

Alpha decay half-lives of $^{171-189}\text{Hg}$ isotopes using Modified Gamow-like model and temperature dependent proximity potential

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Abstract

The alpha decay half-lives for $^{171-189}\text{Hg}$ isotopes have been computed using the Gamow-like model (GLM), modified Gamow-like model (MGLM1), temperature-independent Coulomb and proximity potential model (CPPM), and temperature-dependent Coulomb and proximity potential model (CPPMT). New variable parameter sets were numerically calculated for the $^{171-189}\text{Hg}$ using the modified Gamow-like model (termed MGLM2). The results of the computed standard deviation indicates that the modified Gamow-like model (MGLM2) and the temperature-dependent Coulomb and proximity potential model give the least deviation from available experimental values, and therefore suggests that the two models (MGLM2 and CPPMT) are the most suitable for the evaluation of α -decay half-lives for the Hg isotopes.

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1. Introduction

Alpha decay, first discovered in 1899 by Rutherford, is one of the crucial decay modes for heavy nuclei [1]. It is one of the important decay modes to describe nuclear structure [2] and it can be successfully described by quantum theory [3]. Alpha decay (α -decay) is a crucial decay mode that provides information about the nuclear structure and stability of heavy and superheavy nuclei [4]. It is important in the identification of new heavy and super heavy nuclei (SHN) [5], and in the study of nuclear structure and nuclear force [6]. Investigations of the α -decay half-lives have been carried out both theoretically and

experimentally via various approaches. Some of the theoretical models that have been employed to study the α -decay half-lives are the fission-like model [7], the generalised liquid drop model [8, 9, 10], the effective liquid drop model [11], the modified generalized liquid drop model [6, 12, 13], the Coulomb and proximity potential model [14, 15, 16], the Gamow-like model [1, 4], and the preformed cluster model [17, 18].

Royer [19] developed an analytical formula to calculate the α -decay half-lives by applying a fitting procedure on a set of 373 nuclei. The universal decay law has also been developed by Qi et al. [20, 21]. They introduced the new universal decay law (UDL) to study α and cluster decay modes. The authors made use of the α -like R-matrix theory and the microscopic mecha-

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nism of the charged-particle emission. A generalization of the Viola-Seaborg formula was given by Ren et al. [22] for cluster radioactivity half-lives. Modified versions of the Ren formulas, called new Ren A and new Ren B, have also been presented [23]. The new Ren A included nuclear isospin asymmetry while new Ren B included both nuclear isospin asymmetry and angular momentum.

Gharaei and Zanganeh [24] have studied the half-lives of cluster decay for some isotopes using temperature dependent proximity potential (with the prox. 2010 and prox. Zheng potentials). They found that the width and height of the Coulomb barrier in the temperature-dependent proximity potential are less than its temperature-independent version. Recently, the modified Coulomb and proximity potential model was employed to study the α -decay of $^{186-218}\text{Po}$ [25]. Zdeb et al. [1] proposed a phenomenological model which is based on the Gamow theory for the calculation of α -decay half-lives. In the Gamow-like model, the square well potential is chosen as the nuclear potential, while the potential of a uniformly charged sphere is taken as the Coulomb potential. In Ref. [4], the authors presented the α decay half-lives of nuclei with $Z > 51$ (up to $Z = 120$) using a modified form of the Gamow-like model.

Most of the isotopes of mercury (Hg) are radioactive with half-lives less than a day. Seven of the isotopes are stable. The Hg radioisotope with the longest half-life is ^{194}Hg , with a half-life of 444 years. Some of the applications of Hg isotopes are: in medicine, in nuclear gyroscopes and magnetometers. The radioisotope ^{197}Hg , for example, is useful for diagnostics of kidney and cerebral diseases [26]. The α -decay half-lives of some Astatine isotopes have been studied in Ref. [27] using a modified Coulomb and proximity potential model with one adjustable parameter. The α -decay half-lives of some mercury isotopes have also been studied in Ref. [28] using about 25 different versions of the nuclear potential.

