



The Solution of a Mathematical Model for Dengue Fever Transmission Using Differential Transformation Method

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Abstract

Differential Transformation Method (DTM) is a very effective tool for solving linear and non-linear ordinary differential equations. This paper uses DTM to solve the mathematical model for the dynamics of Dengue fever in a population. The graphical profiles for human population are obtained using Maple software. The solution profiles give the long term behavior of Dengue fever model which shows that treatment plays a vital role in reducing the disease burden in a population.

Keywords: Dengue Fever, Mathematical Model, Differential Transformation Method, Ordinary Differential Equations

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1. Introduction

Dengue fever is an infectious vector borne disease spreading in tropical and subtropical countries with more than 50 million dengue fever cases per year. It is transmitted to humans by the bite of infected aedes mosquitoes. The major vector, *Aedes aegypti*, thrives in tropical regions, mainly in urban areas, closely linked to human populations providing artificial water-holding containers as breeding sites. A second potential vector, *Aedes Albopictus*, resides in temperate regions (North America and Europe), where it may give rise to occasional dengue outbreaks. [1, 2].

As part of awareness campaigns, different kinds of precautions have been suggested towards preventing mosquito's bite. Some of the precautions that can be taken are: to keep home, environment and surrounding clean, to remove all stagnant water and containers, to cover all containers properly to prevent

dengue mosquito breeding there, to use mosquito repellents to avoid mosquito bite, to use mosquitoes net around bed while sleeping etc. [1]. Different studies have shown the importance of mathematical approaches in understanding dengue disease transmission and evaluating the effectiveness and/or cost-effectiveness of control strategies [1].

In recent years, the field of public health has benefited tremendously from the use of mathematical models to study the spread of infectious disease. Most epidemiological models are represented using systems of non-linear ordinary differential equations [3]. Differential Transformation Method is a semi-analytical method of solving both linear and nonlinear system of ordinary differential equations (ODE) to obtain approximate series solutions. This method which was derived from the Taylor's series expansion has been used to solve problems in Mathematics and Physics [4], fractional differential- algebraic equations [5], fourth-order parabolic partial differential equations [6], fractional-order integrodifferential equations [7], differential equation [8] and problems in epidemic models [9].

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2. Materials and Methods

2.1. Model Formulation

Two populations consisting of human and vector population will be considered in this work. The model sub-divides these populations into a number of mutually-exclusive compartments, as given below. The total population of human and vectors is divided into the following mutually exclusive epidemiological classes, namely, susceptible humans ($S_H(t)$), humans with dengue in latent stage ($E_1(t)$), humans with dengue ($I_1(t)$), humans treated of dengue ($R_1(t)$), susceptible vectors ($S_V(t)$), vectors with latent dengue ($E_V(t)$), vectors with dengue ($I_V(t)$).

2.2. Derivation of Model Equations

Let $N_H(t)$ and $N_V(t)$ denote the total number of humans and vectors at time t , respectively. Hence, we have that,

$$N_H(t) = S_H(t) + E_1(t) + I_1(t) + R_1(t)$$

and

$$N_V(t) = S_V(t) + E_V(t) + I_V(t)$$

Susceptible humans are recruited at a rate Λ_H while the susceptible vectors are recruited at a rate Λ_V .

Susceptible humans contract dengue at a rate

$$\lambda_{DV} = \frac{\beta_{VH}(\eta_V E_V + I_V)}{N_H},$$

where $\eta_V < 1$, this accounts for the relative infectiousness of vectors with latent dengue I_V compared to vectors in the I_V class.

Susceptible vectors acquire dengue infection from infected humans at a rate

$$\lambda_{DH} = \frac{\beta_{HV}(\eta_A E_1 + \eta_B I_1)}{N_H},$$

where $\eta_A < \eta_B$ this accounts for the relative infectiousness of humans with latent dengue E_1 compared to humans in the I_1 class [10].

