Published by
NIGERIAN SOCIETY OF PHYSICAL SCIENCES
Available online @ https://journal.nsps.org.ng/index.php/jnsps

J. Nig. Soc. Phys. Sci. 5 (2023) 1453

Journal of the Nigerian Society of Physical Sciences

Modeling and Analysis of a Fractional Visceral Leishmaniosis with Caputo and Caputo–Fabrizio derivatives

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Abstract

Visceral leishmaniosis is one recent example of a global illness that demands our best efforts at understanding. Thus, mathematical modeling may be utilized to learn more about and make better epidemic forecasts. By taking into account the Caputo and Caputo-Fabrizio derivatives, a frictional model of visceral leishmaniosis was mathematically examined based on real data from Gedaref State, Sudan. The stability analysis for Caputo and Caputo-Fabrizio derivatives is analyzed. The suggested ordinary and fractional differential mathematical models are then simulated numerically. Using the Adams-Bashforth method, numerical simulations are conducted. The results demonstrate that the Caputo-Fabrizio derivative yields more precise solutions for fractional differential equations.

DOI:10.46481/jnsps.2023.1453

Keywords: Leishmaniosis, Modelling, Caputo, Caputo–Fabrizio, Sudan.

Article History :
Received: 16 March 2023
Received in revised form: 21 May 2023
Accepted for publication: 19 June 2023
Published: 26 July 2023

Communicated by: B. J. Falaye

1. Introduction

Visceral leishmaniasis, or kala azar, is a lethal vector-borne illness. India, Bangladesh, and Nepal have achieved substantial headway in lowering VL cases. East Africa has made less progress, especially with South Sudan's continuous endemicity and VL outbreaks during the past 40 years. Lack of infrastructure, clinical staff, IDPs, and hunger have hampered VL management in, and longer-term hazards to diagnostic kits and medications. Pentavalent antimonials have been the backbone of VL treatment for decades, and resistance to them, as previously demonstrated in the Indian subcontinent, provides another major barrier to VL treatment and management. To prevent monotherapy and minimize treatment duration, first-line 30-day sodium stibogluconate SS is substituted with a 17-day injectable combination regimen of SSG and PM in WHO recommendations in 2010 and Sudan Ministry of Health guidelines in 2011. Since 2012, AmBisome has been donated to WHO for these purposes. In East Africa, SSG/PM combo treatment had a 5% recurrence rate. Relapse may be due to insufficient cellular immunity following therapy due to HIV, TB, or malnutrition, or inadequate treatment resulting in considerable chronic parasitaemia after initial clinical cure. In places like Sudan, where

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active patient follow-up is difficult and not common, VL recurrence rates are passively evaluated by VL re-treatment admissions as a proportion of overall VL admissions. Passive monitoring shows an increase in re-treatment rates in recent years [1-7].

In the last 30 years, fractional calculus and nonlinear equations has become more well-known and important. Fractional differential equations are used in physics, chemical engineering, mathematical biology, and finance [8-16]. Simulating a fractional model simultaneously with a Caputo derivative and a CF derivative. In addition, modeling and graphing with the fractional derivative is a highly effective technique for demonstrating leishmaniasis using MATLAB. This could be done to better comprehend the infection. Using fractional derivatives as a research strategy for natural occurrences may result in more precise findings than other methods. As a result of this model's use of a non-singleton kernel, the CF derivative has significantly improved predictive abilities.

2. Preliminaries

Definition 2.1. *Riemann-Liouville fractional integral (RLI) operator of order $\alpha > 0$ for a function $y(\tau)$ is given by [17]:*

$$D^{\alpha} y(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} y^n(\tau) d\tau = I^{n-\alpha} y^n(t),\ t > 0. \tag{1}$$

Definition 2.2. *For $y \in H^1(0,t), t > 0, T > 0,\ \alpha \in (0,1]$ Then the CF fractional operator [17] is given by*

$$^{CF}_{0}D^{\alpha}_{t} y(t) = \frac{B(\alpha)}{1-\alpha)} \frac{d}{dt} \int_0^t y(\tau) e^{-\alpha \frac{t-\tau}{1-\alpha}} d\tau,\ 0 < \alpha < 1. \tag{2}$$

In this expression $B(\alpha)$ satisfies the condition $B(0) = B(1) = 1$.

