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On the multiplicity order of spinnable star-like transformation semigroup $T\omega_n^*$

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Abstract

The application of graph theory has gained significant traction within the realm of the algebraic theory of semigroups. This study delves into exploring the geometric properties of the star-like transformation semigroup $\alpha\omega_n^*$, a distinctive category of transformation, and delineates a tropical graph (curve) by elucidating its algebraic and tropical structure. Through this investigation, various tropical properties are established, offering insights into the graph theory aspects of star-like spinnable $T\omega_n^*$ transformation semigroups. Consequently, the objective of this research is to delineate and characterize several tropical and combinatorial functions applicable to $T\omega_n^*$.

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1. Introduction

In the course of studying the algebraic and combinatorial features of transformation semigroups, eminent mathematicians like [1–4], have produced several intriguing findings. Their studies have resulted in useful tools that have been applied to various aspects of combinatorial mathematics. In fact, the study of semigroups of full T_n, partial P_n and partial one-one I_n has been fruitful over the years. It is worth nothing that this research work was inspired by the study of Akinwunmi *et al*. [5].

In the last fifteen years, there have been increasing connections to algebraic geometry and related fields [6]. Algebraic geometry is defined by Maclagan [7] as a branch of geometry spaces defined by polynomial equations. We distinguished star-like tropical algebraic expressions and their operations from the general (known) algebraic expressions by enclosing them in quotes, as tropical geometry, by its very nature, necessitates sound algebraic definitions before any form of forward mathematical motion can occur. This is the convention used by Mikhalkin [8].

Tropical geometry is a relatively young area that bridges algebraic and combinatorial geometry and has links to many other disciplines. It is, in turn, a piecewise-linear variation of algebraic geometry in which polynomial zero sets are turned

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into polynomial complexes [9]. Recently, Brugalle *et al.* [10] made a significant contribution to the theory, especially in the paper that serves as the basis for enumerative algebraic geometry. The tropical geometry is named in honor of Protrka [11], a Brazilian computer scientist.

The primary distinction between tropical geometry and classical geometry is the absence of an identity element in $\mathbb{R}$ in tropical addition, which results in $Max[f,n] = f$ for every $f, n \in \mathbb{R}$. Furthermore, there is no additive "inverse" for $f \in \mathbb{R}$, ruling out the possibility of tropical subtraction. As a result, it is appropriate to investigate tropical geometry using combinatorial methods. It has been demonstrated that numerous algebraically challenging procedures become simpler in a tropical environment because the objects' structures appear to be more straightforward. Due to this, tropical geometry has been widely used, particularly in dynamic programming and physics (string theory) [9].

Star-like polynomials and the geometric positions of their roots, denoted by the notations $Min[f,n]$ and $Max[f,n]$, are the primary subjects of study in tropical geometry. Two integers are tropically added when their tropical products are equal. If the set $\alpha\omega_n^*$ is commutative, associative, and admits the identity element $0_T\omega_n^* = -\infty$, it will be a star-like spinnable $T\omega_n^*$ transformation transformation with reference to tropical sum. Geogebra 5.0 was used to display the tropical curves of the new spinnable star-like $T\omega_n^*$ semigroup, find the roots and multiplicities of the star-like $T\omega_n^*$ elements, and gain certain combinatorial results and the algebraic structure of the new $T\omega_n^*$ semigroup.

Finding the universal relation is necessary due to the star-like sequence's pascal organization and combinatorial character, which in turn underlines the generality of its relevance to both mathematics and science. The study of Mikhalkin [12] shows the relationship of star-like transformation with geometry by generating structure for cyclicpoid transformation. Hence, some combinatorial and tropical star-like functions were characterized on $T\omega_n^*$ in this work. When the tropical space is smooth, the tropical star-like graph (curve) c_n*, which has dimension one, is connected. When c_n* is smooth, the tropical star-like curve is described; otherwise, it is asymmetric.

In the theory of transformation semigroups, the base set X_n needs to be completely ordered in order to define the tropical and combinatorial functions of a star-like transformation. Semigroup theory must be connected to algebraic and discrete mathematics by breaking down a complex transformation into a simpler tropical polynomial, where the composition occurs in a fixed ordered space. In the semigroup CP_n, [13] produced some insightful conclusions. Similarly, [6] examined the combinatorial functions of specific P_n subsemigroups and discovered similar outcomes. Nevertheless, comparable results for the star-like spinnable transformation characterizing the tropical and combinatorial functions on $T\omega_n^*$ have not kept pace with all these discoveries.

2. Preliminary Notes

For completeness, the basic definitions needed are below.