The model equations for dengue disease transmission incorporating treatment as a control measure are given below:

$$S_H = \Lambda_H - \mu_H S_H - \lambda_{DV} S_H$$

$$E_1 = \lambda_{DV} S_H - (\gamma_1 + \mu_H) E_1$$

$$I_1 = \gamma_1 E_1 - (\tau_1 + \mu_H + \delta_{D1}) I_1$$

$$R_1 = \tau_1 I_1 - \mu_H R_1$$

$$S_V = \Lambda_V - \mu_V S_V - \lambda_{DH} S_V$$

$$E_V = \lambda_{DH} S_V - (\gamma_V + \mu_V) E_V$$

$$I_V = \gamma_V E_V - (\mu_V + \delta_{HV}) I_V$$

2.3. Differential Transformation Method (DTM)

An arbitrary function $f(t)$ can be expanded in Taylor series about a point $t = 0$ as

$$f(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{d^k f}{dt^k} \right]_{t=0} \quad (1)$$

The differential transformation of $f(t)$ is defined as

$$F(k) = \frac{1}{k!} \left[\frac{d^k f}{dt^k} \right]_{t=0} \quad (2)$$

Then the inverse differential transform is

$$f(t) = \sum_{k=0}^{\infty} t^k F(k) \quad (3)$$

If $y(t)$ and $g(t)$ are two uncorrelated functions with t where $Y(k)$ and $G(k)$ are the transformed functions corresponding to $y(t)$ and $g(t)$ then, the fundamental mathematical operations performed by differential transform can be proved easily and are listed as follows

Table 1: The Fundamental Mathematical Operations by Differential Transformation Method (DTM). Source: [11, 12]

Transformed Function	Original Function
$y(t) = f(t) \pm g(t)$	$Y(k) = F(k) \pm G(k)$
$y(t) = af(t)$	$Y(k) = aF(k)$
$y(t) = \frac{df(t)}{dt}$	$Y(k) = (k+1)F(k+1)$
$y(t) = \frac{d^2 f(t)}{dt^2}$	$Y(k) = (k+1)(k+2)F(k+2)$
$y(t) = \frac{d^m f(t)}{dt^m}$	$Y(k) = (k+1)(k+2)\dots(k+m)F(k+m)$
$y(t) = 1$	$Y(k) = \delta(k)$
$y(t) = t$	$Y(k) = \delta(k-1)$
$y(t) = t^m$	$Y(k) = \delta(k-m) = \begin{cases} 1, & k = m \\ 0, & k \neq m \end{cases}$
$y(t) = f(t)g(t)$	$Y(k) = \sum_{m=0}^k G(m)f(k-m)$
$y(t) = e^{\lambda t}$	$Y(k) = \frac{\lambda^k}{k!}$
$y(t) = (1+t)^m$	$Y(k) = \frac{(m(m-1)\dots(m-k+1))}{k!}$

2.4. Analytical solution of the model equations using differential transformation method (DTM)

In this section, the Differential Transformation Method (DTM) is employed to solve the system of non-linear differential equations which describe our model for Dengue fever.

Let the model equation be a function $q(t)$, $q(t)$ can be expanded in Taylor series about a point $t = 0$ as

$$q(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{d^k q}{dt^k} \right]_{t=0}, \quad (4)$$

where,

$$q(t) = \{s_H(t), E_1(t), I_1(t), R_1(t), S_V(t), E_V(t), I_V(t)\} \quad (5)$$

The differential transformation of $q(t)$ is defined as

$$Q(t) = \frac{1}{k!} \left[\frac{d^k q}{dt^k} \right]_{t=0} \quad (6)$$

Table 2: Values for parameters used for analytical solutions

Parameter	Description	Values	Unit	Reference
Λ_H, Λ_V	Recruitment rate into the population of susceptible humans and vectors respectively.	500,10000000	Year ⁻¹	[14]
μ_H, μ_V	Natural death for humans, vectors respectively.	0.02041,0.5	Year ⁻¹	[13]
β_{VH}	Effective contact rate for 0.5 dengue from vectors to humans	0.5	Year ⁻¹	[14]
β_{HV}	Effective contact rate for 0.4 dengue from humans to vectors	0.4	Year ⁻¹	[14]
τ_1	Dengue treatment rate for I_1	(0,1)	Ind ⁻¹ Year ⁻¹	[14]
γ_1	Progression rate to active dengue	0.3254	Year ⁻¹	[14]
γ_V	Progression rate to active dengue (vectors)	0.03	Year ⁻¹	[14]
δ_{D_1}	Disease induced death Dengue	0.365	Year ⁻¹	[13]
δ_{HV}	Disease induced death dengue (vectors)	0	Year ⁻¹	[14]
η_V, η_A, η_B	Modification parameters for E_V, E_1, i_1	0.4,1.2,0.5	Year ⁻¹	[13]