Definition 2.3. *Caputo derivative of order $0 \leq n-1 < \alpha < n$ with the lower limit zero for a function $y(\tau)$ is given by [18]:*

$$I^{\alpha} y(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} y(\tau) d\tau,\ t > 0. \tag{3}$$

3. Anthropologic Visceral Leishmaniosis model with Caputo derivative

In this Section, we describe the leishmaniasis model, which includes four subpopulations:susceptible, infectious, Recovered, and Recovered with permanent immunity, for the human population, and two compartments for the reservoir population: susceptible and infected. In addition to that, we have two compartments for sandflies: susceptible and infected. The human population is the only population in the model that has permanent immunity. The positivity, reproduction number, and equilibrium solutions of the model that was established in this work have all been determined to be free of leishmaniasis. Additionally, the leishmaniasis cases, along with their respective localities and global stability properties, have also been determined. We obtain the model formulation by using a new variable:

$$s_h(t) = \frac{S_H}{N_H},\ i_h(t) = \frac{I_H}{N_H},\ p_h(t) = \frac{P_H}{N_H},$$
$$r_h(t) = \frac{R_H}{N_H},\ s_r(t) = \frac{S_R}{N_R},\ I_r(t) = \frac{I_R}{N_R},$$
$$s_V(t) = \frac{S_V}{N_V},\ i_V(t) = \frac{I_V}{N_V},\ s_h(t) = \frac{S_H}{N_H}, m = \frac{N_V}{N_H} \text{ and } N = \frac{N_V}{N_R}.$$

The system of differential equations is given by:

$$\begin{cases} ^{C}_{0}D^{\alpha}_{t} i_h = abmi_v N_h - \left(\alpha_1 + \delta + \frac{A_H}{N_H} - \delta i_h\right) i_h, \\ ^{C}_{0}D^{\alpha}_{t} p_h = (1-\sigma)\alpha_1 i_h - \left(\alpha_2 + \beta + \frac{A_H}{N_H} - \delta i_h\right) p_h, \\ ^{C}_{0}D^{\alpha}_{t} i_r = abni_v s_r - \frac{A_H}{N_H} i_r, \\ ^{C}_{0}D^{\alpha}_{t} i_v = aci_h S_v + acp_h S_v + aci_r S_v - \frac{A_V}{N_V} i_v, \\ ^{C}_{0}D^{\alpha}_{t} s_h = \frac{A_H}{N_H} - \left[abmi_v + \frac{A_H}{N_H} - \delta i_h\right] s_h, \\ ^{C}_{0}D^{\alpha}_{t} r_h = \sigma\alpha_1 i_h + (\alpha_2 + \beta) P_h - \left[\frac{A_H}{N_H} - \delta i_h\right] r_h, \\ ^{C}_{0}D^{\alpha}_{t} S_r = \frac{A_R}{N_R} - abni_v s_r - \frac{A_H}{N_H} s_r, \\ ^{C}_{0}D^{\alpha}_{t} s_v = \frac{A_V}{N_V} - \left[aci_h + acP_h + \frac{A_V}{N_V}\right] s_v, \end{cases} \tag{4}$$

with initial conditions:
$s_h(0) = c_1, i_h(0) = c_2, r_h(0) = c_3, s_r(0) = c_4, I_r(0) = c_5, s_V(0) = c_6,\ \ i_V(0) = c_7$.

4. Anthropologic Visceral Leishmaniosis model with FC derivative

In this Section, we obtain the fractional model formulation under Caputo–Fabrizio derivatives: $s_h(t) = \frac{S_H}{N_H}, i_h(t) = \frac{I_H}{N_H}, p_h(t) = \frac{P_H}{N_H}, r_h(t) = \frac{R_H}{N_H}, s_r(t) = \frac{S_R}{N_R}, I_r(t) = \frac{I_R}{N_R}, s_V(t) = \frac{S_V}{N_V}, i_V(t) = \frac{I_V}{N_V}, s_h(t) = \frac{S_H}{N_H}, m = \frac{N_V}{N_H}$ and $N = \frac{N_V}{N_R}$ The system of differential equations is given by:

$$\begin{cases} ^{FC}_{0}D^{\alpha}_{t} i_h = abmi_v N_h - \left(\alpha_1 + \delta + \frac{A_H}{N_H} - \delta i_h\right) i_h, \\ ^{FC}_{0}D^{\alpha}_{t} p_h = (1-\sigma)\alpha_1 i_h - \left(\alpha_2 + \beta + \frac{A_H}{N_H} - \delta i_h\right) p_h, \\ ^{FC}_{0}D^{\alpha}_{t} i_r = abni_v s_r - \frac{A_H}{N_H} i_r, \\ ^{FC}_{0}D^{\alpha}_{t} i_v = aci_h S_v + acp_h S_v + aci_r S_v - \frac{A_V}{N_V} i_v, \\ ^{FC}_{0}D^{\alpha}_{t} s_h = \frac{A_H}{N_H} - \left[abmi_v + \frac{A_H}{N_H} - \delta i_h\right] s_h, \\ ^{FC}_{0}D^{\alpha}_{t} r_h = \sigma\alpha_1 i_h + (\alpha_2 + \beta) P_h - \left[\frac{A_H}{N_H} - \delta i_h\right] r_h, \\ ^{FC}_{0}D^{\alpha}_{t} S_r = \frac{A_R}{N_R} - abni_v s_r - \frac{A_H}{N_H} s_r, \\ ^{FC}_{0}D^{\alpha}_{t} s_v = \frac{A_V}{N_V} - \left[aci_h + acP_h + \frac{A_V}{N_V}\right] s_v, \end{cases} \tag{5}$$

with initial conditions: $s_h(0) = c_1, i_h(0) = c_2, r_h(0) = c_3, s_r(0) = c_4, I_r(0) = c_5, s_V(0) = c_6,\ \ i_V(0) = c_7$.

5. Stability analysis

In this part we discuss the stability of epidemiological model, the equilibrium points, eigenvalues value and the Jacobian matrix for the model (1).

Table 1: Description of the variables for model.

Variable	Description
$N_H(t)$	Human host population
$N_R(t)$	Reservoir host population
$N_V(t)$	Vector population
$S_H(t)$	Susceptible humans
$P_H(t)$	Recovered and have permanent immunity
$I_H(t)$	Infected humans
$R_H(t)$	Recovery humans
$R_s(t)$	Susceptible reservoir
$I_R(t)$	Infected reservoir
$S_V(t)$	Susceptible sandflies
$I_V(t)$	Infected sandflies

Table 2: Parameters values of the leishmaniasis model.

Parameter	Description	Value	Source
a	Biting rate of sandflies	0.2856 day^{-1}	[16]
b	Progression rate of VL in sandfly	0.22 day^{-1}	[16]
c	Progression rate of VL in human and reservoir	0.0714 day^{-1}	[16]
A_H	Human recruitment rate	10.1009 day^{-1}	Estimated
A_R	Reservoir recruitment rate	19.7795 day^{-1}	Estimated
A_V	Vector recruitment rate	38858.62 day^{-1}	Estimated
μ_h	Natural mortality rate of humans	$4.341e-6$ day^{-1}	[2]
μ_r	Natural mortality rate of reservoirs	0.0017 day^{-1}	[1]
μ_v	Natural mortality rate of vectors	0.0668 day^{-1}	[1]
α_1	Treatment rate of VL	0.02	[2]
α_2	PKDL recovery rate without treatment	0.033	[20]
σ	Recovery rate from VL infection after treatment	0.9	[1]
$1-\sigma$	Developing PKDL rate after treatment	0.1	[1]
δ	Death rate due to VL	0.011	[16]
β	PKDL recovery rate after treatment	0.9	[1]

5.1. Equilibria

The equilibrium points of dynamics (5) are computed solving the nonlinear system.

Table 3: The equilibrium points of the system.

