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# Effect of reduction method on the performance a software defined network system using Gumbel Hougaard family copula distribution

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## Abstract

This study examines a system that consists of three subsystems 1, 2 and 3. These three subsystems are each linked together in series. Subsystem 1's three units are wired in series, subsystem 2's three units are wired in parallel and operating under the 2-out-of-3: G; policy, and subsystem 3's three units are wired in parallel and operating under the 1-out-of-3: G; policy. Units and subsystems failure rates are constant and follow an exponential distribution. The repair rate follow two types of distributions, namely general and Gumbel Hougaard family copula distribution. The system was studied using Laplace transforms and supplementary variable methods. For specific values of the failure and repair rates, availability, reliability, mean time to failure ($MTTF$), and cost analysis have been assessed. As a means to improve the system's overall effectiveness and availability, a reduction strategy is utilized. Tables and graphs are used to display computed results.

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*Keywords:* Reliability, Availability, $MTTF$ , Cost analysis, Gumbel-Hougaard family copula distribution

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## 1. Introduction

Indirectly programmable network control for applications and network services is made possible by software-defined networking (SDN). In order to provide a more flexible and manageable network infrastructure, software defined networks detach the control and forwarding operations of the network from the underlying hardware, such as routers and switches. Network programmability is offered via software-defined networking (SDN) from a central location. Since the complexity of the control plane is offloaded onto the controller, the nodes or data plane devices in SDN simply need to worry about transmitting data packets. Typically, all of the regulations and policies are implemented by the controller. Centralized control allows for more adaptability in identifying and fixing link failures because the controller understands the complete network's structure.

Reliability engineering research has shown time and again that detailed performance analysis can help prevent disasters and save money. One example is the investigation of reliability for a cold standby PCB manufacturing system consisting of two identical units that failed due to the development of flaws in the primary unit by Batra and Malhotra [1]. Gahlot *et al.* [2] examined how a complicated system operating sequentially

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performed under the 2-out-of-3: G and 1-out-of-2: G strategies. Ye *et al*. [3] looked at the reliability of a machine that could be repaired while it was being shocked and degraded by low-quality feedstocks. to look at the connections between machine problems, product quality, and the inspection process.

The research of reliability measures for the system, that consists of two subsystems operating under the 2-out-of-4 rule: G; policy, was the main focus of Kabiru *et al*. [4]. In order to the performance of the complicated system in a series arrangement while accounting for failure and repair, Singh *et al*. [5] used copula and controllers. Yusuf *et al*. [6] solved recursively to provide availability, repairmen's peak demand, and profit function. Ashish *et al*. [7] analyzed a novel efficient and intelligent irrigation system (EIIS) using the concept of cold standby redun-dancy. Singh *et al*. [8] used Copula to analyze the performance of complex systems in series configuration across several failure and repair scenarios. Neama [9] investigated how a dependent system develops its reliability. Yusuf *et al*. [10] studied the RAMD analysis on PV systemplant where PV system mainly consists of five components namelyPV modules, controller, batteries, inverter and Distribution Board. El-Damces *et al*. [11] looked at how a parallel system's equivalence factor in dependability might be affected by fluctuating failure rates. The growth of a complex system with three subsystems was examined by Elsayed *et al*. [12]. Every subsystem consists of three units that use the reduction method and the (1-out-of-3: G) policy.

This paper seeks to examine the reliability and performance of coopration via software defined networks (SDN) in terms of reliability, Mean time to failure ($MTTF$), cost, availability utilizing copula distribution and the effective of the reduction method on Performance of the system in terms of reliability, availability and cost analysis. The system is divided into three subsystems (Figure 1), subsystem 1 (three units in series), subsystem 2 (three controller servers in parallel) follows 2-out-of-3:G operational policy and subsystem 3 (three unis in parallel) follows 1-out-of-3:G operational policy. series connections between all subsystems.The system can go through three distinct states: successful, partially unsuccessful, and failed. The following options may cause the system to enter the failed state:

- Subsystem 1 can fail in any of its three components, yet subsystems 2 and 3 remain functional.
- More than one subsystem 2 unit fail, yet all subsystem 1 and 3 units are in fine working order.
- More than two subsystem 3 units malfunction, yet all subsystems 1 and 2 units function well.

In the following scenarios, the system will be partially failed:

- All units in subsystems 1 and 3 are operational, but in subsystem 2 there is at least one nonworking unit and potentially more than two.
- None of the components from either of the first two subsystems have failed. However, one or both components from the third subsystem may be non-functional.

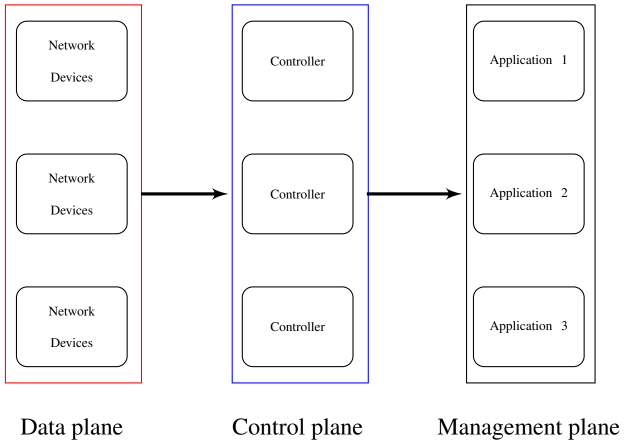


Figure 1. Block diagram for SDN

## 2. Assumptions

The following ideas were made during model analysis (Figure 2):

- At first, everything is running smoothly, including the subsystems.
- It takes three units from subsystem 1, two from subsystem 2, and one from subsystem 3 for the system to be operational.
- The system's capacity will be diminished if one of the subsystems 2 or 3's units malfunctions.
- In the event that one subsystem 1 unit, two subsystem 2 units, and three subsystem 3 units fail, the system will become unworkable.
- When a system's malfunctioning component is in a low-performance or failed condition, it can still be repaired. Once a subsystem fully fails, copula maintenance is necessary. Since no damage is done during the repair, it is claimed that a copula-repaired system performs identically to a brand-new system.
- Once the defective unit has been repaired, the operation can begin.