2.1. Star-like Semigroup

A finite semigroup is said to be star-like if $|\alpha^* f - g| \leq |\alpha^* g - f|$ such that $\mathbb{N} \cup 0 \in \mathbb{R}$ for every $f, g \in \alpha\omega_n^*$, then $\alpha\omega_n^*$ must satisfy the following axioms:

(i) $0\alpha^* = 0$

(ii) $\alpha^* 1^* = 1^* \alpha^*$

(iii) $\alpha(f+g)^* = f\alpha^* + \alpha^* g$ for every $g \in D(\alpha^*)$

(iv) $k^{-1*}(\beta^*) = \alpha^*$, for every $\alpha^*, \beta^* \in \alpha\omega_n^*$

(v) $F(\alpha^*) \leq I(\alpha^*)$, for every $\alpha^* \in \alpha\omega_n^*$

2.2. Tropical semi-field $[\alpha\omega_n^{t*}]$ with star-like properties

Contains the set $(\alpha\omega_n^{t*})$. The following axioms are true if $[\alpha\omega_n^{t*}]$ is equipped with the binary operations $''\cdot''$ and $''+''$:

(i) The operations are closed for every pair $f, n \in (\alpha\omega_n^{t*})$ to the extent that both $f \cdot n$ and $f + n$ are in $(\alpha\omega_n^{t*})$.

(ii) The operations are associative for every pair $f, n \in (\alpha\omega_n^{t*})$ and the equations $(f+n)+g = f+(n+g)$ and $(f+n)+g = f+(n+g)$ hold for all f, n, and $g \in (\alpha\omega_n^{t*})$.

(iii) Every operation has an identity element $1 \in \alpha\omega_n^{t*}$ such a way that it exists and $f \cdot 1 = f(\alpha\omega_n^{t*})$.

(iv) The operation commute: $f \cdot n = n \cdot f$ and $f + n = n + f$ hold for any $f, n \in \alpha\omega_n^{t*}$.

(v) There exist multiplicative inverses: for every element $f \in (\alpha\omega_n^{t*})$ other than zero, exists $f(-1) \in (\alpha\omega_n^{t*})$ such that $f \cdot f^{(-1)} = 1$.

(vi) Multiplication distributes over addition: $\forall\ f, n, g \in (\alpha\omega_n^{t*})$; $f \cdot (n+g) = f \cdot n + f \cdot g$.

2.3. Star-like Curve (c_n^*)

Let $P(T)^*(f,n)_n = \sum_{i}, ja_{i,j} f^i n^j$ be a tropical star-like polynomial, such that c_n^* defined by $P(T)^*(f,n)_n$ is the set of points $(f_0, n_0) \in \mathbb{R}^2$ such that there exists pairs $(i, j) \neq (k, l)$ satisfying $P(T)^*(f_0, n_0)_n = a_{i,j} + if_0 + jn_0 = a_{k,l} + kf_0 + ln_0$.

2.4. Star-like Root (R_n^*)

A spinnable root of a star-like polynomial $P(T)^*(f)_n$ exists if there exists a star-like polynomial $Q(T)^*(f)_n$ such that $P(T)^*(f)_n = (f - f_0), Q(T)^*(f)_n$. All points $f_0^* \in T\omega_n^*$ for which the curve of $P(T)^*(f)_n$ has an angle at f_0 are star-like roots R_n^* of star-like polynomials $P(T)^*(f)_n$.

2.5. Star-like Multiplicity Order (U_{α^*})

Suppose that c_1^* and c_2^* are two tropical star-like graphs (curves) that cross in a countable number of places, all of which are far from the star-like vertices of the two curves. If c_1^* and c_2^* intersect at k, then k is the tropical star-like multiplicity (U_{α^*}) with $c_1^* \bigcap c_2^*$, as well as the area of the star-like triangle opposite k in the double sub-segment of $c_1^* \bigcup c_2^*$.

2.6. Star-like Faces H^*

This is a star-like flat or curving surface of a star-like transformation semigroup.

2.7. Star-like Edges G^*

This is a star-like disk point where two or more star-like faces meet.

2.8. Star-like Vertices V^*

This is a star-like transformation corner where two or more star-like edges meet.

2.9. Star-like Spinnable

Any transformation $\alpha_{i,j}^* \in T\omega_n^*$ is defined as a star-like spinnable if it adheres to the star-like operator and star-like folding principle. This classification is assigned when the faces H^*, edges G^*, and vertices V^* converge at a star-like disk point with a 360-degree angular measure, and the transformation satisfies both combinatorial and tropical properties.

3. Methoology

Since not all transformation semigroups fulfill the star-like operator, it would be more reproducible to apply the new operator to additional transformation semigroup areas. The following outlines the methodical process used to create the tropical graph and the methodology portion of the work:

(i) Decompose any star-like transformation $\alpha_{i,j}^* \in T\omega_n^*$ to a linear or quaratic algebraic equation such that

$$\alpha_{i,j} + \alpha_{i,j}^2 + \alpha_{i,j}^3 + \ldots + \alpha_{i,j}^n = |T\omega_n^*|.$$

(ii) Obtain the maximum and minimum values of $\alpha_{i,j}$.

(iii) Factorize the values obtained in (ii) to obtain the tropical equation.

(iv) Compute the tropical equations in (iii) using Geogebra 5.0 to derive the tropical root and multiplicity.

(v) Use the star-like operator to analyze the roots and multiplicities in (iv) to generate the star-like tropical graph of $\alpha_{i,j}^* \in T\omega_n^*$ in each order of transformation.

