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Simple Motion Pursuit Differential Game

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Abstract

We study a simple motion pursuit differential game of many pursuers and one evader in a Hilbert space l_2. The control functions of the pursuers and evader are subject to integral and geometric constraints respectively. Duration of the game is denoted by positive number θ. Pursuit is said to be completed if there exist strategies u_j of the pursuers P_j such that for any admissible control $v(\cdot)$ of the evader E the inequality $\|y(\tau) - x_j(\tau)\| \leq l_j$ is satisfied for some $j \in \{1, 2, \dots\}$ and some time τ. In this paper, sufficient condition for completion of pursuit were obtained. Consequently strategies of the pursuers that ensure completion of pursuit are constructed.

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1. Introduction

In view of the extensive literature on differential games of several player's with the control functions subjected to either geometric, integral or both constraints. The work in the papers [1-26] and some reference their in attract the attention of many researchers .

In many studies of differential game, motion of each player are explicitly stated and considered to be a system of differential equations of the same order. In the papers [2,5,7-9,13,16], motion of each of the player is considered to obey first order differential equation. In other studies such as Refs. [4,6,12,15,19], players' motions are described by second order differential equations. Whereas in Refs. [1,23,25] motion of the players are described by first and second order differential equations.

In Ref, [24], Rikhsiev studied simple motion differential game of optimal pursuit with one evader and many pursuers on a closed convex subset of the Hilbert space l_2. A sufficient condition for optimality of pursuit time is obtained, when the initial position of the evader belong to the interior of the convex hull of the initial position of the pursuers.

Simple motion differential game of many players with geometric constraints on the control functions of the players is studied in [13]. By using lyapunov function method for an auxiliary problem, they obtained sufficient conditions to find the pursuit time in R^n. Vagin and Petrov in Ref. [22] Studied a pursuit differential game problem with finite number pursuers and one evader in the Hilbert space R^n. Motions of each player is described by n^{th} order differential equation. Control functions of the players are subject to geometric constraints. They

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obtained sufficient condition for completion of pursuit.

The work in Ref. [26] Leong and Ibragimov studied simple motion pursuit differential game with m pursuers and one evader on a closed convex subset of the Hilbert space l_2 . Control functions of the players are subjected to integral constraints. The total resource of the pursuers is assumed to be greater than that of the evader. Strategy of pursuers were constructed sufficient to complete the pursuit from any initial position.

In Ref. [23] pursuit differential game problem for the so-called boy(evader) and crocodile (pursuer) in the space R^n is studied. Boy's motion is described by first order differential equations and that of the crocodile by second order differential equation. Control functions of the pursuer and evader are subject to integral and geometric constraints respectively. They obtained Sufficient conditions of completion of pursuit.

In this piece of research, we study pursuit differential game problem in a Hilbert space l_2, where motions of the pursuers and evader described by first and second order differential equations respectively. Control functions of the pursuers are subject to integral constrains. Whereas, geometric constraint is imposed on the control function of the evader.

2. Statement of the Problem

Consider the space $l_2 = \left\{ \varrho = (\varrho_1, \varrho_2, \dots) : \sum_{k=1}^{\infty} \varrho_k^2 < \infty \right\}$, with inner product
$\langle \cdot, \cdot \rangle : l_2 \times l_2 \to R$ and norm $\| \cdot \| : l_2 \to [0, +\infty)$, defined as follows:

$$\langle x, y \rangle = \sum_{k=1}^{\infty} x_k y_k, \ \|\varrho\| = \left(\sum_{k=1}^{\infty} \varrho_k^2 \right)^{1/2},$$

where $x, y, \varrho \in l_2$, respectively.

We consider a differential game described by the following equations:

$$\begin{cases} P_j : \dot{x}_j = u_j(t), \ x_j(0) = x_{j0}, j \in J, \\ E : \ddot{y} = v(t), \ \dot{y}(0) = y^1, \ y(0) = y^0, \end{cases} \tag{1}$$

where $x_j, x_{j0}, u_i, y, y^0, y^1, v \in l_2$, $u_j = (u_{j1}, u_{j2}, \dots)$ is a control parameter of the pursuer P_j and $v = (v_1, v_2, \dots)$ is that of the evader E. Here and below $J = \{1, 2, \dots\}, \|x_{j0} - y_0\| > l_j$, where $l_j \geq 0$ are given numbers.