## 3. Notations

$t$ / $s$ : Time scale / the variable of Laplace transform
$\delta_1/\delta_2/\delta_3$ : Rates of system-1 failure in units 1-2-3.
$\delta_c/\delta_a$ : Symbolizes the rate with which components of Subsystems 2 and 3 fail.
$\alpha(x)$/ $\alpha(y)$ : Repair rate of sybsystem-2 / sybsystem-3
$\eta(z)$ : Rate of repair for fully broken systems
$P_i(t)$ : The probability of changing from the current state to the $i$th state at any given time $t$.
$P_i(s)$ : Laplace transform of $P_i(t)$
$P_i(x,t)$ : The probability that the system is in state $S_i$,

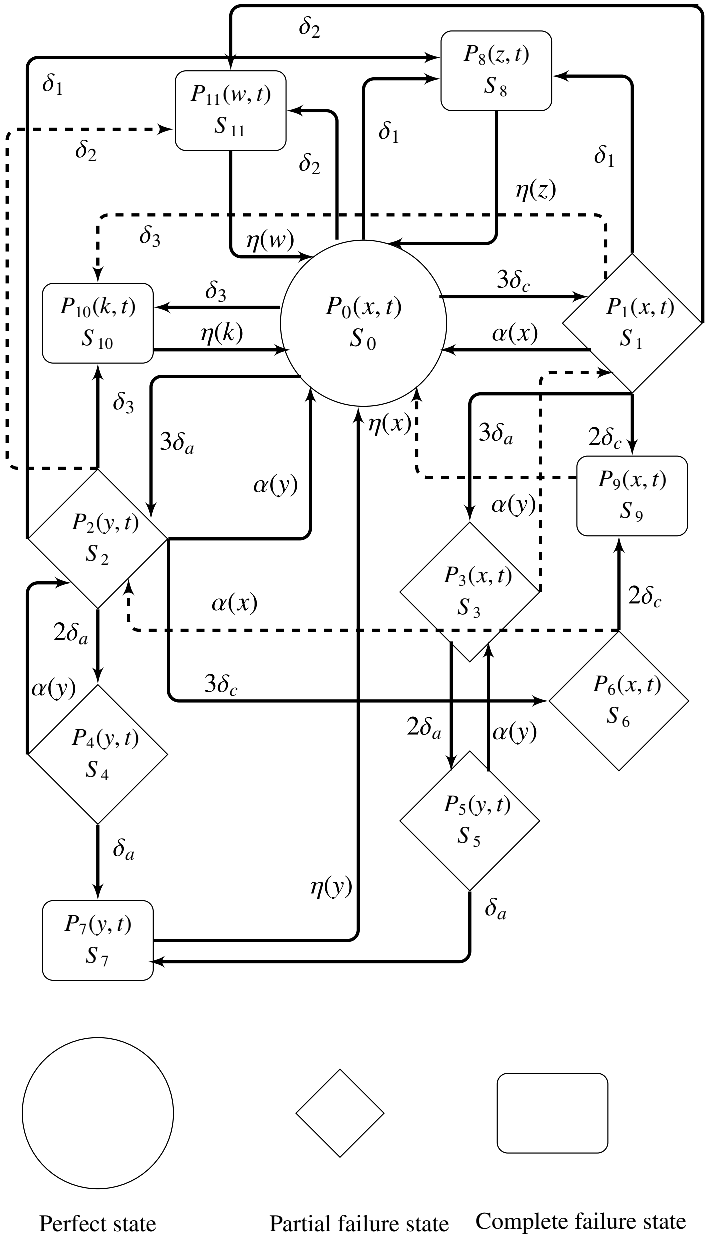


Figure 2. Diagram showing the model's state transitions

Table 1. Describe the current system state.

| State | Description |
|---|---|
| $S_0$ | All of the components are in excellent condition and in this immaculate form. |
| $S_1$ | Due to the loss of the subsystem-2's initial unit, $S_1$ is a degraded condition with a slight partial failure in the subsystem2. The system is under repair |
| $S_2$ | After the failure of any three units in subsystem 3, the system is depicted as degraded yet operable, while all units in subsystems 2 and 1 are in good operational state. Currently, work is being done on it. |
| $S_3$ | Two units have failed, one from subsystem 2 and the other from subsystem 3, causing the system to be degraded at this point. The total repair times are $(x, t)$ and $(y, t)$, respectively. |
| $S_4$ | While the system has been impaired but is still operational due to the failure of two units in subsystem 3, all system units are in good working condition as shown. It's being worked on right now. |
| $S_5$ | The system is currently impaired as a result of three units failing, one from subsystem 2 and the other from subsy -stem 3. the system is undergoing repair. |
| $S_6$ | The system has degraded due to two units failing, one from subsystem 3 and the other from subsystem 2, with total repair times of $(y, t)$ and $(x, t)$, respectively. |
| $S_7$ | Because of the breakdown of more than two Sybsystem 3 units, the entire system is currently inoperable. Repairs are being made using a copula distribution. |
| $S_8$ | A broken system is indicated when Subsystem 1's first component fails. We are using copula distribution to fix the system. |
| $S_9$ | Subsystem 1 is in an entirely failed state due to the breakdown of two units. Copula distribution is utilized to fix the system. |
| $S_{10}$ | Because of the failure of the third unit in subsystem 1, the entire system is now useless. In order to repair the system, copula distribution is being used. |
| $S_{11}$ | Due to a second subsystem 1 unit failing, the system is currently down. To correct the issue, we are using a copula distribution. |