Then the inverse differential transform is

$$q(t) = \sum_{k=0}^{\infty} t^k Q(t). \tag{7}$$

Using the fundamental operations of differential transformation method in Table 1, we obtain the following recurrence relation of equation (1) as

$$S_H(k+1) = \frac{1}{k+1} \left[\Lambda_H - \mu_H S_H(k) - \frac{\beta_{VH}\eta_V}{N_H} \sum_{m=0}^k S_H(m) E_V(k-m) - \frac{\beta_{VH}}{N_H} \sum_{m=0}^k S_H(m) I_V(k-m) \right] \tag{8}$$

$$E_1(k+1) = \frac{1}{k+1} \left[\frac{\beta_{VH}\eta_V}{N_H} \sum_{m=0}^k S_H(m) E_V(k-m) - \frac{\beta_{VH}}{N_H} \sum_{m=0}^k S_H(m) I_V(k-m) - (\gamma_1 + \mu_H) E_1(k) \right] \tag{9}$$

$$I_1(k+1) = \frac{1}{k+1} [\gamma_1 E_1(k) - (\tau_1 + \mu_H + \delta_{D_1}) I_1] \tag{10}$$

$$R_1(k+1) = \frac{1}{k+1} [\tau_1 I_1(k) - \mu_H R_1(k)] \tag{11}$$

$$S_V(k+1) = \frac{1}{k+1} \left[\Lambda_V - \frac{\beta_{HV}\eta_A}{N_H} \sum_{m=0}^k S_V(m) E_1(k-m) - \frac{\beta_{HV}\eta_B}{N_H} \sum_{m=0}^k S_V(m) I_1(k-m) \right] \tag{12}$$

$$E_V(k+1) = \frac{1}{k+1} \left[\frac{\beta_{HV}\eta_A}{N_H} \sum_{m=0}^k S_V(m) E_1(k-m) - \frac{\beta_{HV}\eta_B}{N_H} \sum_{m=0}^k S_V(m) I_1(k-m) - (\gamma_V + \mu_V) E_V(k) \right] \tag{13}$$

$$I_V(k+1) = \frac{1}{k+1} [\gamma_V E_V(k) - (\mu_V + \delta_{HV}) I_V(k)] \tag{14}$$

With the initial conditions

$$S_H(0) = 3503, E_1(0) = 490, I_1(0) = 390, R_1(0) = 87, S_V(0) = 390, E_V(0) = 100, I_V(0) = 190 \tag{15}$$

The parameter values are

$$N_H = 4470, N_V = 610, \Lambda_H = 500, \Lambda_V = 1,000,000, \mu_H = 0.02041, \mu_V = 0.5, \beta_{VH} = 0.5, \beta_{HV} = 0.4, \tau_1 = 0.75, \gamma_1 = 0.3254, \gamma_V = 0.03, \delta_{D_1} = 0.365, \delta_{HV} = 0, \eta_V = 0.4, \eta_A = 1.2, \eta_B = 0.5 \tag{16}$$

We consider $k = 0, 1, 2, 3$. Cases A1 to A3 are the variation of different values of τ_1

Case A1: High Dengue Treatment Rate, $\tau_1 = 0.75$

$$S_H(1) = -237.6147983, S_H(2) = 12320.67062, S_H(3) = -146425.8636, S_H(4) = 1347694.18, E_1(1) = -522.0979067, E_1(2) = 6637.369770, E_1(3) = -82219.53607, E_1(4) = 775491.4995, I_1(1) = -283.36390, I_1(2) = 75.92177345, I_1(3) = 691.1992607, I_1(4) = -6884.757898, R_1(1) = 290.72433, R_1(2) = -109.2283043, R_1(3) = 19.72355993, R_1(4) = 129.4992219, S_V(1) = -14008.26174, S_V(2) = 260942.4174,$$

$$\begin{aligned}
 S_V(3) &= -3239718.545, S_V(4) = 30208553.32, \\
 E_V(1) &= -3515.845638, E_V(2) = 61669.12070, \\
 E_V(3) &= -717387.0983, E_V(4) = 6204920.468, \\
 I_V(1) &= -4742.00, I_V(2) = 86488.76230, \\
 I_V(3) &= -1051663.250, I_V(4) = 9591046.752. \quad (17)
 \end{aligned}$$