In this study, the Gamow-like model, its modified version (the modified Gamow-like model), the temperature-independent Coulomb and proximity potential model (CPPM) and the temperature dependent Coulomb and proximity potential model (CPPMT) have been employed to calculate the α -decay half-lives of $^{171-189}\text{Hg}$ isotopes. The results are compared with the available experimental data. The article is organised as follows. The modified Gamow-like model, the temperature-independent Coulomb and proximity potential model (CPPM) and the temperature-dependent Coulomb and proximity potential model (CPPMT) are described in Section 2 for the calculation of α -decay half-lives. The results are presented and discussed in Section 3 while the conclusion is given in Section 4.

2. Theory

2.1. Modified Gamow-like model

In the modified Gamow-like model, the interaction potential between the alpha particle and daughter nucleus is given by [4]:

$$V(r) = \begin{cases} -V_0, & 0 \leq r \leq R \\ V_H(r) + V_\ell(r), & r \geq R \end{cases} \quad (1)$$

where V_0 is the depth of the square well, the Hulthen type of screened electrostatic Coulomb potential

$$V_H = \frac{aZ_1Z_2e^2}{e^{ar} - 1} \quad (2)$$

the centrifugal potential

$$V_\ell(r) = \frac{(\ell + \frac{1}{2})^2 \hbar^2}{2\mu r^2} \quad (3)$$

Z_1 and Z_2 are the proton numbers of the α particle and daughter nucleus, respectively, a is the screening parameter, and ℓ is the orbital angular momentum that the α particle takes away. The radius of the spherical square well is computed by adding the radii of both the α particle (A_1) and the daughter nucleus (A_2):

$$R = r_0 \left(A_1^{\frac{1}{3}} + A_2^{\frac{1}{3}} \right) \quad (4)$$

where r_0 is a constant, an adjustable parameter.

The α decay half-life is calculated using [1, 4]:

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} 10^h. \quad (5)$$

Here, h is the decay hindrance factor due to the effect of an odd-neutron and/or an odd-proton. The value of h is zero for even-even nuclei. In Ref. [4], the values of a , r_0 , and h were determined to be:

$$a = 7.8 \times 10^{-4}, \quad r_0 = 1.14 \text{ fm}, \quad h = 0.3455. \quad (6)$$

For odd-odd nuclei, $h_{np} = 2h$. The decay constant λ is obtained via:

$$\lambda = \nu P \quad (7)$$

where the penetration probability P is given by

$$P = \exp \left[-\frac{2}{\hbar} \int_R^b \sqrt{2\mu(V(r) - E_k)} dr \right]. \quad (8)$$

Here, $\mu = mA_1A_2/(A_1 + A_2)$ is the reduced mass of the daughter nucleus and the α particle, m is the nucleon mass, $E_k = Q_\alpha(A - 4)/A$ is the kinetic energy of the emitted α particle. The classical turning point b is obtained through the condition $V(b) = E_k$.

In this model, the frequency of assault on the potential barrier is evaluated using

$$\nu = \frac{(G + \frac{3}{2}) \hbar}{1.2\pi\mu R_0^2}, \quad (9)$$

where the parent nucleus radius R_0 is obtained via

$$R_0 = 1.28A^{1/3} - 0.76 + 0.8A^{-1/3}. \quad (10)$$

The main quantum number G is obtained using

$$G = \begin{cases} 22 & N > 126 \\ 20 & 82 < N \leq 126 \\ 18 & N \leq 82 \end{cases}. \quad (11)$$

2.2. The Coulomb and Proximity Potential Model (CPPM)

The total interaction potential between the emitted α particle and the daughter nucleus in the CPPM contains (for both the touching configuration and for separated fragments) the nuclear, the Coulomb and the centrifugal terms [29]:

$$V_T(r) = V_{\text{prox}}(z) + V_C(r) + \frac{\hbar\ell(\ell+1)}{2\mu r^2} \quad (12)$$

where μ is the reduced mass of the interaction system and ℓ is the angular momentum. The Coulomb potential $V_C(r)$ is defined as:

$$V_C(r) = Z_1 Z_2 e^2 \begin{cases} \frac{1}{r} & \text{for } r \geq R_C \\ \frac{1}{2R_C} \left[3 - \left(\frac{r}{R_C}\right)^2 \right] & \text{for } r \leq R_C \end{cases}. \quad (13)$$