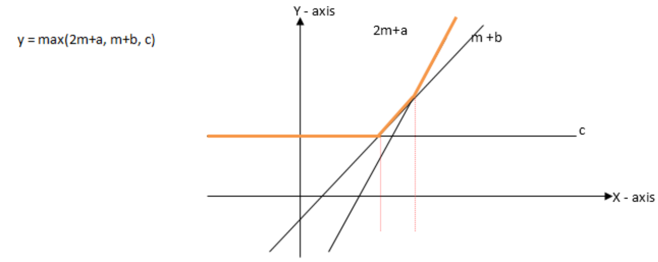

Figure 1. Tropical graph of $\alpha^* \in T\omega_n^*$.

4. Main Results

The results obtained in this work show spinnable features in the application of transformation semigroups to tropical algebra with some tropical and combinatorial properties.

Let $T\omega_n^*$ be a star-like spinnable transformation semigroup and $X_n = 1, 2, 3, \ldots$ be a separate finite n-element set:

$$|\alpha^* f - g| \leq |\alpha^* g - f|. \quad (1)$$

For every $f \in D(\alpha^*)$ and $g \in I(\alpha^*)$ where $i \geq n \geq 1$; $n \in \mathbb{N}$. $A \longrightarrow X_n$, where $A = f_1, f_2, f_3, \ldots f_n$ is a subset of $T\omega_n^*$, and the element of α^* was acquired, is then transformed into an array, which is

$$\alpha^* = \begin{pmatrix} f_1 & f_2 & f_3 & \ldots & f_n \\ \alpha f_1 & \alpha f_2 & \alpha f_3 & \ldots & \alpha f_n \end{pmatrix}. \quad (2)$$

The tropical star-like polynomial $P(T)^*(f) = \sum_{i=0}^{d} a_i^* f$ where $a_i \in P(T)^*$ from the structure of $T\omega_n^*$ star-like transformation semigroup was given in equation (3):

$$''a + f + 3f^{a+1} + \ldots + a_i f^{a_i+1}'' = Max\,[a, f, (a+1)f + 3,$$
$$\ldots, (a_i + 1)f + a_i]. \quad (3)$$

The tropical curve of a star-like transformation $\alpha^* : A \longrightarrow X_n$ was generated using Geogebra 5.0 and is shown in Figure 1. The tropical lines reflect some of the well-known geometric features of 'classical' lines in the plane. Suppose that $\alpha^* \in T\omega_n^*$ is a star-like spinnable transformation, and if $D(\alpha^*) = I(\alpha^*)$ under the composition of mapping, equation (4) lists all the elements of $T\omega_3^*$:

$$|T\omega_3^*| = \begin{cases} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \\ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \end{pmatrix}, \\ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, \\ \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \end{pmatrix}. \end{cases} \quad (4)$$

The classical and tropical polynomial of equation (4) was obtained in each case:

$$\begin{aligned}
\alpha_1^* &= \begin{array}{cc} & f \\ \begin{array}{c} f \\ 1 \end{array} & \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{array} \\
&= f^2 + f \\
&= ''f^2 + f'' \\
P(T)^*(\alpha_1^*) &= Max\,[2f, f].
\end{aligned}$$

$$\begin{aligned}
\alpha_2^* &= \begin{array}{cc} & \begin{array}{cc} f & 1 \end{array} \\ \begin{array}{c} f \\ 1 \end{array} & \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \end{array} \\
&= f^2 + 2f + f + 2 \\
&= ''f^2 + 3f + 2'' \\
P(T)^*(\alpha_2^*) &= Max\,[2f, 3 + f, 2].
\end{aligned}$$

$$\begin{aligned}
\alpha_3^* &= \begin{array}{cc} & \begin{array}{cc} f & 1 \end{array} \\ \begin{array}{c} f \\ 1 \end{array} & \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \end{array} \\
&= f^2 + 2f + 2f + 1 \\
&= ''f^2 + 4f + 1'' \\
P(T)^*(\alpha_3^*) &= Max\,[2f, 4 + f, 1].
\end{aligned}$$

$$\begin{aligned}
\alpha_4^* &= \begin{array}{cc} & \begin{array}{cc} f & 1 \end{array} \\ \begin{array}{c} f \\ 1 \end{array} & \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \end{array} \\
&= f^2 + 2f + 2f + 2 \\
&= ''f^2 + 4f + 2'' \\
P(T)^*(\alpha_4^*) &= Min\,[2f, 4 + f, 2].
\end{aligned}$$

$$\begin{aligned}
\alpha_5^* &= \begin{array}{cc} & \begin{array}{ccc} f^2 & f & 1 \end{array} \\ \begin{array}{c} f \\ 1 \end{array} & \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \end{pmatrix} \end{array} \\
&= f^3 + 2f^2 + 3f + f^2 + 3f + 3 \\
&= ''f^3 + 3f^2 + 6f + 3'' \\
P(T)^*(\alpha_5^*) &= Min\,[3f, 3 + 2f, 6 + f, 3].
\end{aligned}$$

$$\begin{aligned}
\alpha_6^* &= \begin{array}{cc} & \begin{array}{ccc} f^2 & f & 1 \end{array} \\ \begin{array}{c} f \\ 1 \end{array} & \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \end{array} \\
&= f^3 + 2f^2 + 3f + f^2 + 2f + 3 \\
&= ''f^3 + 3f^2 + 5f + 3'' \\
P(T)^*(\alpha_6^*) &= Max\,[3f, 3 + 2f, 5 + f, 3].
\end{aligned}$$