In the space l_2, we define a ball (respectively, sphere) of radius r and center at x_0 by $B(x_0, r) = \{x \in l_2 : \|x - x_0\| \leq r\}$ (respectively, by $S(x_0, r) = \{x \in l_2 : \|x - x_0\| = r\}$).

Definition 2.1. *A function $u_j(t) = (u_{j1}(t), u_{j2}(t), \dots)$ with Borel measurable coordinates such that*

$$\int_0^{\theta} \|u_j(t)\|^2 dt \leq \rho_j^2, \tag{2}$$

where ρ_j is given positive number, is called an admissible control of the j^{th} pursuer.

Definition 2.2. *A function $v(t) = (v_1(t), v_2(t), \dots)$ with Borel measurable coordinates such that*

$$\|v(t)\| \leq \sigma, \ \ t \geq 0, \tag{3}$$

is called admissible control of the evader.

If the pursuers P_j and evader E chose their admissible controls $u_j(\cdot)$ and $v(\cdot)$ respectively, the solutions to the dynamic equations (1) are given by:

$$x_j(t) = x_{j0} + \int_0^t u_j(s) ds, \tag{4}$$

$$y(t) = y^0 + t y^1 + \int_0^t \int_0^r v(s) ds dr. \tag{5}$$

It is not difficult to see that

$$\int_0^t \int_0^r v(s) ds dr = \int_0^t (t - s) v(s) ds \tag{6}$$

Therefore equation (5) becomes

$$y(t) = y^0 + t y^1 + \int_0^t (t - s) v(s) ds, \tag{7}$$

One can readily see that $x_j(\cdot), y(\cdot) \in C(0, \theta; l_2)$, where $C(, 0, \theta; l_2)$ is the space of functions

$$h(t) = (h_1(t), h_2(t), \cdots, h_k(t), \cdots) \in l_2, \quad t \geq 0,$$

such that the following conditions hold:
(1) $h_j(t), \ 0 \leq t \leq \theta, \ \ j = 1, 2, \cdots$, are absolutely continuous functions;
(2) $h(t), \ 0 \leq t \leq \theta$, is a continuous function in the norm of l_2.

With this instead of differential game described by (1) we can consider an equivalent differential game with the same control functions described by:

$$\begin{cases} P : \dot{x}_j = u(t), \ x_j(0) = x_{j0} \\ E : \dot{y} = (\theta - t) v(t), \ y(0) = y^1 \theta + y^0 = y_0. \end{cases} \tag{8}$$

Indeed, if the evader uses an admissible control $v(t) = (v_1(t), v_2(t), \dots)$, then according to (1), we have

$$y(\theta) = y^0 + y^1 \theta + \int_0^{\theta} \int_0^r v(s) ds dr = y^0 + y^1 \theta + \int_0^{\theta} (\theta - t) v(t) dt, \tag{9}$$

and the same result can be obtained by (8)

$$y(\theta) = y_0 + \int_0^{\theta} (\theta - t) v(t) dt = y^0 + y^1 \theta + \int_0^{\theta} (\theta - t) v(t) dt, \tag{10}$$

Also, the same argument can be made for the pursuer P_j, therefore in the distance $\|y(\theta) - x_j(\theta)\|$ we can take either the solution of (1) or the solution of (8).