$i = 1, ..., 11$. $P_i(x, t)$ represents that the system is in repair with repair variables $x$.
$K_1$ / $K_2$ : Revenue / service cost per unit time.
$E_p(t)$ : Expected profit during the interval. $[0, t)$
$\eta(x)$ : An expression of the joint probability from a failed state $S_j$, $j = 8, ..., 11$, to good state $S_0$ according to the Gumbel-Hougaard family of copula is given as:
$\mu_0(x) = C_\theta\big(u_1(x), u_2(x)\big) = exp[x^\theta + \{log\phi(x)\}^\theta]^{\frac{1}{\theta}}$, $1 < \theta < \infty$, where $u_1 = \phi(x)$ and $u_2 = e^x$.

## 4. Mathematical formulation and solution

This mathematical model is related to the following set of difference-differential equations by use of factor probability and argument consistency.

$$\Big[\frac{d}{dt} + \delta_1 + \delta_2 + \delta_3 + 3\delta_a + 3\delta_c\Big]P_0(t) = \int_0^\infty \alpha(x)P_1(x,t)dx + \int_0^\infty \alpha(y)P_2(y,t)dy + \int_0^\infty \eta(y)P_7(y,t)dy + \int_0^\infty \eta(z)P_8(z,t)dz + \int_0^\infty \eta(x)P_9(x,t)dx + \int_0^\infty \eta(k)P_{10}(k,t)dk + \int_0^\infty \eta(w)P_{11}(w,t)dw, \quad (1)$$

$$\Big[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\delta_1+\delta_2+\delta_3+3\delta_a+2\delta_c+\alpha(x)\Big]P_1(x,t)=0, \quad (2)$$

$$\Big[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\delta_1+\delta_2+\delta_3+2\delta_a+3\delta_c+\alpha(y)\Big]P_2(y,t)=0, \quad (3)$$

$$\Big[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+2\delta_a+\alpha(y)\Big]P_3(y,t)=0, \quad (4)$$

$$\Big[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\delta_a+\alpha(y)\Big]P_4(y,t)=0, \quad (5)$$

$$\Big[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\delta_a+\alpha(y)\Big]P_5(y,t)=0, \quad (6)$$

$$\Big[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\delta_c+\alpha(x)\Big]P_6(x,t)=0, \quad (7)$$

$$\Big[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\eta(y)\Big]P_7(y,t)=0, \quad (8)$$

$$\Big[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\eta(z)\Big]P_8(z,t)=0, \quad (9)$$

$$\Big[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\eta(x)\Big]P_9(x,t)=0, \quad (10)$$

$$\Big[\frac{\partial}{\partial t}+\frac{\partial}{\partial k}+\eta(k)\Big]P_{10}(k,t)=0, \quad (11)$$

$$\Big[\frac{\partial}{\partial t}+\frac{\partial}{\partial w}+\eta(w)\Big]P_{11}(w,t)=0, \quad (12)$$

Boundary conditions:

$$P_1(0,t) = 3\delta_c P_0(t)+\int_0^\infty \alpha(y)P_3(y,t)dy, \quad (13)$$

$$P_2(0,t) = 3\delta_a P_0(t)+\int_0^\infty \alpha(y)P_4(y,t)dy \quad (14)$$

$$+\int_0^\infty \alpha(x)P_6(x,t)dx, \quad (15)$$

$$P_3(0,t) = 3\delta_a P_1(0,t)+\int_0^\infty \alpha(y)P_5(y,t)dy, \quad (16)$$

$$P_4(0,t) = 2\delta_a P_2(0,t), \quad (17)$$

$$P_5(0,t) = 2\delta_a P_3(0,t), \quad (18)$$

$$P_6(0,t) = 3\delta_c P_2(0,t), \quad (19)$$

$$P_7(0,t) = \delta_a\Big[P_4(0,t)+P_5(0,t)\Big], \quad (20)$$

$$P_8(0,t) = \delta_1\Big[P_0(t)+P_1(0,t)+P_2(0,t)\Big], \quad (21)$$

$$P_9(0,t) = 2\delta_c\Big[P_1(0,t)+P_6(0,t)\Big], \quad (22)$$

$$P_{10}(0,t) = \delta_3\Big[P_0(t)+P_1(0,t)+P_2(0,t)\Big], \quad (23)$$

$$P_{11}(0,t) = \delta_2\Big[P_0(t)+P_1(0,t)+P_2(0,t)\Big]. \quad (24)$$

Other state transition probabilities at t = 0 are zero, and the initial condition $P_o(0) = 1$ is also true. By using the Laplace transformation of (1) - (23), we can solve the partial differential equations as follows:

$$\bar{P}_0(s) = \frac{1}{D(s)} \quad (25)$$

$$\bar{P}_1(s) = \frac{(1-\bar{S}_\alpha(s+\delta_1+\delta_2+\delta_3+3\delta_a+2\delta_c))}{D(s)(s+\delta_1+\delta_2+\delta_3+3\delta_a+2\delta_c)} \times\Big[3\delta_c+\frac{9\delta_a\delta_c\bar{S}_\alpha(s+2\delta_a)}{A}\Big] \quad (26)$$