$$\begin{aligned}
\alpha_7^* &= \begin{array}{cc} & \begin{array}{ccc} f^2 & f & 1 \end{array} \\ \begin{array}{c} f \\ 1 \end{array} & \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \end{pmatrix} \end{array} \\
&= f^3 + 2f^2 + 3f + 2f^2 + 2f + 3 \\
&= ''f^3 + 4f^2 + 5f + 3'' \\
P(T)^*(\alpha_7^*) &= Min\,[3f, 4 + 2f, 5 + f, 3].
\end{aligned}$$

$$\begin{aligned}
\alpha_8^* &= \begin{array}{cc} & \begin{array}{ccc} f^2 & f & 1 \end{array} \\ \begin{array}{c} f \\ 1 \end{array} & \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \end{pmatrix} \end{array} \\
&= f^3 + 2f^2 + 3f + 2f^2 + 3f + 2 \\
&= ''f^3 + 4f^2 + 6f + 2'' \\
P(T)^*(\alpha_8^*) &= Min\,[3f, 4 + 2f, 6 + f, 2].
\end{aligned}$$

$$\begin{aligned}
\alpha_9^* &= \begin{array}{cc} & \begin{array}{ccc} f^2 & f & 1 \end{array} \\ \begin{array}{c} f \\ 1 \end{array} & \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 3 \end{pmatrix} \end{array} \\
&= f^3 + 2f^2 + 3f + 2f^2 + 3f + 3 \\
&= ''f^3 + 4f^2 + 6f + 3'' \\
P(T)^*(\alpha_9^*) &= Min\,[3f, 4 + 2f, 6 + f, 3].
\end{aligned}$$

$$\begin{aligned}
\alpha_{10}^* &= \begin{array}{cc} & \begin{array}{ccc} f^2 & f & 1 \end{array} \\ \begin{array}{c} f \\ 1 \end{array} & \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \end{array} \\
&= f^3 + 2f^2 + 3f + 3f^2 + 2f + 1 \\
&= ''f^3 + 5f^2 + 5f + 1'' \\
P(T)^*(\alpha_{10}^*) &= Max\,[3f, 5 + 2f, 5 + f, 1].
\end{aligned}$$

$$\begin{aligned}
\alpha_{11}^* &= \begin{array}{cc} & \begin{array}{ccc} f^2 & f & 1 \end{array} \\ \begin{array}{c} f \\ 1 \end{array} & \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 3 \end{pmatrix} \end{array} \\
&= f^3 + 2f^2 + 3f + 3f^2 + 2f + 3 \\
&= ''f^3 + 5f^2 + 5f + 3'' \\
P(T)^*(\alpha_{11}^*) &= Min\,[3f, 5 + 2f, 5 + f, 3].
\end{aligned}$$

$$\begin{aligned}
\alpha_{12}^* &= \begin{array}{cc} & \begin{array}{ccc} f^2 & f & 1 \end{array} \\ \begin{array}{c} f \\ 1 \end{array} & \begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & 1 \end{pmatrix} \end{array} \\
&= f^3 + 2f^2 + 3f + 3f^2 + 3f + 1 \\
&= ''f^3 + 5f^2 + 6f + 1'' \\
P(T)^*(\alpha_{12}^*) &= Min\,[3f, 5 + 2f, 6 + f, 1].
\end{aligned}$$

$$\begin{aligned}
\alpha_{13}^{*} &= \begin{matrix} & f^2 & f & 1 \\ f & \multicolumn{3}{c}{\left(\begin{matrix} 1 & 2 & 3 \\ 3 & 3 & 2 \end{matrix}\right)} \\ 1 & & & \end{matrix} \\
&= f^3 + 2f^2 + 3f + 3f^2 + 3f + 2 \\
&= ''f^3 + 5f^2 + 6f + 2'' \\
P(T)^*(\alpha_{13}^{*}) &= Min\,[3f, 5 + 2f, 6 + f, 2].
\end{aligned}$$

$$\begin{aligned}
\alpha_{14}^{*} &= \begin{matrix} & f^2 & f & 1 \\ f & \multicolumn{3}{c}{\left(\begin{matrix} 1 & 2 & 3 \\ 3 & 3 & 3 \end{matrix}\right)} \\ 1 & & & \end{matrix} \\
&= f^3 + 2f^2 + 3f + 3f^2 + 3f + 3 \\
&= ''f^3 + 5f^2 + 6f + 3'' \\
P(T)^*(\alpha_{14}^{*}) &= Min\,[3f, 5 + 2f, 6 + f, 3].
\end{aligned}$$

