$ is density Rayleigh number,
$R_D = \frac{\rho_0\alpha(T_0-T_1)gk_1 d}{\mu\kappa_m}$ is Rayleigh Darcy Number
$R_n = \frac{(\rho_p-\rho_0)(\varphi_1-\varphi_0)gk_1 d}{\mu\kappa_m}$ is nanoparticle Rayleigh number,
$T_a = \left(\frac{2\Omega\rho d^2}{\mu}\right)^2$ is Taylor number,
$L_n = \frac{\kappa_m}{D_B}$ is Lewis number,
$N_A = \frac{D_T(T_0-T_1)}{D_B T_1(\varphi_1-\varphi_0)}$ is nanofluid modified diffusivity ratio,
$N_B = \frac{\varepsilon(\rho c)_p(\varphi_1-\varphi_0)}{(\rho c)_f}$ is modified nanoparticle-density increment.
The reduced non-dimensional boundary conditions are:

$$\begin{cases} w=0, \quad T=1, \quad \varphi=0 \quad at \quad z=0 \\ w=0, \quad T=0, \quad \varphi=1 \quad at \quad z=1 \end{cases} \tag{10}$$

3. Basic States and it's Solutions

The time independent basic states for nanofluid are expressed as (Nield *et al.* [18, 19] and Chand *et al.* [49]):

$$\begin{cases} \boldsymbol{q}(u,v,w) = \boldsymbol{0} \quad \Rightarrow \quad u=v=w=0, \\ p = p_b(z), \quad T = T_b(z), \quad \varphi = \varphi_b(z) \end{cases} \tag{11}$$

The basic variable represented by subscript b. Using equation (11) in 6), (7,8), (9), these equations reduce to

$$0 = -\frac{d}{dz}p_b(z) - R_m(1+\delta h(z)) + R_D(1+\delta h(z))T_b(z) - R_n(1+\delta h(z))\varphi_b(z) \tag{12}$$

$$\frac{d^2}{dz^2}\varphi_b(z) + N_A\frac{d^2}{dz^2}T_b(z) = 0 \tag{13}$$

$$\frac{d^2}{dz^2}T_b(z) + \frac{N_B}{L_n}\frac{d}{dz}\varphi_b(z)\frac{d}{dz}T_b(z) + \frac{N_A N_B}{L_n}\left(\frac{d}{dz}T_b(z)\right)^2 = 0 \tag{14}$$

Solving equation 13 with boundary conditions equation 10, we get

$$\varphi_b(z) = (1-N_A)z + (1-T_b)N_A \tag{15}$$

Using (15) in equation (14), we have

$$\frac{d^2}{dz^2}T_b(z) + \frac{(1-N_A)N_B}{L_n}\frac{d}{dz}T_b(z) + \frac{N_A N_B}{L_n}\left(\frac{d}{dz}T_b(z)\right)^2 = 0$$

Neglecting the higher order term, we have

$$\frac{d^2}{dz^2}(T_b(z)) + \frac{(1-N_A)N_B}{L_n}\frac{d}{dz}(T_b(z)) = 0 \tag{16}$$

Using boundary conditions (10), the solution of differential equation 16 is

$$T_b(z) = \frac{e^{-\frac{(1-N_A)N_B}{L_n}z}\left[1 - e^{-\frac{(1-N_A)N_B}{L_n}(1-z)}\right]}{1 - e^{-\frac{(1-N_A)N_B}{L_n}}} \tag{17}$$

According to Buongiorno [2] hypothesis, the approximated solutions for equations 15 and 17 are given as

$$T_b = 1 - z, \quad and \quad \varphi_b = z \tag{18}$$

These approximated solutions 18 agrees well with the results obtained by Nield *et al.* [18, 19, 20], Sheu [21] and Chand *et al.* [49].