$$\bar{P}_2(s) = \frac{(1-\bar{S}_\alpha(s+\delta_1+\delta_2+\delta_3+2\delta_a+3\delta_c))}{D(s)(s+\delta_1+\delta_2+\delta_3+2\delta_a+3\delta_c)} \times\Big[\frac{3\delta_a}{B}\Big] \quad (27)$$

$$\bar{P}_3(s) = \frac{(1-\bar{S}_\alpha(s+2\delta_a))}{D(s)(s+2\delta_a)}\Big[\frac{9\delta_a\delta_c}{A}\Big] \quad (28)$$

$$\bar{P}_4(s) = \frac{(1-\bar{S}_\alpha(s+\delta_a))}{D(s)(s+\delta_a)}\Big[\frac{6\delta_a^2}{B}\Big] \quad (29)$$

$$\bar{P}_5(s) = \frac{(1-\bar{S}_\alpha(s+\delta_a))}{D(s)(s+\delta_a)}\Big[\frac{18\delta_a^2\delta_c}{A}\Big] \quad (30)$$

$$\bar{P}_6(s) = \frac{(1-\bar{S}_\alpha(s+2\delta_c))}{D(s)(s+2\delta_c)}\Big[\frac{9\delta_a\delta_c}{B}\Big] \quad (31)$$

$$\bar{P}_7(s) = \frac{6\delta_a^3(1-\bar{S}_\eta(s))}{D(s)(s)}\Big[\frac{1}{B}+\frac{3\delta_c}{A}\Big] \quad (32)$$

$$\bar{P}_8(s) = \frac{(1-\bar{S}_\eta(s))}{D(s)(s)}\Bigg[\delta_1+\frac{3\delta_1\delta_a}{B} +\frac{3\delta_1\delta_c(1-2\delta_a\bar{S}_\alpha(s+\delta_a))}{A}\Bigg] \quad (33)$$

$$\bar{P}_9(s) = \frac{(1-\bar{S}_\eta(s))}{D(s)(s)}\Bigg[\frac{6\delta_c^2(1-2\delta_a\bar{S}_\alpha(s+\delta_a))}{A} +\frac{18\delta_a\delta_c^2}{B}\Bigg] \quad (34)$$

$$\bar{P}_{10}(s) = \frac{(1-\bar{S}_\eta(s))}{D(s)(s)}\Bigg[\delta_3+\frac{3\delta_3\delta_a}{B} +\frac{3\delta_3\delta_c(1-2\delta_a\bar{S}_\alpha(s+\delta_a))}{A}\Bigg] \quad (35)$$

$$\bar{P}_{11}(s) = \frac{(1-\bar{S}_\eta(s))}{D(s)(s)}\Bigg[\delta_2+\frac{3\delta_2\delta_a}{B} +\frac{3\delta_2\delta_c(1-2\delta_a\bar{S}_\alpha(s+\delta_a))}{A}\Bigg], \quad (36)$$

where

$$D(s) = s\Big[1+\frac{(1-\bar{S}_\alpha(s+\delta_1+\delta_2+\delta_3+3\delta_a+2\delta_c))}{(s+\delta_1+\delta_2+\delta_3+3\delta_a+2\delta_c)}\Big[3\delta_c +\frac{9\delta_a\delta_c\bar{S}_\alpha(s+2\delta_a)}{A}\Big]+\frac{3\delta_a(1-\bar{S}_\alpha(s+\delta_1+\delta_2+\delta_3+2\delta_a+3\delta_c))}{B(s+\delta_1+\delta_2+\delta_3+2\delta_a+3\delta_c)} +\frac{9\delta_a\delta_c(1-\bar{S}_\alpha(s+2\delta_a))}{A(s+2\delta_a)}+\frac{(1-\bar{S}_\alpha(s+\delta_a))}{(s+\delta_a)}\Big[\frac{6\delta_a^2}{B}+\frac{18\delta_a^2\delta_c}{A}\Big] +\frac{(1-\bar{S}_\alpha(s+2\delta_c))}{(s+2\delta_c)}\Big[\frac{9\delta_a\delta_c}{B}\Big]+\frac{(1-\bar{S}_\eta(s))}{(s)}\Big[\delta_1+\delta_2+\delta_3$$

$$+\frac{3\delta_c(6\delta_a^3+G(\delta_1+2\delta_c+\delta_3+\delta_2))}{A} + \frac{3\delta_a(2\delta_a^2+\delta_1+6\delta_c^2+\delta_3+\delta_2)}{B}\Big]\Big], \quad (37)$$

$$A = 1-\delta_a\Big[3\bar{S}_\alpha(s+2\delta_a)+2\bar{S}_\alpha(s+\delta_a)\Big], \quad (38)$$

$$B = 1-2\delta_a\bar{S}_\alpha(s+\delta_a)-3\delta_c\bar{S}_\alpha(s+2\delta_c), \quad (39)$$

$$G = 1-2\delta_a\bar{S}_\alpha(s+\delta_a). \quad (40)$$

The following Laplace transformations indicate the probability of the system switching between operational and failed states at any given moment:

$$\bar{P}_{up}(s) = \bar{P}_0(s)+\bar{P}_1(s)+\bar{P}_2(s)+\bar{P}_3(s)+\bar{P}_4(s)+\bar{P}_5(s)+\bar{P}_6(s). (41)$$

$$\bar{P}_{down}(s) = 1-\bar{P}_{up}(s). \quad (42)$$

## 5. Analytical study of the model

### 5.1. Availability analysis

The fix would be consistent with the general and Gumbel-Hougaard copula distribution families. Setting

$$\bar{S}_\eta(s) = \frac{exp[x^\theta+\{log\alpha(x)\}^\theta]^{\frac{1}{\theta}}}{s+exp[x^\theta+\{log\alpha(x)\}^\theta]^{\frac{1}{\theta}}}, \bar{S}_\alpha(s) = \frac{\alpha}{s+\alpha}.$$