All points R_n^* of $T\omega_n*$ for which the graph (curve) of $P(T)^*(f)$ has a diagonal at R_n^* are the star-like roots of a tropical polynomial $P(T)^*(f)$. Given the multiplicity U_α^* of $P(T)^*(f)$ as the difference in slope between any two segments meeting at roots $R_i(n)^*$ corresponding corners, thus, the tropical graph (curve) of $|T\omega_3^*|$ was created using the roots R_n^* and multiplicities U_α^* in

$$M(r_i^*) = |r_i^* - u_{\alpha_i+1}^*| \leq |\alpha^* f - g| \leq |\alpha^* g - f| = |u_{\alpha_i}^* - u_{\alpha_i+1}^*|,$$

where r_i^* is $\in R_n^*$ and u_i^* is $\in U_\alpha^*$ of $T\omega_n^*$. Since multiplicity order of tropical properties for small size of n, where $\begin{pmatrix} n \\ U_\alpha^* \end{pmatrix}$ is the number of U_α^* subsets of a n-dimensional set; $n \geq 0$, then $\begin{pmatrix} n \\ U_\alpha^* \end{pmatrix} = 0$ if $U_\alpha^* \geq n$, or For any $n \geq 1$, the star-like recurrence relation for U_α^* of $T\omega_n^*$ is given as:

$$\begin{pmatrix} n \\ U_\alpha^* \end{pmatrix} = \begin{pmatrix} n-1 \\ U_\alpha^* \end{pmatrix} + \begin{pmatrix} n-1 \\ U_\alpha^* - 1 \end{pmatrix},$$

with initial conditions $\begin{pmatrix} 0 \\ U_\alpha^* \end{pmatrix} = 0$ if $U_\alpha^* \neq 0$ and $\begin{pmatrix} 0 \\ 0 \end{pmatrix} = 1$. The star-like recurrence relation allows for efficient calculation of minuscule multiplicities through the tropical route, such that the zeroth row of the tropical path has height one and only contains the number 1. Below that, the first row has two 1, one below and to the left of the 1 in the zeroth row. The second row has three entries: a 1 below and to the left of the leftmost 1 in the first row, a 1 below and to the right of the rightmost 1 in the first row, and a 2 in the center. Each successive row comprises a constant element of 1 and one more item than the former row, beginning with a 1 below and to the left of the leftmost with $U_\alpha^* \in T\omega_n^*$. see Figure 2.

Consider $U_\alpha^* \geq 0$ sing the factorial star-like operator, defined combinatorially as the number of ways to arrange n distinct values of U_α^* and algebraically by $n! = n(n-1)(n-2)\ldots(3)(2)(1)$ for $n \geq 1$ with $0! = 1$

$$\begin{pmatrix} n \\ U_\alpha^* \end{pmatrix} = \frac{n(n-1)\ldots(n-(U_\alpha^*-1))}{U_\alpha^*!} = \frac{n!}{U_\alpha^*!}(n-U_\alpha^*)!.$$

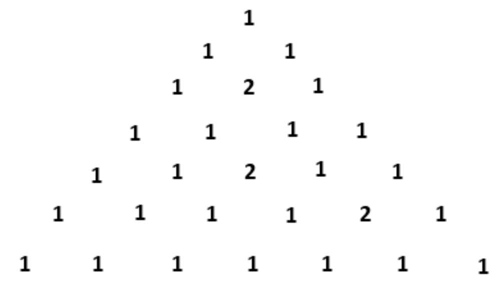

Figure 2. Multiplicity order: $U_\alpha^* \in T\omega_n^*$.

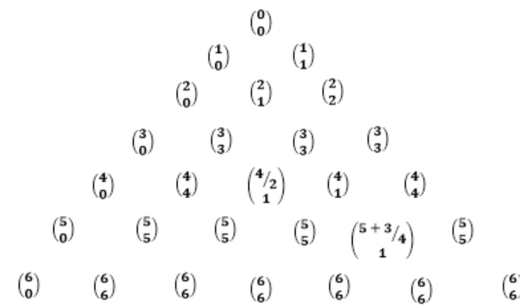

Figure 3. Pascal combinatorial sequence: $U_\alpha^* \in T\omega_n^*$.

see Figure 3.

As a result, $n(n-1)\ldots n(U_\alpha^* - 1)$ is clearly the number of ordered lists of U_α^* with star-like identity difference 1 occurring in multiplicity order for all $n \in \mathbb{N}$.

Lemma 1:
Suppose $\alpha\omega_n^* = T\omega_n^*$, then

$$|\, T\omega_n^* \,| = \frac{X[(x^3) - 91(x^2 + 292(x) - 389] + 183}{3}.$$

for all $x, n \in N_i \geqslant 2$.