4. Perturbation Solutions

superimposing infinitesimal perturbation on the basic states in ordered to examine the stability of the system, the basic states equation 11 are written in following form (Nield *et al.* [18, 19, 20] and Chand *et al.* [49])

$$\begin{cases} \boldsymbol{q}(u,v,w) = \boldsymbol{0} + \boldsymbol{q}'(u,v,w), \quad p = p_b + p' \\ T = T_b + T' = (1-z) + T', \quad \varphi = \varphi_b + \varphi' = z + \varphi' \end{cases} \tag{19}$$

Using (19) in equations (6, 7,8,9), and linearize by ignoring the products of primes and for convenience discarding primes ($'$) . We obtain the reduced equations (6,7,8,9) as

$$\nabla.\boldsymbol{q} = 0 \tag{20}$$

$$0 = -\nabla p - \frac{1}{1+\lambda}\boldsymbol{q} - R_n(1+\delta h(z))\varphi\hat{k} + R_D(1+\delta h(z))T\hat{k} + \sqrt{T_a}\left(\boldsymbol{q}\times\hat{k}\right) \tag{21}$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}w = \frac{N_A}{L_n}\nabla^2 T + \frac{1}{L_n}\nabla^2\varphi \tag{22}$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T - 2\frac{N_A N_B}{L_n}\frac{\partial T}{\partial z} + \frac{N_B}{L_n}\left(\frac{\partial T}{\partial z} - \frac{\partial\varphi}{\partial z}\right) \tag{23}$$

and Boundary Conditions are

$$\varphi = 0, \quad T = 0, \quad w = 0 \quad at \quad z = 0 \ and \ z = 1 \tag{24}$$

It should be noted that R_m is unrelated in equations 21,22 and 23, it is simply the basic static pressure gradient measurement. Operating equation 21 with $\hat{k}.curl.curl$, we get (i.e. Making use of result $curl.curl = grad.div - \nabla^2$)

$$\frac{1}{1+\lambda}\nabla^2 w = -R_n(1+\delta h(z))\nabla_H^2\varphi + R_D(1+\delta h(z))\nabla_H^2 T - \sqrt{T_a}\frac{\partial\xi}{\partial z} \tag{25}$$

Now eliminating p from equation (21), i.e. by operating it with $i\frac{\partial}{\partial y}$ and further with $-j\frac{\partial}{\partial x}$, respectively and further solving, we get

$$\xi = (1+\lambda)\sqrt{T_a}\frac{\partial w}{\partial z} \tag{26}$$

Now, using (26) in equation (25), we have

$$\frac{1}{1+\lambda}\left[\frac{1}{1+\lambda}\nabla^2 w + R_n(1+\delta h(z))\nabla_H^2\varphi - R_D(1+\delta h(z))\nabla_H^2 T\right] + T_a\frac{\partial^2 w}{\partial z^2} = 0 \tag{27}$$

5. Stability Analysis by Normal Mode

The disturbances analysing by normal mode analysis as follow (Chandrasekhar [7]):

$$[w, T, \varphi] = [W(z), \Theta(z), \Phi(z)]\, exp(ik_x x + ik_y y + nt) \tag{28}$$

where growth rate is represented as n and the wave number along x and y directions are k_x and k_y, respectively. Using equation 28 in equations 22, 23 and 27, we get

$$\frac{1}{1+\lambda}\left[\frac{1}{1+\lambda}\left(D^2-a^2\right)W + R_D(1+\delta h(z))a^2\Theta - R_n(1+\delta h(z))a^2\Phi\right] + T_a D^2 W = 0 \tag{29}$$

$$\frac{1}{\varepsilon}.W - \frac{N_A}{L_n}(D^2-a^2)\Theta + \left[\frac{n}{\sigma} - \frac{(D^2-a^2)}{L_n}\right]\Phi = 0 \tag{30}$$

$$W + \left[(D^2-a^2) + \frac{N_B}{L_n}D - 2\frac{N_A N_B}{L_n}D - n\right]\Theta - \frac{N_B}{L_n}D\Phi = 0 \tag{31}$$

where $D = \frac{d}{dz}$ and $-a^2 = k_x^2 + k_y^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\nabla^2 = \frac{d^2}{dz^2} - a^2 = D^2 - a^2$. The a is the dimensionless resulting wave number. The boundary conditions by considering normal mode are written as Chandrasekhar [7] (free- free Boundary Condition)

$$W = D^2W = \Theta = \Phi = 0 \quad at \quad z = 0 \;\; and \;\; z = 1 \tag{32}$$

Assume that the solutions for W, Θ and Φ are of the form (Chandrasekhar [7])

$$W = W_0\, sin(\pi z), \quad \Theta = \Theta_0\, sin(\pi z), \quad \Phi = \Phi_0\, sin(\pi z) \tag{33}$$