We have studied the following two cases for the availability of system.
*Case I:* Using various parameter values as $\delta_1 = 0.02, \delta_2 = 0.03, \delta_3 = 0.03, \delta_a = 0.025, \delta_c = 0.035, \theta = 1, \alpha = 1, x = y = 1$ and $\alpha(x) = \alpha(y) = 1$ in equation (41), and then taking inverse Laplace transform, we have availability of the system,

$$\begin{aligned}P_{up}(t) &= 0.968501+0.0401169e^{-2.83285t}-0.00736907e^{-1.3617t} \\ &- 0.0000102754e^{-1.23146t} - 0.000114346e^{-1.04506t} \\ &- 0.0000477605e^{-1.03916t} - 0.00101205e^{-0.954528t} \\ &- 0.0000641764e^{-0.905826t} - 1.56216(10^{-13})e^{-0.235t} \\ &+ 5.98848(10^{-14})e^{-0.225t} + 2.389(10^{-15})e^{-0.07t} \\ &- 7.916899(10^{-15})e^{-0.05t} + 3.70097(10^{-15})e^{-0.025t}. \quad (43)\end{aligned}$$

*Case II:* Reduction method
By multiplying the failure rates of the system's components by a factor *rho* such that $0 < \rho < 1$, we have been able to boost system availability. Using the inverse Laplace transformation and the following parameter values ($\delta1 = 0.02, \delta2 = 0.03, \delta3 = 0.03, \delta a = 0.025, \delta c = 0.035, \theta = 1, \alpha = 1, x = y = 1$, and $\alpha(x) = \alpha(y) = 1$), we may gain the increased availability of the system.

$$\begin{aligned}P_{up}(t) &= 0.987911+0.0134429e^{-2.75536t}-0.00114909e^{-1.14601t} \\ &- 1.52928(10^{-6})e^{-1.09258t} - 0.0000157255e^{-1.01784t} \\ &- 9.37697(10^{-6})e^{-1.01556t} - 0.000168718e^{-0.979336t} \\ &- 9.52708(10^{-6})e^{-0.962258t} - 9.06119(10^{-14})e^{-0.094t} \\ &+ 9.29511(10^{-14})e^{-0.09t} - 1.99697(10^{-15})e^{-0.028t} \\ &- 1.53729(10^{-15})e^{-0.02t} + 1.28667(10^{-15})e^{-0.01t}. \quad (44)\end{aligned}$$

Table 2. Comparing the availability of the original and enhanced systems.

| Time | Availability *CaseI* | Availability *CaseII* |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 0.968498 | 0.988324 |
| 2 | 0.967974 | 0.987821 |
| 3 | 0.968316 | 0.987867 |
| 4 | 0.968443 | 0.987896 |
| 5 | 0.968483 | 0.987906 |
| 6 | 0.968495 | 0.987909 |
| 7 | 0.968499 | 0.98791 |
| 8 | 0.9685 | 0.987911 |
| 9 | 0.968501 | 0.987911 |
| 10 | 0.968501 | 0.987911 |

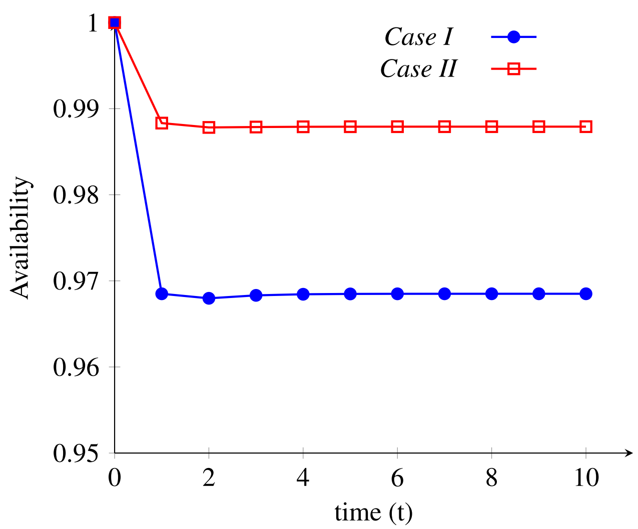


Figure 3. Availability analysis of two cases.

Now, if we change $t = 0$ to 10 in (43) and (44) above, we get Table 2 and the associated Figure 3, which show how availability shifts with time in two different scenarios.

### 5.2. Reliability Analysis

For certain levels of failure rates, the reliability can be derived by setting all repair rates in equation (41) to zero. The same two scenarios that were covered in the discussion of availability are covered here as well.