Then the closed form of the solution where $k = 0, 1, 2, 3$ can be written as

$$\begin{aligned}
 S_H(t) &= \sum_{k=0}^{\infty} S_H(k) t^k = 3503 - 237.6147983t \\
 &\quad + 12320.67062t^2 - 146425.8636t^3 \\
 &\quad + 1347694.180t^4 \\
 E_1(t) &= \sum_{k=0}^{\infty} E_1(k) t^k = 490 - 522.0979067t \\
 &\quad + 6637.369770t^2 - 82219.53607t^3 \\
 &\quad + 775491.4995t^4 \\
 I_1(t) &= \sum_{k=0}^{\infty} I_1(k) t^k = 390 - 283.36390t \\
 &\quad + 75.92177345t^2 + 691.1992607t^3 \\
 &\quad - 6884.757898t^4 \\
 R_1(t) &= \sum_{k=0}^{\infty} R_1(k) t^k = 87 + 290.72433t \\
 &\quad - 109.2283043t^2 + 19.72355993t^3 \\
 &\quad + 129.4992219t^4 \\
 S_V(t) &= \sum_{k=0}^{\infty} S_V(k) t^k = 390 - 14008.26174t \\
 &\quad + 260942.4174t^2 - 3239718.545t^3 \\
 &\quad + 30208553.32t^4 \\
 E_V(t) &= \sum_{k=0}^{\infty} E_V(k) t^k = 100 - 3515.845638t \\
 &\quad + 61669.12070t^2 - 717387.0983t^3 \\
 &\quad + 6204920.468t^4 \\
 I_V(t) &= \sum_{k=0}^{\infty} I_V(k) t^k = 130 - 4742.00t \\
 &\quad + 86488.76230t^2 - 1051663.250t^3 \\
 &\quad + 9591046.752t^4 \quad (18)
 \end{aligned}$$

Case A2: Moderate Dengue Treatment Rate, $\tau_1 = 0.5$

$$\begin{aligned}
 S_H(1) &= -237.6147983, S_H(2) = 12320.67062, \\
 S_H(3) &= -146421.4193, S_H(4) = 1347571.999, \\
 E_1(1) &= -522.0979067, E_1(2) = 6637.369770, \\
 E_1(3) &= -82223.98033, E_1(4) = 775613.8858, \\
 I_1(1) &= -185.86390, I_1(2) = -2.662451550, \\
 I_1(3) &= 720.7191613, I_1(4) = -6848.453788, \\
 R_1(1) &= 193.22433, R_1(2) = -48.43782929, \\
 R_1(3) &= -0.1142032264, R_1(4) = 90.09047788
 \end{aligned}$$

$$\begin{aligned}
 S_V(1) &= -14008.26174, S_V(2) = 260933.9106, \\
 S_V(3) &= -3239404.789, S_V(4) = 30202664.15, \\
 E_V(1) &= -3515.845638, E_V(2) = 61660.61400, \\
 E_V(3) &= -71707.62417, E_V(4) = 619913.9432, \\
 I_V(1) &= -4742.00, I_V(2) = 86488.76230, \\
 I_V(3) &= -1051663.335, I_V(4) = 9591049.860. \quad (19)
 \end{aligned}$$