The attainability domain of the pursuer P_i from the initial position x_{i0} up to the time θ is the ball $B(x_{j0}, \rho_i \sqrt{\theta})$. Indeed, by Cauchy-Schwartz inequality we have

$$\left\| x_j(\theta) - x_{j0} \right\|$$

$$\begin{aligned}
&= \left\| x_{j0} + \int_0^\theta u_j(s)ds - x_{j0} \right\| \\
&= \left\| \int_0^\theta u_j(s)ds \right\| \\
&\le \int_0^\theta \|u_j(s)\|ds \\
&\le \left(\int_0^\theta 1^2 ds\right)^{\frac{1}{2}} \left(\int_0^\theta \|u_j(s)ds\|^2\right)^{\frac{1}{2}} \\
&\le \rho_j \sqrt{\theta}
\end{aligned}$$

On the other hand, let $\bar{x} \in B(x_{j0}, \rho_j\sqrt{\theta})$. If the pursuer P_j uses the control $u_j(s) = \frac{\bar{x}-x_{j0}}{\theta}, 0 \le s \le \theta$, then we have

$$x_j(\theta) = x_{j0} + \int_0^\theta u_j(s)ds = x_{j0} + \int_0^\theta \frac{\bar{x}-x_{j0}}{\theta}ds = x_{j0} + \bar{x} - x_{j0} = \bar{x}.$$

In a similar fashion we can show that the attainability domain of the evader E from the initial position y_0 up to the time θ is the ball $B(y_0, \sigma\frac{\theta^2}{2})$.

Definition 2.3. *A strategy of the j^{th} pursuer is a function $U_j(t, x_j, y, v)$, $U_j : [0,\infty) \times l_2 \times l_2 \times l_2 \to l_2$, such that the system*

$$\begin{cases} \dot{x}_j = U_j(t, x, y, v(t)),\ x_j(0) = x_{j0}, \\ \ddot{y} = v(t),\ \ \dot{y}(0) = y^1,\ y(0) = y^0 \end{cases}$$

has a unique solution $(x_j(\cdot), y(\cdot))$, and that $x_j(\cdot), y(\cdot) \in C(0,\theta; l_2)$, for an arbitrary admissible control $v = v(t)$, $0 \le t \le \theta$, of the evader E. A strategy U_j is said to be admissible if each control formed by this strategy is admissible.

Definition 2.4. *The system described by (1) in which the controls $u_j(\cdot)$ and $v(\cdot)$ satisfy the inequalities (2) and (3) respectively is called game G.*

Definition 2.5. *Pursuit is said to be completed in l-catch sense in the game G if there exist strategies u_j of the pursuer P_j such that for any admissible control $v(\cdot)$ of the evader E the inequality $\|y(\tau) - x_j(\tau)\| \le l_j$ is satisfied for some $j \in \{1, 2, \dots\}$ and some time $\tau \in [0, \theta]$.*

Research problem: *In the game G, find sufficient condition for completion of pursuit.*

We define the half space

$$\Phi_j = \left\{ \alpha \in l_2 : 2\langle y_0 - x_{j0}, \alpha\rangle \le \theta\left(\rho_j^2 - \sigma^2\frac{\theta^3}{3}\right) + \|y_0\|^2 - \left\|x_{j0}\right\|^2 \right\}.$$

3. MAIN RESULT

In this section, we present the main result of the paper.

Theorem 3.1. *If $y(\theta) \in \Phi_j$, then pursuit can be completed in the game G.*

Proof To prove this theorem, we first introduce dummy pursuer with state variable z and motion described by the following equation.

$$\dot{z}(t) = w(t),\ z(0) = x_{j0},$$

where the control function $w(t)$ is such that

$$\left(\int_0^\theta \|w(t)\|^2\,dt\right)^{\frac{1}{2}} \le \bar{\rho} = \rho_j + \frac{l_j}{\sqrt{\theta}}.$$

Clearly, $\bar{\rho} > \rho_j$ and $(\bar{\rho} - \rho_j)\sqrt{\theta} = l_j$ for all $j \in J$.