These solutions in (33) satisfy the boundary conditions (32). Substituting solution (33) into equations (29,30,31) and integrating each equations individually within limits $z = 0$ to $z = 1$, we gain the following matrix equation

$$\begin{bmatrix} \frac{J}{1+\lambda} + (1+\lambda)\pi^2 T_a & -a^2R_D(1+\delta h(z)) & a^2R_n(1+\delta h(z)) \\ 1 & -(J+n) & 0 \\ \frac{1}{\varepsilon} & \frac{N_A}{L_n}J & \frac{J}{L_n} + \frac{n}{\sigma} \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{34}$$

where $J = \pi^2 + a^2$ is the entire wave number. The eigenvalue to the system of linear equation 34 is given as

$$R_D = \left[\frac{J}{1+\lambda} + (1+\lambda)\pi^2 T_a\right]\frac{(J+n)}{a^2(1+\delta h(z))} - \frac{\left[\frac{N_A}{L_n}J + \frac{(J+n)}{\varepsilon}\right]}{\frac{J}{L_n} + \frac{n}{\sigma}}R_n \tag{35}$$

6. Stationary Convection

For steady state, put $n = 0$ in equation 35, we obtain

$$R_D = \frac{(\pi^2+a^2)^2}{a^2(1+\lambda)(1+\delta h(z))} + \frac{(\pi^2+a^2)(1+\lambda)\pi^2 T_a}{a^2(1+\delta h(z))} - \left(N_A + \frac{L_n}{\varepsilon}\right)R_n \tag{36}$$

The Rayleigh Darcy Number for stationary convection reveal by the equation 36 is a function of a, λ, $\delta h(z)$, T_a, N_A, L_n, ε, R_n.

In non-appearance of Jeffrey's nanofluid ($\lambda = 0$), the equation 36 reduces to

$$R_D = \frac{(\pi^2+a^2)^2}{a^2(1+\delta h(z))} + \frac{(\pi^2+a^2)\pi^2 T_a}{a^2(1+\delta h(z))} - \left(N_A + \frac{L_n}{\varepsilon}\right)R_n \tag{37}$$

Equation 37 agrees well with the results obtained by Chand *et al.* [49] for stationary convection.
In non-appearance of Jeffrey's nanofluid ($\lambda = 0$) and rotation ($T_a = 0$), the equation 36 reduces to

$$R_D = \frac{(\pi^2+a^2)^2}{a^2(1+\delta h(z))} - \left(N_A + \frac{L_n}{\varepsilon}\right)R_n \tag{38}$$

Equation 38, agrees well with the results obtained by Pradhan *et al.* [40]. In non-appearance of Jeffrey's nanofluid ($\lambda = 0$), rotation ($T_a = 0$) and constant gravity ($\delta h(z) = 0$), then the equation 36 reduces to

$$R_D = \frac{(\pi^2+a^2)^2}{a^2} - \left(N_A + \frac{L_n}{\varepsilon}\right)R_n \tag{39}$$

Equation 39, agrees well with the results obtained by Nield *et al.* [18] and Chand *et al.* [49]. According to Nield *et al.* [18], the critical value of equation 36 is accomplished at $a = \pi$, so

Table 1: On the onset of Stationary Convection

$(R_D)_c$		Variation of z in graphs	Constant Variables							Variable Gravity's impact on Stationary Convection			
			λ	T_a	L_n	R_n	ε	N_A	δ	$h(z) = z^2 - 2z$	$h(z) = -z^2$	$h(z) = -z$	$h(z) = z$
λ	0.3 0.6 0.9	0 - 1		100	500	-1	0.6	5	0.5	Stabilising	Stabilising	Stabilising	Destabilising
T_a	100 200 300	0 - 1	0.6		500	-1	0.6	5	0.5	Stabilising	Stabilising	Stabilising	Destabilising
L_n	100 500 1000	0 - 1	0.6	100		-1	0.6	5	0.5	Stabilising	Stabilising	Stabilising	Destabilising
R_n	-1 -0.5 -0.1	0 - 1	0.6	100	500		0.6	5	0.5	Destabilising	Destabilising	Destabilising	Destabilising
ε	0.3 0.6 0.9	0 - 1	0.6	100	500	-1		5	0.5	Destabilising	Destabilising	Destabilising	Destabilising
N_A	1 5 10	0 - 1	0.6	100	500	-1	0.6		0.5	Stabilising	Stabilising	Stabilising	Destabilising