Here, Z_1 and Z_2 are, respectively, the charge numbers of the daughter and emitted nuclei. The radial distance $R_C = 1.24(R_1 + R_2)$. The term $V_{\text{prox}}(z)$ represents the proximity potential and $z = r - C_1 - C_2$ is the distance between the near surfaces of the fragments, where r is the distance between the fragment centres [16]. The presence of the proximity potential causes a reduction in the height of the potential barrier. The proximity potential has been used in the preformed cluster model by Malik et al. [30]. The proximity potential V_{prox} can be obtained by calculating the strength of the nuclear interactions between the daughter and emitted α particle:

$$V_{\text{prox}}(z) = 4\pi b\gamma\bar{R}\Phi\left(\frac{z}{b}\right) \text{ MeV} \quad (14)$$

where the term $b\gamma\bar{R}$ depends on the geometry and shape of the two nuclei, and the mean curvature radius \bar{R} is given as

$$\bar{R} = \frac{C_1 C_2}{C_1 + C_2}. \quad (15)$$

The Süsmann central radii of fragments C_1 and C_2 are computed using

$$C_i = R_i \left[1 - \left(\frac{b}{R_i}\right)^2 + \dots \right] \quad (16)$$

where the diffuseness of nuclear surface $b \approx 1$ fm and R_i are given by

$$R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3} \text{ fm} \quad (i = 1, 2). \quad (17)$$

The universal function $\Phi\left(\epsilon = \frac{z}{b}\right)$ is given in the form [24]:

$$\Phi(\epsilon) = \begin{cases} -\frac{1}{2}(\epsilon - 2.54)^2 - 0.0852(\epsilon - 2.54)^3 & \epsilon \leq 1.2511 \\ -3.437 \exp(-\epsilon/0.75) & \epsilon \geq 1.2511 \end{cases} \quad (18)$$

The nuclear surface energy coefficient γ is defined as

$$\gamma = 1.460734 \left[1 - 4 \left(\frac{N-Z}{N+Z} \right)^2 \right] \text{ MeV/fm}^2 \quad (19)$$

where N and Z denote the neutron and proton numbers of the parent nucleus, respectively. The Prox. 2010 potential has been used in this work.

According to the WKB approximation [24, 31, 32], the penetration probability P of the emitted α nucleus through the potential barrier is calculated using:

$$P = \exp \left[-\frac{2}{\hbar} \int_{R_{in}}^{R_{out}} \sqrt{2\mu [V(r) - Q]} dr \right] \quad (20)$$

where the classical turning points R_{in} and R_{out} are determined from

$$V(R_{in}) = V(R_{out}) = Q \quad (21)$$

and the reduced mass μ is calculated using $\mu = mA_1A_2/A$, where m is the nucleon mass, A_1 and A_2 denote the mass numbers of the emitted and daughter nuclei, respectively, and A is the parent nucleus mass number. The α -decay half-life is then computed using

$$T_{1/2} = \frac{\ln 2}{\nu P} \quad (22)$$

where the assault frequency ν has been taken to be 10^{20} s^{-1} .

2.2.1. Temperature-Dependent Proximity Potential

The thermal effects are studied by using the temperature dependent forms of the parameters R , γ and b . They are given by [24]:

$$R_i(T) = R_i(T=0) \left[1 + 0.0005T^2 \right] \text{ fm} \quad (i = 1, 2) \quad (23)$$

$$\gamma(T) = \gamma(T=0) \left[1 - \frac{T - T_b}{T_b} \right]^{3/2} \quad (24)$$

$$b(T) = b(T=0) \left[1 + 0.009T^2 \right] \quad (25)$$

where T_b is the temperature associated with near Coulomb barrier energies and $b(T=0) = 1$. In this work, we have adopted an alternative form of the temperature dependent surface energy coefficient in the form $\gamma(T) = \gamma(0)(1 - 0.07T)^2$ [33]. The temperature T (in MeV) can be obtained from

$$E^* = E_{kin} + Q_{in} = \frac{1}{9}AT^2 - T \quad (26)$$

where E^* is the excitation energy of the parent nucleus and A is its mass number, and Q_{in} denotes the entrance channel Q-value of the system. The kinetic energy of the emitted α particle E_{kin} is obtained from