E_i	Equilibria
E_1	$(0, 0, 0, 0, 1, 0, 711.58, 1)$
E_2	$(32.5436, 0.1132, 711.5801, 22.7897, -29.7254, -1.9314, 0, 0)$
E_3	$(-31.1459, -0.0488, 711.58, -21.8109, 33.9641, -1.7693, 0, 0)$
E_4	$(85.0303, -72.8759, 711.5801, 59.5451, -82.2121, -71.0578, 0, 0)$
E_5	$(2.8182, 0.0062, -2.8244, 0, 0, -1.8244, 714.4046, 0)$
E_6	$(2.8182, 0.0062, 711.5801, 252.9373, 0, -1.8244, 0, 0)$
E_7	$(0, 0, 0, 0, -7.2892e11, 7.2892e11, 0, 0)$
E_8	$(0, 0, 0, 5.1155e8, 0, 0, 0, 0)$

5.2. The Jacobian matrix for the model:

Here, we talk about this epidemiological model stability. The disease-free equilibrium point is given as $E_1 = (0, 0, 0, 0, 1, 0, 711.58, 1)$ and the endemic equilibrium points $E_8 = (0, 0, 0, 5.1155e8, 0, 0, 0, 0)$.

$$J(E_1) = \begin{bmatrix} -0.031 & 0 & 0 & 0.0157 & 0 & 0 & 0 & 0 \\ 0.002 & -0.933 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4.297e^{-8} & 15907.35 & 0 & 0 & 0 & 0 \\ 0.02 & 0.02 & 0.02 & -3.438e^{-11} & 0 & 0 & 0 & 0 \\ 0.011 & 0 & 0 & -0.0157 & -4.297e^{-8} & 0 & 0 & 0 \\ 0.018 & 0.0933 & 0 & 0 & 0 & -4.297e^{-8} & 0 & 0 \\ 0 & & 0 & -15907.35 & 0 & 0 & -4.297e^{-8} & 0 \\ -0.02 & -0.02 & 0 & 0 & 0 & 0 & 0 & -3.438e^{-11} \end{bmatrix} \tag{6}$$

$$J(E_2) = \begin{bmatrix} -0.031 & 0 & 0 & 0 & 8035456.6 & 0 & 0 & 0 \\ 0.002 & -0.933 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4.297e^{-8} & -0.0006 & 0 & 0 & 11435742383.06 & 0 \\ 0.000002 & 0.00002 & 0.00002 & -3.438e^{-11} & 0 & 0 & 0 & 14.225 \\ 0 & 0 & 0 & 0 & -8035456.6 & 0 & 0 & 0 \\ 0.028 & 0.0933 & 0 & 0 & 0 & -0.00000003 & 0 & 0 \\ 0 & 0 & 0 & 0.0006 & 0 & 0 & -11435742383.06 & 0 \\ -0.00002 & -0.0002 & 0 & 0 & 0 & 0 & 0 & -0.000000027 \end{bmatrix} \tag{7}$$

Table 4: Variable values.

	Eigenvalues	Stability
λ	$(17.833, 0, 0, 0, 0, 0, 0, -17.833)$	Unstable
λ^*	$(-3.438e^{-11}, -2.8e^{-8}, -2.8e^{-8},$ $-4.297e^{-8}, -0.31, -0.933, -035456.6, -1.14e^{10})$	Stable

5.3. The Basic Reproduction Number

The basic reproduction number is a baseline statistic in epidemiology and is represented by R_0, which stands for the predicted value of the secondary infections rate per time unit. Using the equation's fractional model (1), we have fours infected classes, rewrite the system of Equation 1 for the susceptible and infected classes in the general form:

$$\frac{dx}{dt} = f(x) - v(x), \tag{8}$$

where

$$f(x) = \begin{pmatrix} abmi_v s_h \\ 0 \\ abmi_v s_r \\ ac(i_h + p_h + i_r)s_v \end{pmatrix},$$
$$\text{and} \quad v(x) = \begin{pmatrix} (\alpha_1 + \delta + \mu_h)\, i_h \\ (\alpha_2 + \beta + \mu_h)p_h - (1-\sigma)\alpha_1 i_h \\ \mu_r i_r \\ \mu_v i_v \end{pmatrix}. \tag{9}$$

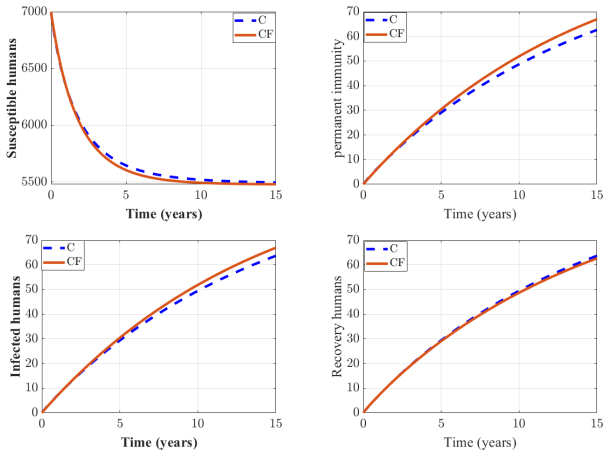

Figure 1: Systems of fractional orders model for α=0.99.