We construct the strategy of the dummy pursuer as follows:

$$w(t) = \begin{cases} \frac{y_0 - x_{j0}}{\theta} + (\theta - t)v(t), & 0 \le t \le \theta, \\ 0, & t > \theta. \end{cases} \tag{11}$$

To show that the strategy (11) is admissible, we use the fact that $y(\theta) \in \Phi_j$. This means

$$2\langle y_0 - x_{j0}, y(\theta)\rangle \le \theta\left(\rho_j^2 - \sigma^2\frac{\theta^3}{3}\right) + \|y_0\|^2 - \|x_{j0}\|^2 \tag{12}$$

In accordance with the inequality (12) and using the state equation of the evader (10), we have:

$$\begin{aligned}
&2\left\langle y_0 - x_{j0}, \int_0^\theta (\theta - t)v(t)dt \right\rangle \qquad (13) \\
&= 2\langle y_0 - x_{j0}, y(\theta) - y_0\rangle \\
&= 2\langle y_0 - x_{j0}, y(\theta)\rangle - 2\langle y_0 - x_{j0}, y_0\rangle \\
&= 2\langle y_0 - x_{j0}, y(\theta)\rangle - 2\|y_0\|^2 + 2\langle x_{j0}, y_0\rangle \\
&\le \theta\left(\rho_j^2 - \sigma^2\frac{\theta^3}{3}\right) + \|y_0\|^2 - \|x_{j0}\|^2 - 2\|y_0\|^2 + 2\langle x_{j0}, y_0\rangle \\
&= \theta\left(\rho_j^2 - \sigma^2\frac{\theta^3}{3}\right) - \|y_0\|^2 - \|x_{j0}\|^2 + 2\langle x_{j0}, y_0\rangle \\
&= \theta\left(\rho_j^2 - \sigma^2\frac{\theta^3}{3}\right) - \left(\|y_0\|^2 + \|x_{j0}\|^2 - 2\langle x_{j0}, y_0\rangle\right) \\
&= \theta\left(\rho_j^2 - \sigma^2\frac{\theta^3}{3}\right) - \|y_0 - x_{j0}\|^2. \qquad (14)
\end{aligned}$$

Using inequality (13), we have

$$\begin{aligned}
&\int_0^\theta \|w(t)\|^2 dt \\
&= \int_0^\theta \left\| \frac{y_0 - x_{j0}}{\theta} + (\theta - t)v(t) \right\|^2 dt \\
&= \int_0^\theta \left(\left\|\frac{y_0 - x_{j0}}{\theta}\right\|^2 + 2\left\langle \frac{y_0 - x_{j0}}{\theta}, (\theta - t)v(t) \right\rangle + \|(\theta - t)v(t)\|^2 \right) dt \\
&= \int_0^\theta \frac{\|y_0 - x_{j0}\|^2}{\theta^2} ds + 2\int_0^\theta \left\langle \frac{y_0 - x_{j0}}{\theta}, (\theta - t)v(t) \right\rangle dt \\
&\quad + \int_0^\theta (\theta - t)^2 \|v(t)\|^2 dt \\
&\le \frac{\|y_0 - x_{j0}\|^2}{\theta} + \frac{2}{\theta}\left\langle y_0 - x_{j0}, \int_0^\theta (\theta - t)v(t)dt \right\rangle + \sigma^2 \int_0^\theta (\theta - t)^2 dt \\
&\le \frac{\|y_0 - x_{j0}\|^2}{\theta} + \frac{1}{\theta}\left(\theta\left(\rho_j^2 - \sigma^2\frac{\theta^3}{3}\right) - \|y_0 - x_{j0}\|^2\right) + \sigma^2\frac{\theta^3}{3} \\
&= \rho_j^2 < \bar{\rho}
\end{aligned}$$

Therefore the strategy (11) is admissible.

Suppose that the dummy pursuer z uses the strategy (11). One can easily see that $w(\theta) = y(\theta)$. Indeed,

$$\begin{aligned} z(\theta) &= x_{j0} + \int_0^\theta \left(\frac{y_0 - x_{j0}}{\theta} + (\theta - t)v(t) \right) ds \\ &= x_{j0} + \int_0^\theta \left(\frac{y_0 - x_{j0}}{\theta} \right) dt + \int_0^\theta (\theta - t)v(t)dt \\ &= x_{j0} + y_0 - x_{j0} + \int_0^\theta (\theta - t)v(t)dt \\ &= y(\theta). \end{aligned}$$