$$E_{kin} = (A_d/A_p) Q. \quad (27)$$

3. Results and Discussions

The α -decay half-lives of the Mercury isotopes ($Z = 80$) within the mass range $171 \leq A \leq 189$ have been calculated using the Gamow-like model (GLM), modified Gamow-like model (MGLM1) using the variable parameters of Ref. [4], the temperature-independent Coulomb and proximity potential model (CPPM), and the temperature-dependent Coulomb and proximity potential model (CPPMT). The proximity 2010 potential has been employed in the CPPM and CPPMT calculations. We have also calculated the α -decay half-lives of the isotopes using the modified Gamow-like model (MGLM2) with new parameter values. The new values for the parameters a , r_0 , and h in the modified Gamow-like model (MGLM2) were obtained through least squares fitting procedure. The values of the three adjustable parameters obtained for the Hg isotopes are

$$r_0 = 1.2457 \text{ fm}, \quad h = 0.1891, \quad a = -2.159 \times 10^{-3} \quad (28)$$

The database have been taken from the NUBASE2016 [34, 35, 36]. The reaction Q_α value has been computed via [29]:

$$Q_\alpha = \Delta M_P - \Delta M_D - \Delta M_\alpha + k(Z_P^\varepsilon - Z_D^\varepsilon) \quad (29)$$

where ΔM_P , ΔM_α , and ΔM_D denote the mass excesses of the parent nucleus, the alpha particle, and the daughter nucleus, respectively. The term $k(Z_P^\varepsilon - Z_D^\varepsilon)$ denotes the screening effect of atomic electrons [37]; $k = 8.7 \text{ eV}$, $\varepsilon = 2.517$ for $Z \geq 60$, and $k = 13.6 \text{ eV}$, $\varepsilon = 2.408$ for $Z < 60$ [38].

The angular momentum ℓ are obtained from the selection rule given by [23, 39, 40]:

$$\ell = \begin{cases} \delta_j & \text{for even } \delta_j \text{ and } \pi_d = \pi_p \\ \delta_j + 1 & \text{for odd } \delta_j \text{ and } \pi_d = \pi_p \\ \delta_j & \text{for odd } \delta_j \text{ and } \pi_d \neq \pi_p \\ \delta_j + 1 & \text{for even } \delta_j \text{ and } \pi_d \neq \pi_p \end{cases} \quad (30)$$

Here $\delta_j = |j_p - j_d|$, where j_d , π_d , j_p , π_p are the spin and parity values of the daughter and parent nuclei, respectively.

The alpha-decay half-lives computed for the 19 Hg isotopes ($^{171-189}\text{Hg}$) are shown in Table 1. The first four columns show, respectively, the mass number (A), the calculated Q_α values, the calculated temperature and the experimental α -decay half-lives (Expt.) ($\log [T_{1/2}(s)]$). The last five columns of the Table show the computed α -decay half-lives using the GLM, MGLM1, MGLM2, CPPM, and CPPMT, respectively. All the models give reasonable values of the half-lives when compared to the available experimental results.

The root mean square standard deviation σ has been evaluated using the formula:

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N \left[\left(\log_{10} T_{1/2,i}^{\text{Theory}} - \log_{10} T_{1/2,i}^{\text{Expt}} \right)^2 \right]} \quad (31)$$

where $T_{1/2,i}^{\text{Theory}}$ are the half-lives obtained using the five models and $T_{1/2,i}^{\text{Expt}}$ are the experimental half-lives. The standard deviation have been calculated in order to compare the agreement

between the experimental half-lives and theoretically calculated half-lives using the various models. The computed standard deviations (σ) using the different models are shown in Table 2. The second column of the Table shows the calculated standard deviations for the Hg isotopes. From the Table, the MGLM2 has the least standard deviation with a value of 0.5013 followed by the GLM with a standard deviation value of 0.5885, less than the standard deviation of CPPM. The MGLM1, CPPM, and CPPMT have respective standard deviation values of 0.6130, 0.6866, and 0.5949. One observes that the CPPMT has a lower standard deviation value compared with the CPPM. This shows the importance of using temperature-dependent form of the proximity potential model. All the models give standard deviation values less than 0.7. However, among the five models, the GLM and MGLM2 seem to be the most suitable for the determination of the α -decay half-lives of the Hg isotopes. It should be noted these Hg isotopes were studied in Ref. [28] using about various proximity potential models. However, the authors considered experimental values of $^{171-177}\text{Hg}$ only. This makes it difficult to compare the standard deviations.