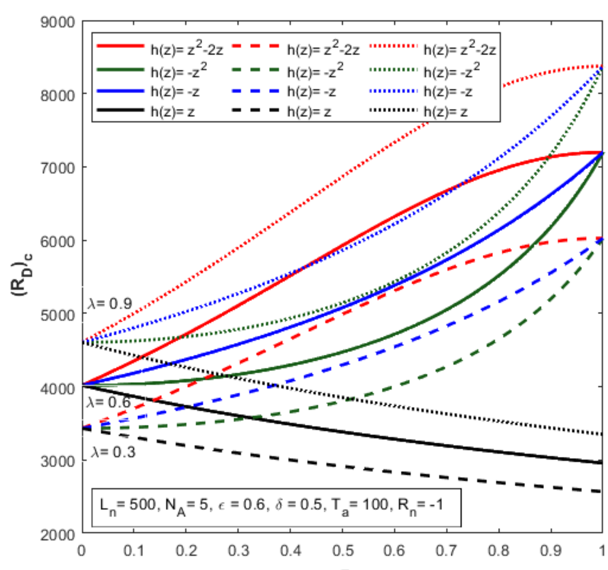

Figure 2: Variability of $(R_D)_c$ w.r.t. z for distinct values of $h(z)$ by taking distinct values of λ

for stationary convection the critical Rayleigh-Darcy Number is specified as

$$(R_D)_c = \frac{4\pi^2}{(1+\lambda)(1+\delta h(z))} + \frac{2\pi^2(1+\lambda)T_a}{1+\delta h(z)} - \left(N_A + \frac{L_n}{\varepsilon}\right)R_n \quad (40)$$

In non-appearance of rotation ($T_a = 0$), Jeffrey's nanofluid ($\lambda = 0$), nanoparticles and at constant gravity ($\delta h(z) = 0$), we

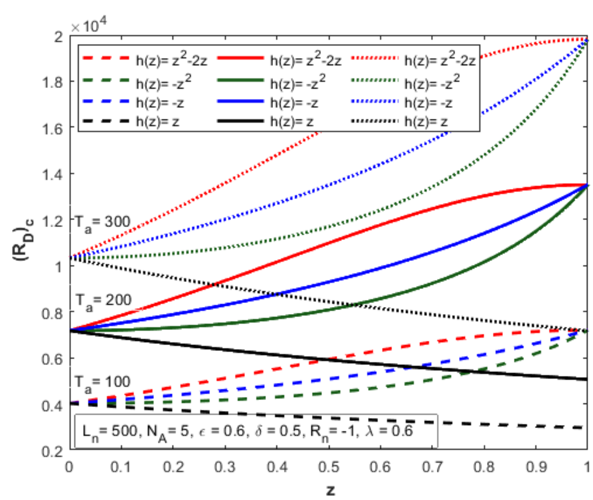

Figure 3: Variability of $(R_D)_c$ w.r.t. z for distinct values of $h(z)$ by taking distinct values of T_a

obtained the Rayleigh Darcy Number given as

$$(R_D)_c = 4\pi^2$$

This agrees well with the results obtained by Lapwood [12] for regular field.

7. Oscillatory Convection

Here, possibility for oscillatory convection is considered. For oscillatory convection, put $n = in_i$ in equation 35, we have

$$R_D = \left[\frac{J}{1+\lambda} + (1+\lambda)\pi^2 T_a\right]\frac{(J + in_i)}{a^2(1+\delta h(z))}$$

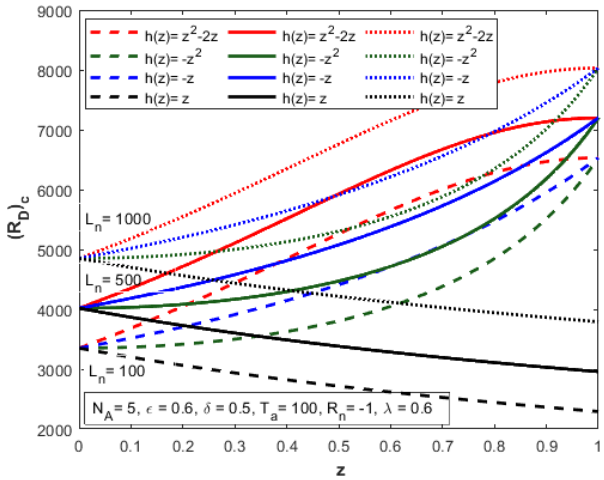

Figure 4: Variability of $(R_D)_c$ w.r.t. z for distinct values of $h(z)$ by taking distinct values of L_n