Then the closed form of the solution where $k = 0, 1, 2, 3$ can be written as

$$\begin{aligned}
 S_H(t) &= \sum_{k=0}^{\infty} S_H(k) t^k = 3503 - 237.6147983t \\
 &\quad + 12320.67062t^2 - 146421.4193t^3 \\
 &\quad + 1347571.999t^4 \\
 E_1(t) &= \sum_{k=0}^{\infty} E_1(k) t^k = 490 - 522.0979067t \\
 &\quad + 6637.369770t^2 - 82223.98033t^3 \\
 &\quad + 775613.8858t^4 \\
 I_1(t) &= \sum_{k=0}^{\infty} I_1(k) t^k = 390 - 185.86390t \\
 &\quad - 2.662451550t^2 + 720.7191613t^3 \\
 &\quad - 6848.453788t^4 \\
 R_1(t) &= \sum_{k=0}^{\infty} R_1(k) t^k = 87 + 193.22433t \\
 &\quad - 48.43782929t^2 - 0.1142032264t^3 \\
 &\quad + 90.09047788t^4 \\
 S_V(t) &= \sum_{k=0}^{\infty} S_V(k) t^k = 390 - 14008.26174t \\
 &\quad + 260933.9106t^2 - 3239404.789t^3 \\
 &\quad + 30202664.15t^4 \\
 E_V(t) &= \sum_{k=0}^{\infty} E_V(k) t^k = 100 - 3515.845638t \\
 &\quad + 61660.61400t^2 - 71707.62417t^3 \\
 &\quad + 619913.9432t^4 \\
 I_V(t) &= \sum_{k=0}^{\infty} I_V(k) t^k = 130 - 4742.00t \\
 &\quad + 86488.76230t^2 - 1051663.335t^3 \\
 &\quad + 9591049.860t^4 \quad (20)
 \end{aligned}$$

Case A3: Low Dengue Treatment Rate, $\tau_1 = 0.25$

$$\begin{aligned}
 S_H(1) &= -237.6147983, S_H(2) = 12320.67062, \\
 S_H(3) &= -1464169750, S_H(4) = 1347571.999, \\
 E_1(1) &= -522.0979067, E_1(2) = 6637.369770, \\
 E_1(3) &= -82223.98033, E_1(4) = 775613.8858, \\
 I_1(1) &= -88.36390, I_1(2) = -56.87167655, \\
 I_1(3) &= 720.7191613, I_1(4) = -6848.453788, \\
 R_1(1) &= 95.72433, R_1(2) = -12.02235429,
 \end{aligned}$$

$$\begin{aligned}
R_1(3) &= -0.1142032264, R_1(4) = 90.09047788 \\
S_V(1) &= -14008.26174, S_V(2) = 260925.4039, \\
S_V(3) &= -3239404.789, S_V(4) = 30202664.15, \\
E_V(1) &= -3515.845638, E_V(2) = 61652.10730, \\
E_V(3) &= -71707.62417, E_V(4) = 619913.9432, \\
I_V(1) &= -4742.00, I_V(2) = 86488.76230, \\
I_V(3) &= -1051663.335, I_V(4) = 9591049.860. \quad (21)
\end{aligned}$$

Then the closed form of the solution where $k = 0, 1, 2, 3$ can be written as

$$\begin{aligned}
S_H(t) &= \sum_{k=0}^{\infty} S_H(k) t^k = 3503 - 237.6147983t \\
&\quad + 12320.67062t^2 - 146416.9750t^3 \\
&\quad + 1347450.373t^4, \\
E_1(t) &= \sum_{k=0}^{\infty} E_1(k) t^k = 490 - 522.0979067t \\
&\quad + 6637.369770t^2 - 82228.42463t^3 \\
&\quad + 775735.7158t^4, \\
I_1(t) &= \sum_{k=0}^{\infty} I_1(k) t^k = 390 - 88.36390t \\
&\quad - 56.87167655t^2 + 731.9789850t^3 \\
&\quad - 6805.559035t^4, \\
R_1(t) &= \sum_{k=0}^{\infty} R_1(k) t^k = 87 + 95.72433t \\
&\quad - 12.02235429t^2 - 4.657514297t^3 \\
&\quad + 45.77245152t^4, \\
S_V(t) &= \sum_{k=0}^{\infty} S_V(k) t^k = 390 - 14008.26174t \\
&\quad + 260925.4039t^2 - 323909.2451t^3 \\
&\quad + 30196827.38t^4, \\
E_V(t) &= \sum_{k=0}^{\infty} E_V(k) t^k = 100 - 3515.845638t \\
&\quad + 61652.10730t^2 - 717666.8033t^3 \\
&\quad + 6193410.4t^4, \\
I_V(t) &= \sum_{k=0}^{\infty} I_V(k) t^k = 130 - 4742.00t \\
&\quad + 86488.76230t^2 - 1051663.420t^3 \\
&\quad + 9591052.958t^4. \quad (22)
\end{aligned}$$

2.5. Numerical Simulation and Graphical Representation of the Solutions of the Model Equations

The numerical simulation which illustrates the analytical solution of the Model is demonstrated using Maple software. This is achieved by using some set of parameter values given in the Table 2. The following initial conditions for the human populations $S_H(0) = 3503$, $E_1(0) = 490$, $I_1(0) = 390$, $R_1(0) = 87$, $S_V(0) = 390$, $E_V(0) = 100$, $I_V(0) = 190$ are considered.