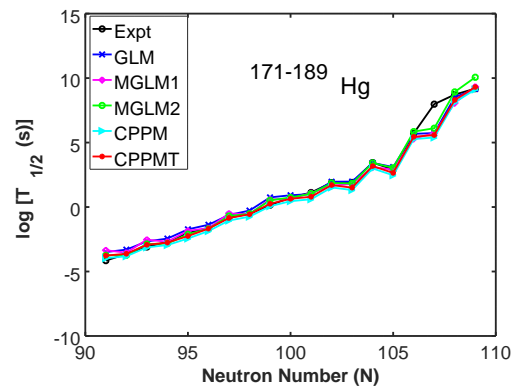


Figure 1. Comparison of the calculated α -decay half-lives of Hg isotopes between the various models and experiment.

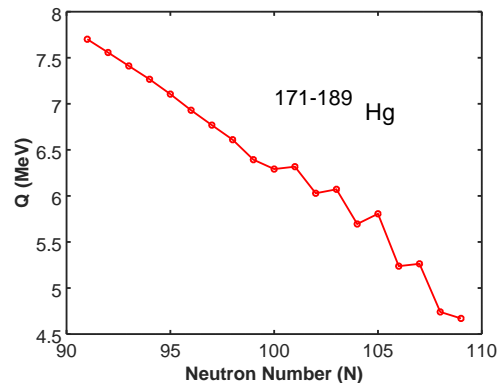


Figure 2. Plot of the calculated Q_α values against Neutron number (N) for the Hg isotopes.

The calculated half-lives $\log [T_{1/2}(s)]$ for the Hg isotopes using the five models have been plotted against the neutron

Table 1. Calculated α -decay half-lives, $\log [T_{1/2}(s)]$, of Hg ($Z = 80$) using GLM, MGLM1, MGLM2, CPPM, CPPMT.

A	$Q_\alpha(MeV)$	T (MeV)	$\log [T_{1/2}(s)]$					
			Expt.	GLM	MGLM1	MGLM2	CPPM	CPPMT
171	7.7011	0.9218	-4.1549	-3.4908	-3.3518	-3.7442	-3.9665	-3.7652
172	7.5571	0.9107	-3.6364	-3.3033	-3.5380	-3.7354	-3.8178	-3.6243
173	7.4111	0.8994	-3.0969	-2.6643	-2.5327	-2.8705	-3.1276	-2.9279
174	7.2661	0.8882	-2.6990	-2.4462	-2.6902	-2.8297	-2.9492	-2.7573
175	7.1051	0.8761	-1.9957	-1.7283	-1.8488	-2.1095	-2.4407	-2.2496
176	6.9301	0.8630	-1.6467	-1.3748	-1.6321	-1.6953	-1.8640	-1.6738
177	6.7686	0.8508	-0.8246	-0.6132	-0.5042	-0.6952	-1.0459	-0.8497
178	6.6105	0.8387	-0.5237	-0.2744	-0.5467	-0.5267	-0.7492	-0.5606
179	6.3935	0.8229	0.1461	0.7478	0.5927	0.5200	0.0658	0.2534
180	6.2916	0.8142	0.7321	0.9145	0.6241	0.7394	0.4550	0.6418
181	6.3175	0.8135	1.1249	1.0050	1.0923	1.0300	0.6013	0.7944
182	6.0288	0.7930	1.8947	1.9638	1.6562	1.8607	1.5193	1.7046
183	6.0717	0.7935	1.9049	1.9747	1.8009	1.8322	1.3184	1.5035
184	5.6951	0.7672	3.4442	3.4213	3.0873	3.4225	2.9941	3.1775
185	5.8061	0.7723	2.9129	3.1016	2.9081	3.0392	2.4621	2.6457
186	5.2376	0.7327	5.7140	5.6765	5.2962	5.8501	5.2691	5.4498
187	5.2628	0.7324	7.9777	5.7360	5.7394	6.1107	5.3940	5.5811
188	4.7405	0.6945	8.7218	8.5148	8.0682	8.9273	8.1268	8.3040
189	4.6699	0.6876	9.1818	9.1566	9.3338	10.0638	9.1191	9.3085

Table 2. Calculated root means square standard deviation (σ) using the different models.