Using the strategy of the dummy pursuer, we define the strategies of the real pursuers P_j, $j \in J$ as follows:

$$u_j(t) = \frac{\rho_j}{\bar{\rho}} w(t). \quad (15)$$

The admissibility of this strategies follows from the fact that the control $w(\cdot)$ is admissible. Therefore it is left to show that

$$\left\| y(\theta) - x_j(\theta) \right\| \le l_j.$$

Indeed, using Cauchy-Schwartz inequality we have

$$\begin{aligned} \left\| y(\theta) - x_j(\theta) \right\| &= \left\| z(\theta) - x_j(\theta) \right\| \\ &= \left\| x_{j0} + \int_0^\theta w(t)dt - x_{j0} - \int_0^\theta u_j(t)dt \right\| \\ &= \left\| \int_0^\theta w(t)dt - \int_0^\theta \frac{\rho_j}{\bar{\rho}} w(t)dt \right\| \\ &\le \int_0^\theta \left\| \left(1 - \frac{\rho_j}{\bar{\rho}} \right) w(t) \right\| dt \\ &= \left(\frac{\bar{\rho} - \rho_j}{\bar{\rho}} \right) \int_0^\theta \| w(t) \| \, dt \\ &\le \left(\frac{\bar{\rho} - \rho_j}{\bar{\rho}} \right) \left[\left(\int_0^\theta 1^2 dt \right)^{\frac{1}{2}} \left(\int_0^\theta \| w(t) \|^2 \, dt \right)^{\frac{1}{2}} \right] \\ &\le \left(\frac{\bar{\rho} - \rho_j}{\bar{\rho}} \right) \bar{\rho} \sqrt{\theta} \\ &= (\bar{\rho} - \rho_j) \sqrt{\theta} = l_j. \end{aligned}$$

This complete the prove of the theorem.

Illustrative Example

Let $\rho_j = 5$, $\sigma = 8$, $\theta = 1$ in the game G. We consider the following initial positions $x_{j0} = (0, 0, \dots, 3, 0, \dots)$, $y_0 = (0, 0, \dots)$ of the players, where the number 3 is j^{th} coordinate of the point x_j. Observe that $\rho_j \sqrt{\theta} = 5$, $\sigma \frac{\theta^2}{2} = 4$, $\left\| x_{j0} - y_0 \right\| = \left(0^2 + 0^2 + \cdots + 3^2 + 0^2 + \dots \right)^{\frac{1}{2}} = 3 > 0$. We show that $y(\theta) \in \Phi_j$. It is suffices to show that the inclusion

$$B(0, \sigma \frac{\theta^2}{2}) \subset \bigcup_{j \in J} B(x_{j0}, \rho_j \sqrt{\theta}),$$

holds, where 0 is the origin. Indeed, let $z = (z_1, z_2, \dots)$ be arbitrary nonnegative point of the ball $B(0, 4) : \sum_{j=1}^\infty z_j^2 \le 16$. Then

$$\begin{aligned} \left\| z - x_{j0} \right\| &= \left(z_1^2 + \cdots + z_{j-1}^2 + (3 - z_j)^2 + z_{j+1}^2 + \dots \right)^{\frac{1}{2}} \\ &= \left(\sum_{j=1}^\infty z_j^2 + 9 - 6z_j \right)^{\frac{1}{2}} \\ &\le \left(16 + 9 - 6z_j \right)^{\frac{1}{2}} \\ &= \left(25 - 6z_j \right)^{\frac{1}{2}} \\ &\le 5. \end{aligned}$$

This means that hypothesis of our theorem is satisfied, therefore pursuit can be completed in the game G.

4. Conclusion

We have studied a simple motion pursuit differential game problem in which countable number of pursuers chase one evader in the Hilbert space l_2. Control function of the pursuers and evader are subject to integral and geometric constraints respectively. Pursuers' motions are described by 1^{st} order differential equations and that of the evader by 2^{nd} order differential equation. In this piece of research the strategies of the pursuers are constructed and sufficient condition for completion of pursuit were obtained. For further research, value of the game and optimality of the pursuit times can also be investigated.

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