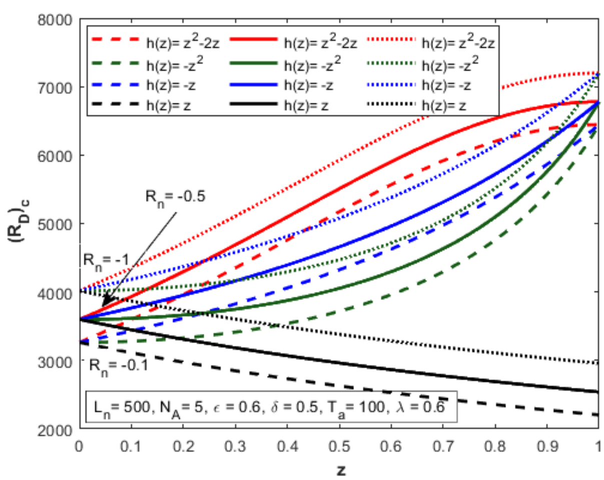

Figure 5: Variability of $(R_D)_c$ w.r.t. z for distinct values of $h(z)$ by taking different values of R_n

$$-\frac{\left[\frac{N_A}{L_n}J + \frac{(J+in_i)}{\varepsilon}\right]}{\frac{J}{L_n} + \frac{in_i}{\sigma}}R_n \tag{41}$$

By equating the real and imaginary components of equation 41, we get

$$\begin{aligned}\frac{a^2 J R_D(1+\delta h(z))}{L_n} + \left(\frac{N_A}{L_n} + \frac{1}{\varepsilon}\right) J R_n a^2 (1+\delta h(z)) = \\ \left[\frac{J}{1+\lambda} + (1+\lambda)\pi^2 T_a\right]\frac{J^2}{L_n} \\ -\left[\frac{J}{1+\lambda} + (1+\lambda)\pi^2 T_a\right]\frac{n_i^2}{\sigma}\end{aligned} \tag{42}$$

and

$$\begin{aligned}\frac{R_D}{\sigma} + \frac{R_n}{\varepsilon} - \frac{J}{a^2(1+\delta h(z))} \\ \times\left[\frac{J}{1+\lambda} + (1+\lambda)\pi^2 T_a\right]\left(\frac{1}{\sigma} + \frac{1}{L_n}\right) = 0\end{aligned} \tag{43}$$

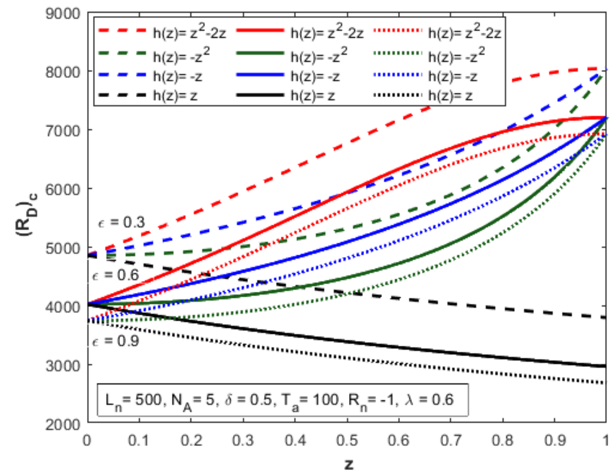

Figure 6: Variability of $(R_D)_c$ w.r.t. z for distinct values of $h(z)$ by taking different values of ε