Proof
Let $T\omega_n^* \subset \alpha\omega_n^*$ be star-like spinnable transformation, there exist maximum degree $k \in T\omega_n^*$ such that $\alpha\omega_n{}^*k = \emptyset$ whenever $|\, T\omega_n^* \,| = \emptyset$. By the star-like operator,

$$|T\omega_n^*| \leq |U_\alpha^*|$$

such that $\alpha_u^* \in T\omega_u^*$ (star-like transformation with finite multiplicity order) is bijective. Adopting star-like folding principle, it was observed that nth order of $D(\alpha^*)$ can be selected from the X_n in $\begin{pmatrix} n \\ u \end{pmatrix}$ ways where multiplicity relation $U_{\alpha^*} \geqslant 2$ such that

$$U_\alpha^* = \omega_0(x)^4 + \omega_1(x)^3 + \omega_2(x)^2 + \omega_3(x) + \omega_4. \quad (5)$$

generate star-like algebraic system

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 16 & 8 & 4 & 2 & 1 \\ 81 & 27 & 9 & 3 & 1 \\ 256 & 64 & 16 & 4 & 1 \\ 625 & 125 & 25 & 5 & 1 \end{pmatrix} \begin{pmatrix} \omega_0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \\ 28 \\ 97 \\ 346 \end{pmatrix}, \quad (6)$$

to obtain

$$\omega_0 = \frac{11}{3}, \omega_1 = -\frac{19}{3}, \omega_2 = \frac{292}{3}, \omega_3 = -\frac{389}{3}, \omega_4 = 61.$$

Substitute the values of ω_n^* in equation (6) to get

$$T\omega_n^* \mid = \frac{u[(u)^3 - 91(u)^2 + 292(u) - 389] + 183}{3}, \tag{7}$$

Where $U_\alpha^* = \mid T\omega_n^* \mid, (n-1) = u$ for all $u, n \in X_n, n \geqslant 2$.

Lemma 2:
Suppose $U_\alpha^* = T\omega_n^*$ then $\mid T\omega_n^* \mid = \binom{2^f}{f-n+1} n, f \in T\omega_n^*$ for all $n, f \leq 1$.
Proof:
Assume there is
$\alpha^* \in T\omega_n^*$, then a star-like multiplicity disk operator can be selected from X_n in $\binom{2f}{f-n+1}$ ways in a one-to-one model. According to Lemma(1), the order of multiplicity class of $T\omega_n^*$ is

$$\frac{f[(f^3) - 6(f)^2 + 3(f) - 18] + 24}{\beta^3 + \beta^2}.$$

with $\omega_0 = -\frac{1}{2}, \omega_1 = -\frac{1}{2}, \omega_2 = \frac{23}{12}, \omega_3 = -\frac{3}{2}$ and $\omega_4 = 2$ which equivalent to

$$\binom{2f}{f-n+1}.$$

Lemma 3:
If $L(p,q)$ denote number of sequence path from $(0,0)$, to (p,q) with row-p and column-q, any given $\alpha^* \in T\omega_n^*$ form pascal combinatorial array of sequence.
Proof:
Suppose $\alpha^* \in T\omega_n^*$, with x-row and y-column of sequence, for all $N_i = \{i, i+1, i+3, ...\}, (i = \{0, 1, 2, ...\})$. Then if L(p,0)=K, L(0,q)= K where (0,0) is the Star-like origin of all path of the pascal combinatorial sequence obtained by $\alpha^* \in T\omega_n^*$, gives

$$L(p,q) = L(p, q-k) + L(p-k, q).$$

such that,

$$\binom{p+q}{p} + \binom{p+q}{q} = \frac{p+q}{p!q!}, \tag{8}$$

Where k is the star-like arbitrary constant.

Theorem 4:
The following statement is true for any star-like transformation α^* in $T\omega_n^*$:

(i) Any $\alpha^* \in T\omega_n^*$ has a maximum element $k(\alpha^*)$.

(ii) $\mid T\omega_n^* \mid = \bigcup_{q^*=2}^{n} \binom{n+2q^*}{3q^*}$.

(iii) $F(n, q^*, k^*) = \left(2^{(n-q^*)+k^*} - 1\right) : n, k^* \geqslant q^* \geqslant 2$.

Proof:
$(i) \Longrightarrow (ii)$
Suppose $F(n, q^*, k^*)$ are a star-like function with a distinct non negative integer $X_i = \{i, i+1, i+2, ...\}, i = \{0, 1, 2, ...\}$ such that $T\omega_n^*$ contains a transformation $\alpha^* \in T\omega_n^*$. Then $i \in X_i$ of $I(\alpha^*)$ can be chosen in $\binom{q^*}{n}$ ways.

Since α^* is a star-like bijective map, $T\omega_n^*$ is replaceable. If $I(\alpha^*) = 0 \mid q^*(\alpha^*) \mid = 1$ but if $n = q^* = k^*$; $\mid q^*(\alpha^*) \mid = q^* + 1$ for each value of $\alpha^* \in T\omega_n^*$ such that

$$\mid T\omega_n^* \mid = \binom{n+2q^*}{3q^*}. \tag{9}$$

$(ii) \Longrightarrow (iii)$ If $\alpha \in T\omega_n^*$ is star-like, there exist finitely many elements such that $D(\alpha^*) = I(\alpha^*)$ where $\lambda_n^* \bigcap r_n$ form a total of

$$\bigcup_{n=1}^{k} \left(\frac{2^{(n-q^*)+1}}{2^{(n-k^*)+1}} - 1\right) \binom{n+2q^*}{3q^*} = F(n, q^*, k^*). \tag{10}$$

$(iii) \Longrightarrow (i)$
Suppose $k(\alpha^*)$ denote maximum element in $I(\alpha^*)$ and $T\omega_n^* \subseteq \alpha\omega_n^*$ with the composition of a star-like transformation.