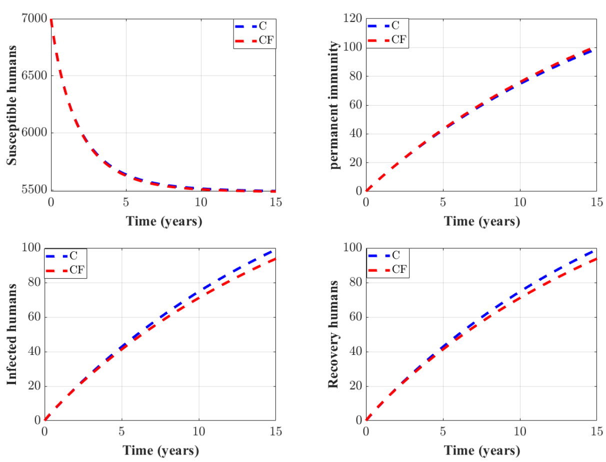

Figure 2: Systems of fractional orders model for α=1 (First part)

Now, the Jacobian of $f(x)$ and $v(x)$ of the disease free equilibrium point is:

$$F = \begin{pmatrix} 0 & 0 & 0 & abm \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & abm \\ ac & ac & ac & 0 \end{pmatrix},$$
$$\text{and} \quad V = \begin{pmatrix} \alpha_1 + \delta + \mu_h & 0 & 0 & 0 \\ -(1-\sigma)\alpha_1 & \alpha_2 + \beta + \mu_h & 0 & 0 \\ 0 & 0 & \mu_r & 0 \\ 0 & 0 & 0 & \mu_v \end{pmatrix} \tag{10}$$

we have

$$R_0 = \rho\left(FV^{-1}\right) = \sqrt{\frac{ac\Big[\mu_r abm\,(\alpha_2 + \delta + \mu_h + (1-\sigma)\,\alpha_1) + abn(\alpha_1 + \delta + \mu_h)(\alpha_2 + \delta + \mu_h)\Big]}{\mu_v\, \mu_r(\alpha_1 + \delta + \mu_h)(\alpha_2 + \delta + \mu_h).}} \tag{11}$$

Lemma 5.1. *The disease-free equilibrium E_0 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

6. Numerical Simulation and Graphical Representations

This section is devoted to finding the approximate solutions of the proposed models (4) and (5) under fractional operators of Caputo and Caputo-Fabrizio, respectively. We simulate our model using some highly reliable numerical techniques. The finite difference scheme for the initial value problem yields the following numerical techniques for the underlying operators:

$$^{c}x_{r+1} = x_0 + \frac{(\Delta t)^{\omega}}{\Gamma(\omega+1)} \sum_{k=0}^{r} \left[(r-k+1)^{\omega} - (r-k)^{\omega}\right] F(x_k) + O\left(\Delta t^2\right),$$
$$^{\mathrm{CF}}x_{r+1} = x_0 + (1-\delta)F(x_r) + \delta \Delta t \sum_{k=0}^{r} F(x_k) + O\left(\Delta t^2\right), \tag{12}$$

Table 1 shows a description of the variables in the model. Figures 1 and 2 were obtained with the Caputo (4) and CF methods (5) using the parameters in Table 2. Tables 3 & 4 show a summary of equilibrium points and the corresponding eigenValues of the Jacobian matrix.

7. Conclusion

A fractional model was simulated by using a Caputo derivative as well as a CF derivative simultaneously. In addition, modeling and graphing with the aid of the fractional derivative is a very effective approach that can be used to show leishmaniasis with the use of MATLAB. This may be done in order to better understand the infection. When doing research on natural events, using fractional derivatives as a strategy might lead to more precise findings than other approaches. Due to the fact that this model employs a non-singleton kernel, the CF derivative has much enhanced prediction capabilities. This research was carried out in the hope that it will be a useful resource for future applications and explorations of simulation by using a Caputo derivative, and to investigate new methods such as those in Refs. [18-29]

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