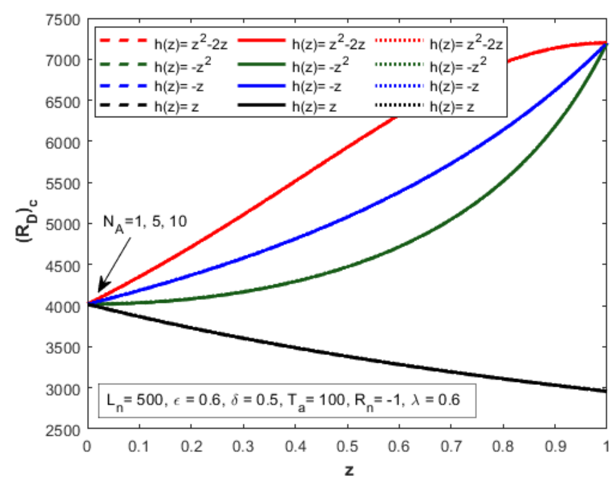

Figure 7: Variability of $(R_D)_c$ w.r.t. z for distinct values of $h(z)$ by taking different values of N_A

where $J = \pi^2 + a^2$. The frequency of the oscillatory mode is calculated as follows

$$\begin{aligned}\frac{n_i^2 L_n}{a^2\sigma} = \frac{J^2}{a^2} - \frac{J R_D(1+\delta h(z))}{\left[\frac{J}{1+\lambda} + (1+\lambda)\pi^2 T_a\right]} \\ -\frac{J(1+\delta h(z))}{\left[\frac{J}{1+\lambda} + (1+\lambda)\pi^2 T_a\right]}\left(N_A + \frac{L_n}{\varepsilon}\right)R_n\end{aligned} \tag{44}$$

In order for n_i to be real it is necessary that

$$\frac{J(1+\delta h(z))}{\left[\frac{J}{1+\lambda} + (1+\lambda)\pi^2 T_a\right]}\left[R_D + \left(N_A + \frac{L_n}{\varepsilon}\right)R_n\right] \leqslant \frac{J^2}{a^2} \tag{45}$$

where $J = \pi^2 + a^2$. The equations 43,44,45 becomes as the absence of the Jeffrey nanofluid ($\lambda = 0$), rotation ($T_a = 0$) and constant gravity ($\delta h(z) = 0$)

$$\frac{R_D}{\sigma} + \frac{R_n}{\varepsilon} - \frac{(\pi^2 + a^2)^2}{a^2}\left(\frac{1}{L_n} + \frac{1}{\sigma}\right) = 0 \tag{46}$$

$$\frac{n_i^2 L_n}{a^2\sigma} = \frac{(\pi^2+a^2)^2}{a^2} - R_D - \left(N_A + \frac{L_n}{\varepsilon}\right) R_n \tag{47}$$

and

$$R_D + \left(N_A + \frac{L_n}{\varepsilon}\right) R_n \leqslant \frac{(\pi^2+a^2)^2}{a^2} \tag{48}$$

The above result obtained in 46,47,48 are good agreement of results obtained by Nield *et al.* [18] and Chand *et al.* [36, 49]. According to Nield *et al.* [18], the critical value of the wave number is accomplished at $a = \pi$, therefore, by setting $a = \pi$ in equations 46,47,48, we obtain the result for the stability boundary case as

$$\frac{R_D}{\sigma} + \frac{R_n}{\varepsilon} = 4\pi^2 \left(\frac{1}{L_n} + \frac{1}{\sigma}\right) \tag{49}$$

$$\frac{n_i^2 L_n}{a^2\sigma} = 4\pi^2 - \left[R_D + \left(N_A + \frac{L_n}{\varepsilon}\right) R_n\right] \tag{50}$$

and

$$\left[R_D + \left(N_A + \frac{L_n}{\varepsilon}\right) R_n\right] \leqslant 4\pi^2 \tag{51}$$

These results obtained in equations 49,50,51 are same as that of obtained by Nield *et al.* [18] for particular case.

8. Results and Discussion

Variable gravity factors' impacts on density Rayleigh number, nanoparticle Rayleigh number, Lewis number, porosity of porous media, modified diffusivity ratio, and rotation on stationary convection have been graphed, and their stabilising or destabilising effect has been explored below. The variable gravity parameters are as follow: $h(z) = z^2 - 2z$, $h(z) = -z^2$, $h(z) = -z$ and $h(z) = z$.

Figure 2 shows the graph for $(R_D)_c$ with respect to z for distinct values of $\lambda = 0.3, 0.6, 0.9$ by fixing other parameters as $L_n = 500, N_A = 5, \varepsilon = 0.6, \delta = 0.5, T_a = 100, R_n = -1$. It is discovered that when the gravity parameter changes, such as when it becomes $h(z) = z^2 - 2z$, $h(z) = -z^2$, $h(z) = -z$ it stabilises, however when it becomes $h(z) = z$, it destabilises. These match those in Straughan [42] for the variable gravity parameter.