$$r^* = \begin{pmatrix} u_1 & u_2 & u_0 \\ k_1 & k_2 & k_0 \end{pmatrix} \in k(\alpha^*). \tag{11}$$

Such that there exists another element $\gamma^* \in k(\alpha$ with $r^* \leq \gamma^*$and$I_0 < \gamma^*$. Obviously, r^* and γ^* are bijective, then

$$(r^* I\omega_n^*) \bigcap (\gamma^* I\omega_n^*) = k(\alpha^*).$$

Theorem 5:
Let $U_\alpha^* i$ be a set of star-like multiplicity order and $\alpha^* \in T\omega_n^*$ such that $D(\alpha^*) \subseteq I(\alpha^*)$, then $V^*(T\omega_n^*)$ is $\frac{1}{2}V^* = \frac{G^* - H^*}{2} + \beta^*; G^* \in D(T\omega_n^*), H^* \in I(T\omega_n^*)$.
Proof:
Suppose $T\omega_n^*$ is a star-like spinnable transformation with a composite relation,

$$x_i^* + y_j^* + z_k^* = T\omega_n^*,$$

such that for any star-like multiplicity order U_α^*, there exists a star-like disk operator $\beta^* = 2$, with

$$H^* + V^* = G^* + 2. \tag{12}$$

Then

$$\mid T\omega_n^* \mid = |\alpha^* f - g| \leq |\alpha^* g - f|. \tag{13}$$

Where

$$\frac{1}{2}V^* = |\alpha^* g - f| \leq |\alpha^* f - g|. \tag{14}$$

Since $T\omega_n^*$ satisfy (1), it shows that

$$\frac{1}{2}V^* = \frac{G^* - H^*}{2} + \beta^*. \tag{15}$$

Therefore $\frac{1}{2}U_\alpha^* i(T\omega_n^*)$ implies $V^* = G^* - H^* + \beta^*$ which gives star-like vertices of any $\alpha_n^* \in T\omega_n^*$ transformation multiplicity with disk operator β^*.

Theorem 6:
Suppose $T\omega_n^*$ is a star-like replaceable transformation with $U_\alpha^* \in T\omega_n^*$ then any $\beta^* \in U_\alpha^*$ is equal if and only if

(i) U_α^* is spinnable and

(ii) U_α^* is replaceable.

Proof
$(i) \Longrightarrow (ii)$
Let $U_\alpha^* \in T\omega_n^*$ be the order of multiplicity of star-like replaceable transformations. If any $U_\alpha^* i$ is spinnable then we need to show that it is replaceable with $\beta^* \in U_\alpha^*$ such that for all sides of any spinnable transformation with star-like coordinate

$$i = j = k \in \beta^* \subseteq U_\alpha^*.$$

gives a star-like model of $T\omega_n^*$ shown in Figure 4.

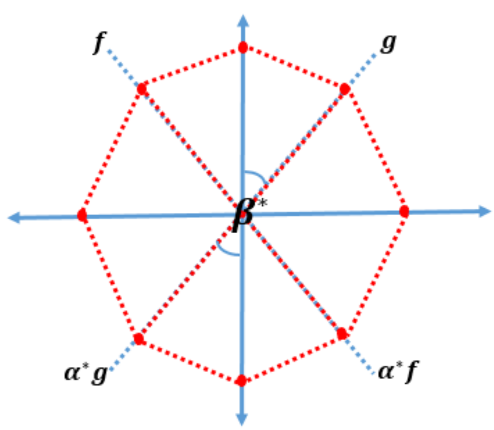

Figure 4. Star-like spinnable model.

By virtue of V^* of $\alpha_n^* \in T\omega_n^*$ such that $U_\alpha^* n = \alpha_n^*$ we have

$$V^* = \frac{1}{2} i \times j \times k. \tag{16}$$

And

$$\frac{1}{2} = |\alpha^* g - f| \leq |\alpha^* f - g|. \tag{17}$$

Then $U_\alpha^* T\omega_n^*$ is spinnable.
$(ii) \Longrightarrow (i)$
Suppose $U_\alpha^* = T\omega_n^*$ then $|\ T\omega_n^*\ | = \emptyset$. By the choice of U_α^* to be the angular order of a star-like multiplicity relation of any $\alpha^* \in T\omega_n^*$ the disk operator $\beta_n^* \in U_\alpha^*$ is a star-like bijective with finitely many replaceable relations U_n of order n, such that

$$U_\alpha^* = \omega_0(u)^4 + \omega_1(u)^3 + \omega_2(u)^2 + \omega_3(u) + \omega_4$$

generate replaceable star-like multiplicity angle of $T\omega_n^*$ in Figure 5

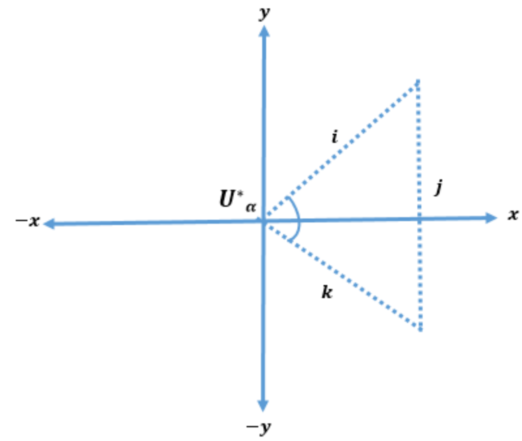

Figure 5. Star-like multiplicity angle of $T\omega_n^*$.

where $U_\alpha^* =|\ T\omega_n^*\ |$. Therefore, adopting star-like folding principle , the order of domain of $U_\alpha^* \omega_n^*$ can be chosen for $X_n = \{1, 2, ...\}$ in $\binom{n}{U_{\alpha_n}^* - 1}$ ways, which also contain finitely many replaceable star-like transformations.