Model	σ (Hg)
GLM	0.5885
MGLM1	0.6130
MGLM2	0.5013
CPPM	0.6866
CPPMT	0.5949

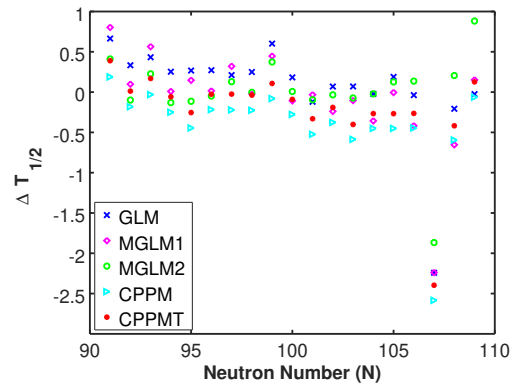


Figure 4. Plot of the ΔT against Neutron number (N) for the Hg isotopes using the different models.

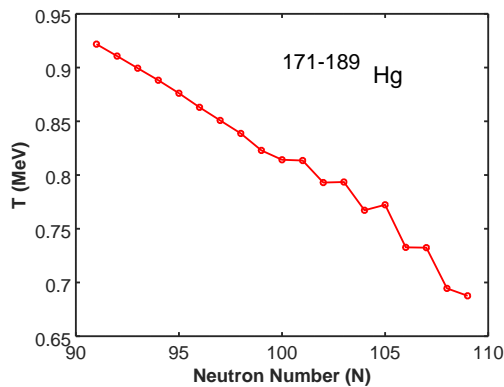


Figure 3. Plot of the calculated Temperature (T in MeV) against Neutron number (N) for the Hg isotopes.

number in Figure 1. From the Figure, the half-lives can be seen to increase with increase in neutron number for the Hg isotopes. Aside $N = 107$ (corresponding to $A = 187$), the models give very good descriptions of the half-lives. It should be noted that near double magicity of the parent nucleus ^{202}Hg ($Z = 80, N = 122$) was suggested in Ref. [28]. Figure 2 shows plots of the Q_α values against neutron number for the Hg isotopes while the calculated temperature of the Hg plotted against the neutron number in Figure 3. The temperature values decrease with increase in neutron number which is equivalent to increase in the half-lives. That is, the calculated temperature is inversely proportional to the α -decay half-lives.

The difference between the theoretical and experimental α -decay half-lives have been computed using

$$\Delta T_{1/2} = \log_{10} \left[T_{1/2}^{\text{theor}} / T_{1/2}^{\text{expt}} \right]. \quad (32)$$

This factor ($\Delta T_{1/2}$) has been plotted for all the models used in this work in Figure 4. It can be observed that most of the points are near zero and within ± 0.5 . The Figure shows that the MGLM2 model gives the lowest $|\Delta T_{1/2}|$ values while the CPPM model gives the highest $|\Delta T_{1/2}|$ values. This agrees with the results shown in Table 2.

4. Conclusion

In this work, the α -decay half-lives of Hg isotopes in the mass range $171 \leq A \leq 189$ have been studied using the Gamow-like model (GLM), modified Gamow-like model (MGLM1), the temperature-independent Coulomb and proximity potential model (CPPM), and the temperature-dependent Coulomb and proximity potential model (CPPMT). The Prox. 2010 proximity potential has been employed for the CPPM and CPPMT calculations. For the set of isotopes, new parameter values were obtained through a least squares scheme using the modified Gamow-like model (termed MGLM2). It is shown that the modified Gamow-like model (MGLM2) with new local parameter values and the GLM are the most suitable methods (among the methods considered here) for calculating the α -decay half-lives for the Hg isotopes. In general, all the models give α -decay half-lives that are in good agreement with the experimental data with the maximum standard deviation values less than 0.7.

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