*Case I:* Using the values of various parameters as $\delta_1 = 0.02, \delta_2 = 0.03, \delta_3 = 0.03, \delta_a = 0.025, \delta_c = 0.035$. After entering in all of these numbers into (41), the system's dependability is calculated using the inverse Laplace transformation:

$$\begin{aligned}R(t) &= 0.127357e^{-0.473674t} + 0.000261585e^{-0.230773t} \\ &+ 0.174506e^{-0.0819933t} + 0.145205e^{-0.0587927t}\end{aligned}$$

Table 3. Variation of reliability with respect to time in various cases.

| Time | Reliability *CaseI* | Reliability *CaseII* |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 0.911036 | 0.966337 |
| 2 | 0.842435 | 0.935999 |
| 3 | 0.787218 | 0.908743 |
| 4 | 0.740997 | 0.883329 |
| 5 | 0.700998 | 0.86021 |
| 6 | 0.665459 | 0.838818 |
| 7 | 0.633247 | 0.818902 |
| 8 | 0.603622 | 0.800255 |
| 9 | 0.576097 | 0.782702 |
| 10 | 0.550336 | 0.766098 |

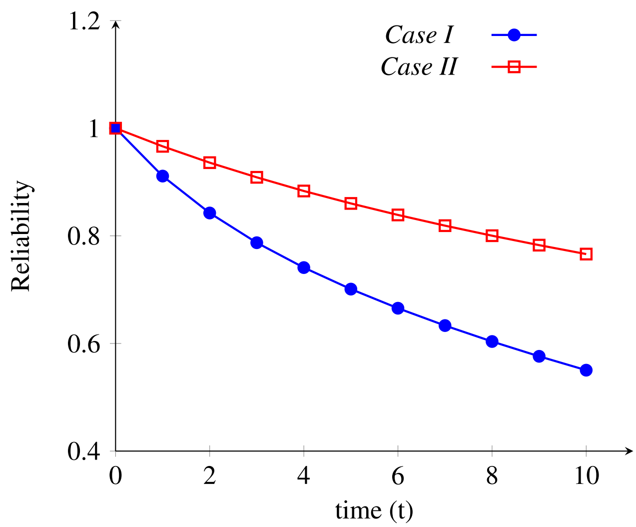


Figure 4. Reliability analysis of two cases.

$$+ 0.402824e^{-0.0390278t} + 0.149846e^{-0.0230376t}. \quad (45)$$

*Case II:* It is expected that the failure rates of the system's units are decreased by a factor of $\rho$ such that $0 < \rho < 1$ when the reduction approach is applied. If we set $\rho = 0.4$ and $\delta_1 = 0.02, \delta_2$=0.03, $\delta_3 = 0.03, \delta_a = 0.025, \delta_c = 0.035$, and so on, we can predict that the system will exhibit the following behavior: By plugging these numbers into (41) and doing the inverse Laplace transformation, we can calculate the system's reliability:

$$R(t) = 0.108202e^{-0.183232t} + 0.000227366e^{-0.0923024t} + 0.0701865e^{-0.0295817t} + 0.178017e^{-0.0221234t} + 0.56058e^{-0.015812t} + 0.0827866e^{-0.00961557t}. \quad (46)$$

The reliability variance over time for two situations is shown in the Table 3 and the associated Figure 4.

### 5.3. Mean Time to Failure ($MTTF$) Analysis

Setting the total number of fixes to zero and the limit as $s$ approaches zero in (41) allows us to determine the system's

Table 4. Variation of $MTTF$ with respect to failure rates.

| Failure rates | $MTTF$ $(\delta_1)$ | $MTTF$ $(\delta_2)$ | $MTTF$ $(\delta_3)$ | $MTTF$ $(\delta_a)$ | $MTTF$ $(\delta_c)$ |
|---|---|---|---|---|---|
| 0.1 | 5.70218 | 5.92611 | 5.92611 | 6.23364 | 5.97549 |
| 0.2 | 4.12612 | 4.24425 | 4.24425 | 5.35957 | 4.50606 |
| 0.3 | 3.22363 | 3.296 | 3.296 | 4.94086 | 3.78019 |
| 0.4 | 2.64157 | 2.69026 | 2.69026 | 4.69275 | 3.34807 |
| 0.5 | 2.23597 | 2.27089 | 2.27089 | 4.5282 | 3.06156 |
| 0.6 | 1.93754 | 1.96378 | 1.96378 | 4.41097 | 2.85771 |
| 0.7 | 1.70895 | 1.72937 | 1.72937 | 4.32317 | 2.70528 |
| 0.8 | 1.52835 | 1.54469 | 1.54469 | 4.25495 | 2.587 |
| 0.9 | 1.38212 | 1.39548 | 1.39548 | 4.2004 | 2.49255 |
| 1 | 1.26132 | 1.27245 | 1.27245 | 4.15578 | 2.4154 |

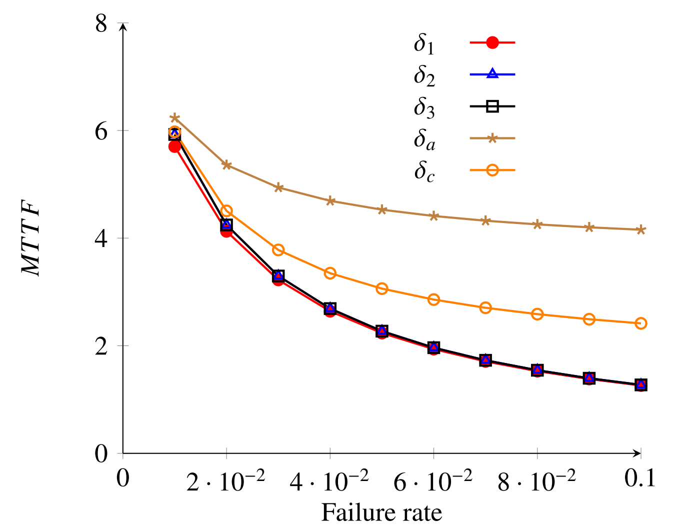


Figure 5. $MTTF$ versus rates of failure

$MTTF$:

$$MTTF = \lim_{s\to 0} \bar{P}_{up}(s).$$

The formula for $MTTF$ can be derived by defining the limit as $s$ goes to zero and treating all repairs in equation (41) as zero.