Figure 3 depicts the graph for $(R_D)_c$ with respect to z for various values of $T_a = 100, 200, 300$ setting other parameters like $L_n = 500, N_A = 5, \varepsilon = 0.6, \delta = 0.5, R_n = -1, \lambda = 0.6$. It is found that T_a has a stabilising impact when the gravity parameters are $h(z) = z^2 - 2z$, $h(z) = -z^2$, $h(z) = -z$, but a destabilising effect when the gravity parameter is $h(z) = z$. This is in good accord with the finding reported by Chand *et al.* [49], which states that reducing the gravity parameter has a stabilising impact on stationary convection while raising the gravity parameter has a destabilising effect.

Figure 4 depicts the curve for $(R_D)_c$ with respect to z for distinct values of $L_n = 100, 500, 1000$ while holding other parameters constant like $N_A = 5, \varepsilon = 0.6, \delta = 0.5, T_a = 100, R_n = -1, \lambda = 0.6$. It is found that L_n has a stabilising impact when the gravity parameters are $h(z) = z^2 - 2z$, $h(z) = -z^2$, $h(z) = -z$, but a destabilising effect when the gravity parameter is $h(z) = z$. This is in excellent accord with the finding from Chand *et al.* [49] that reducing the gravity parameter stabilises stationary convection while raising the gravity parameter destabilises it.

Figure 5 shows that $(R_D)_c$ decreases with increase in R_n $(as = -1, -0.5, -0.1)$. Thus R_n has destabilizing effect for all variable gravity parameter on stationary convection. Figure 6 shows that $(R_D)_c$ decreases with increase in ε $(as = 0.3, 0.6, 0.9)$. Thus ε has destabilizing effect for all variable gravity parameter on stationary convection.

Figure 7 depicts the graph for $(R_D)_c$ with respect to z for various values of $N_A = 1, 5, 10$ setting other parameters like $L_n = 500, \varepsilon = 0.6, \delta = 0.5, T_a = 100, R_n = -1, \lambda = 0.6$. It is found that N_A has a stabilising impact when the gravity parameters are $h(z) = z^2 - 2z$, $h(z) = -z^2$, $h(z) = -z$, but a destabilising effect when the gravity parameter is $h(z) = z$.

9. Conclusion

This article investigates the thermal instability of spinning Jeffrey nanofluids in porous media with changing gravity. The problem is examined for free-free boundary conditions using Galerkin technique and normal mode analysis. Equation 40 is the essential Rayleigh-Darcy number for stationary convection, and it has been studied whether this number stabilises or destabilises stationary convection with regard to changing gravity. Equation 45 yields the adequate condition for the oscillatory mode's frequency, while equation 44 also finds the oscillatory mode's frequency.

In Table 1, the varied gravity's effects in stationary convection are illustrated visually by changing one parameter at a time while keeping the other parameters constant by assigning them certain constant values.

The main conclusions from Table 1 are:

1. Jeffrey parameter (λ), Taylor number (T_a), modified diffusivity ratio (N_A) and Lewis number (L_n) have stabilizing effect on stationary convection when variable gravity parameters varies as $h(z) = z^2 - 2z$, $h(z) = -z^2$, $h(z) = -z$ whereas have destabilizing effect when variable gravity parameter varies as $h(z) = z$. In other words, stationary convection has a stabilising impact for lowering the variable gravity parameters and destabilising the system for raising the variable gravity parameters.

2. When the variable gravity parameter fluctuates as: $h(z) = z^2 - 2z$, $h(z) = -z^2$, $h(z) = -z$ and $h(z) = z$, the nanoparticle Rayleigh number (R_n), porosity of porous medium (ε) destabilise the system.

3. Variable gravity perameters $h(z) = z^2 - 2z$, $h(z) = -z^2$, $h(z) = -z$ delay the motion's onset.

4. N_B has no effect on $(R_D)_c$.

5. The variable gravity parameter $h(z) = z^2 - 2z$ has more stabilizing impact on stationary convection rather than other variable gravity parameters taken in this paper.

6. The sufficient condition for the oscillatory mode's frequency is obtained and is represented by equation 45.

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