Theorem 7:
Let $\beta^* \in T\omega_n^*$ then $|R_n^*| = \binom{w-u}{u-1} = \binom{w-(u-1)}{w-u}$.

Proof:
let $X_n = \{1, 2, 3, ...\}$ then $D(\beta^*) \subseteq X_n$ where $\beta^* \subseteq U_\alpha^* i$. If

$$F(w, u) = |\ \beta^* \in T\omega_n^* : R_n^*(\beta^*)\ |=|\ I(\beta^*)\ |= u$$

Then $R_0^* \in D(\beta^*)$ such that

$$\beta_{x_0}^* \leq \beta_{ny_0}^* \Longrightarrow \beta_{x_0}^* \leq y_0. \tag{18}$$

Implies

$$\beta_{nx_0y_0}^* = \binom{e^0}{R_0^*}$$

So, x_0 has $w - e + 1$ disk operator degree of freedom with order

$$|\ \beta^*\ |= \binom{w-u}{u-1},$$

where $w = u = 2$ which gives $\binom{w-u}{u-1} = 1$. Since for any choice of star-like multiplicity order, is a subset of all star-like transformations, then $\beta^* \in T\omega_n^* : R_n^*(\beta_n^*) = u$. Therefore, irrespective of the value of $w \geq 2$ whenever $u = (w-1)$, there is exactly 2 star-like root order with many disk operators.

$$|\ R_n^*\ |= \binom{w-(u-1)}{w-u}.$$

Theorem 8:
Let U_α^* be the multiplicity order of tar-like transformation with interior angles, $\lambda_i^*, ...\lambda_n^*$, Then

$$Area(U_\alpha^*) = (\bigcup_{n=1}^{\infty} \lambda_n) - (n-2)\pi.$$

Proof:
Let R_n^* be a star-like root in U_α^* with origin in the interior of U_α^* projecting the boundary of U_α^* on $T\omega_n^*$ using the star-like function

$$f(G^*, H^*, V^*) = \frac{(G^*, H^*, V^*)}{\sqrt{G^{n*} + H^{n*} + V^{n*}}}. \tag{19}$$

By star-like folding principle, V^* of $\beta^* \in U_\alpha^*$ go to part on $T\omega_n^*$, H^* go to part of great circles in β^* and G^* go to polygon while the union of $(U_{\alpha_1}^*, + \cdots U_{\alpha_n}^*)$ multiplicities degree converge on $T\omega_n^*$. Then

$$Area(U_{\alpha_1}^*)+Area(U_{\alpha_2}^*)+\cdots+Area(U_{\alpha_n}^*) = Area(T\omega_n^*). \tag{20}$$

Let R_i^* be the number of star-like roots of $U^*{}_{\alpha i}$ and $\lambda_{ij} for j = 1, \cdots n_i$ be the interior angles such that

$$\frac{1}{2}\,V^* = \frac{G^* - H^*}{2} + \beta^* \; where \; \beta^* = 2n.$$

implies;

$$V^* = G^* - H^* + 4n. \tag{21}$$

Since $R_i^* \subseteq U^*{}_{\alpha i}$ and λ_{ij} for $j = 1, \cdots R_i^*$ then

$$\bigcup_{R_i^*=1}^{i,j} (\bigcup_{j=1}^{n} \lambda_{ij} - R_i^* \pi + 2n\pi) = 4n\pi$$

in which two star-like tropical corners share one edge gives

$$\bigcup_{i=j}^{n} n_j \pi = 2n\pi G^*. \tag{22}$$

Also, if the sum of angles at every star-like multiplicity order is $2n\pi$ then

$$\bigcup_{i=1}^{n} \bigcup_{j=1}^{n} \lambda_{ij} = 2n\pi V^*. \tag{23}$$

Therefore,

$$2n\pi V^* - 2n\pi G^* + 2n\pi H^* \leq Area(U_\alpha^*) = (\bigcup_{n=1}^{\infty} \lambda_n) - (n-2)\pi = 4n\pi.$$

5. Conclusion

The research converted the star-like transformation semigroup $T\omega_n^*$ into a star-like polynomial and produced a tropical algebra, which was then used to build a star-like graph. The output of Theorems 5 through 8 is predicated on multiplicity star-like order, whose form was thought to be solid, stiff, and spinnable at various axes with a constant star-like angle unless the root rotates. The paper suggests using the findings to solve issues in code theory, genetics, and other physical sciences.

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