$$MTTF = \frac{1}{2(\delta_1 + \delta_2 + \delta_3 + 3(\delta_a + \delta_c))}\Big[2 + \delta_c\Big(9 + \frac{6}{\delta_1 + \delta_2 + \delta_3 + 3\delta_a + 2\delta_c}\Big) + \delta_a\Big(21 + 36\delta_c + \frac{4}{\delta_1 + \delta_2 + \delta_3 + 2\delta_a + 3\delta_c}\Big)\Big]. \quad (47)$$

Failure rates were identified by setting $\delta_1 = 0.02, \delta_2 = 0.03, \delta_3 = 0.03, \delta_a = 0.025, \delta_c = 0.035$, and then systematically changing each of these values from 0.01 to 0.10 in (47). The disparity between the mean time to failure ($MTTF$) and the failure rates is seen in Table 4 and Figure 5.

### 5.4. Cost Analysis

We may calculate the expected benefit of the system for the time period [0, t) if the service facility is always accessible as

Table 5. Expected profit where the repair follows cupola distribution

| Time | $k_2 = 0.5$ | $k_2 = 0.4$ | $k_2 = 0.3$ | $k_2 = 0.2$ | $k_2 = 0.1$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0.477003 | 0.577003 | 0.677003 | 0.777003 | 0.877003 |
| 2 | 0.944951 | 1.14495 | 1.34495 | 1.54495 | 1.74495 |
| 3 | 1.41312 | 1.71312 | 2.01312 | 2.31312 | 2.61312 |
| 4 | 1.88151 | 2.28151 | 2.68151 | 3.08151 | 3.48151 |
| 5 | 2.34998 | 2.84998 | 3.34998 | 3.84998 | 4.34998 |
| 6 | 2.81847 | 3.41847 | 4.01847 | 4.61847 | 5.21847 |
| 7 | 3.28696 | 3.98696 | 4.68696 | 5.38696 | 6.08696 |
| 8 | 3.75546 | 4.55546 | 5.35546 | 6.15546 | 6.95546 |
| 9 | 4.22396 | 5.12396 | 6.02396 | 6.92396 | 7.82396 |
| 10 | 4.69246 | 5.69246 | 6.69246 | 7.69246 | 8.69246 |

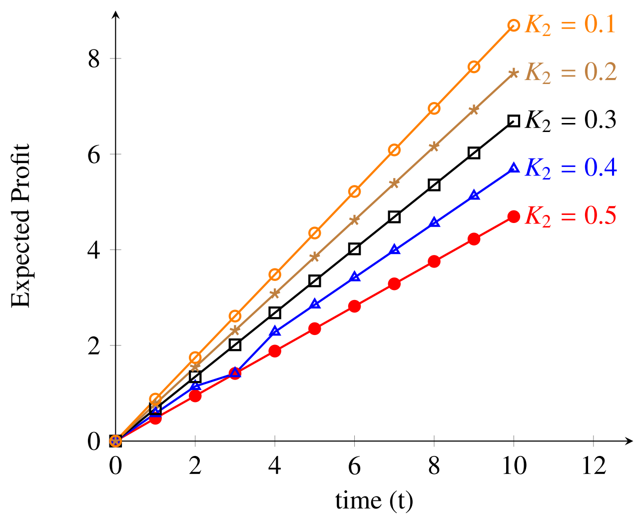


Figure 6. An estimate of the expected profit under the copula repair strategy.

follows:

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t. \quad (48)$$

Like availability and reliability we have studied two cases. *Case I:* $K_1$ and $K_2$ represent the revenue and service cost per unit time, respectively. Using the identical values for the parameters in equation(43) and equation(48) as in case I of subsection (5.1), one obtains the same result as in (49). For this reason,

$$\begin{aligned} E_p(t) = \Big(&0.00745477 - 0.0141613e^{-2.83285t} \\ &+ 0.00541169e^{-1.3617t} + 8.34404(10^{-6})e^{-1.23146t} \\ &+ 0.000109416e^{-1.04506t} + 0.0000459607e^{-1.03916t} \\ &+ 0.00106026e^{-0.954528t} + 0.0000708485e^{-0.905826t} \\ &+ 6.6475(10^{-13})e^{-0.235t} - 2.66155(10^{-13})e^{-0.225t} \\ &- 3.41286(10^{-14})e^{-0.07t} + 1.58338(10^{-13})e^{-0.05t} \\ &- 1.48039(10^{-13})e^{-0.025t} + 0.968501t\Big)K_1 - K_2 t. \quad (49) \end{aligned}$$

Table 5 and Figure 6 show the estimated profit for $K_1 = 1$ and $K_2 = 0.1, 0.2, 0.3, 0.4,$ and 0.5 and $t = 0$ to 10 units of time. *Case II:* When employing the same set of parameters as in Case II of subsection (5.1) and equations (44) and (48). From this, one can derive equation (50):

$$\begin{aligned} E_p(t) = \Big(&0.00366789 - 0.00487883e^{-2.75536t} \\ &+ 0.00100268e^{-1.14601t} + 1.39969(10^{-6})e^{-1.09258t} \\ &+ 0.00001545e^{-1.01784t} + 9.23326(10^{-6})e^{-1.01556t} \\ &+ 0.000172278e^{-0.979336t} + 9.90076(10^{-6})e^{-0.962258t} \\ &+ 9.63957(10^{-13})e^{-0.094t} - 1.03279(10^{-12})e^{-0.09t} \\ &+ 7.13203(10^{-14})e^{-0.028t} + 7.68644(10^{-14})e^{-0.02t} \\ &- 1.28667(10^{-13})e^{-0.01t} + 0.987911t\Big)K_1 - K_2 t. \quad (50) \end{aligned}$$

Table 6. Expected profit where the repair follows cupola distribution with $\rho = 0.4$

| Time | $k_2 = 0.5$ | $k_2 = 0.4$ | $k_2 = 0.3$ | $k_2 = 0.2$ | $k_2 = 0.1$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0.491665 | 0.591665 | 0.691665 | 0.791665 | 0.891665 |
| 2 | 0.979601 | 1.1796 | 1.3796 | 1.5796 | 1.7796 |
| 3 | 1.46744 | 1.76744 | 2.06744 | 2.36744 | 2.66744 |
| 4 | 1.95533 | 2.35533 | 2.75533 | 3.15533 | 3.55533 |
| 5 | 2.44323 | 2.94323 | 3.44323 | 3.94323 | 4.44323 |
| 6 | 2.93114 | 3.53114 | 4.13114 | 4.73114 | 5.33114 |
| 7 | 3.41905 | 4.11905 | 4.81905 | 5.51905 | 6.21905 |
| 8 | 3.90696 | 4.70696 | 5.50696 | 6.30696 | 7.10696 |
| 9 | 4.39487 | 5.29487 | 6.19487 | 7.09487 | 7.99487 |
| 10 | 4.88278 | 5.88278 | 6.88278 | 7.88278 | 8.88278 |

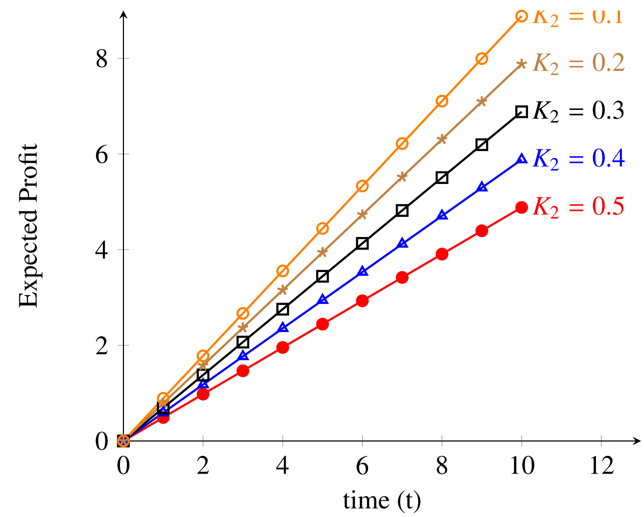


Figure 7. Expected Profit from developing system with roh=0.4

## 6. Discussion and Conclusions

A complex system with three subsystems was evaluated in terms of its reliability traits. The supplementary variable technique has been used to derive explicit expressions. The reduction method has been utilized as a successful strategy for increasing system reliability by reducing the failure rate of some of the system components.

The data on the complex repairable system's availability evolves over time when rates of failure are set at $\delta_1 = 0.02, \delta_2 = 0.03, \delta_3 = 0.03, \delta_a = 0.025$, and $\delta_c = 0.035$ is shown in Table 2 and Figure 3. We can see that the system's availability declines as time t's value rises and eventually stabilizes after a suitably extended period of time. Utilizing the reduction strategy as case II, we lower the system's unit failure rates via a factor $\rho$ such that $0 < \rho < 1$ in order to increase the system availability. Therefore, It seems reasonable to conclude that the technique of reduction strategy is an option worth considering.
The system's reliability is assessed in the same two scenarios as its availability. The original system's reliability decreases over time when failure rates are deliberately varied, as in cases I and II. When applied to the original system, a reduction strategy with a factor $\rho$ such that $0 < \rho < 1$ reduces the rate of failure per unit of the system, hence increasing its reliability. Table 3 and Figure 4 show that the reduction technique enhances the reliability of the original system when comparing the two cases, Case I and Case II.

The system $MTTF$ is depicted in Figure 5 and Table 4 with respect to variations in the failure rates $\delta_1, \delta_2, \delta_3, \delta_a$, and $\delta_c$, respectively, with the assumption that all other parameters remain constant. The system's $MTTF$ is decreasing in terms of a number of failure rates. Mean Time to Failure ($MTTF$) for the entire system is longest for subsystem 3 and shortest for subsystem 1.

The expected profit has been computed (Tables 5 and 6), and the results are illustrated by the graph given revenue cost is fixed at $K_1 = 1$ and service cost are at $K_2 = 0.5, 0.4, 0.3, 0.2$, and 0.1 (Figures 6 and 7). It demonstrates that for smaller values of $K_2$, predicted profit increased over time, whereas for greater values of service cost, expected profit declined. For this reason, when service expenses are low, profits are greater than when they are high. The expected profit of the system is estimated in two cases, just like its availability, and displayed in Table 5, Figure 6, Table 6, and Figure 7. As in cases I and II, it is determined that by employing a reduction procedure with a factor $\rho$ such that $0 < \rho < 1$. By comparing the system's expected profit in instance I to the system's expected profit in case II while applying the reduction approach, it can be proven that the reduction technique raises the system's expected profit.

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