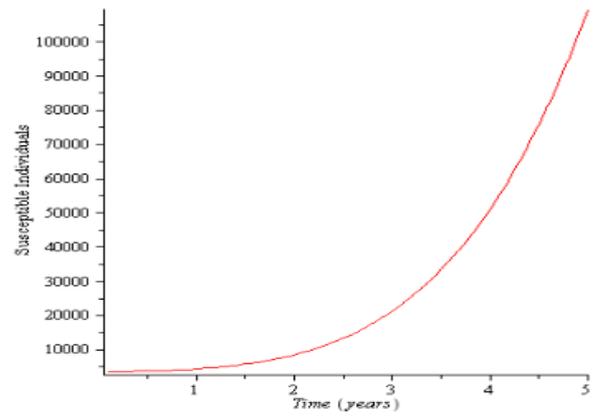


Figure 1: Solution of Susceptible Population Using DTM

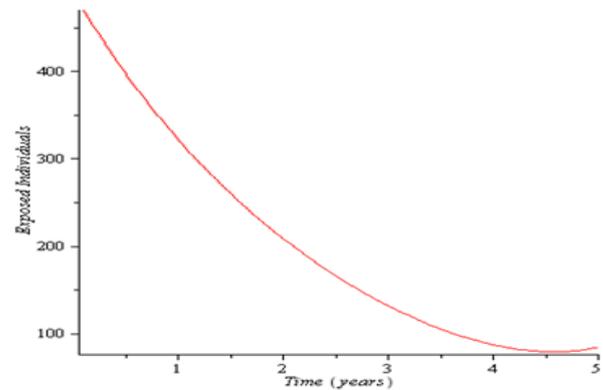


Figure 2: Solution of Exposed Population Using DTM

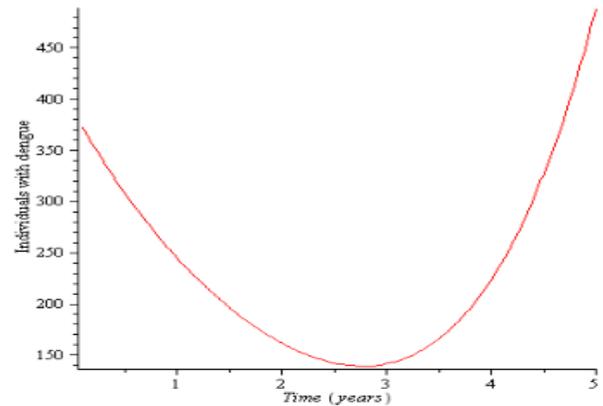


Figure 3: Solution of Human Population with Dengue Using DTM

3. Discussion of results

The Figures 1 to 6 give the numerical profiles of the solutions (17), (19) and (21) using DTM. Figure 1 shows increase in the population of susceptible individuals while Figure 2 indicates a decreasing population of the exposed owing to the progression out of exposed class to class of human with Dengue. Figure 3 implies a decrease in the population of human infected with dengue which later increases. Figure 4 implies the treated

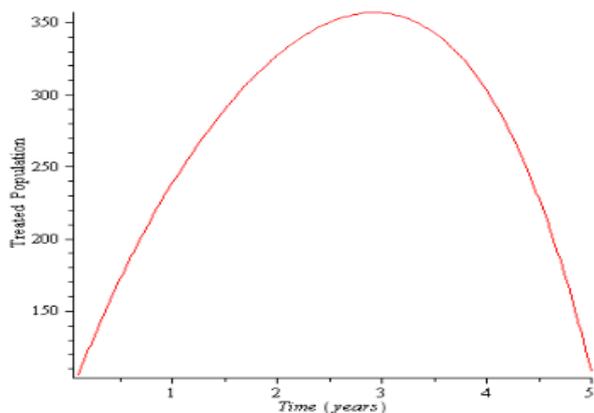


Figure 4: Solution of Treated Human Population Using DTM

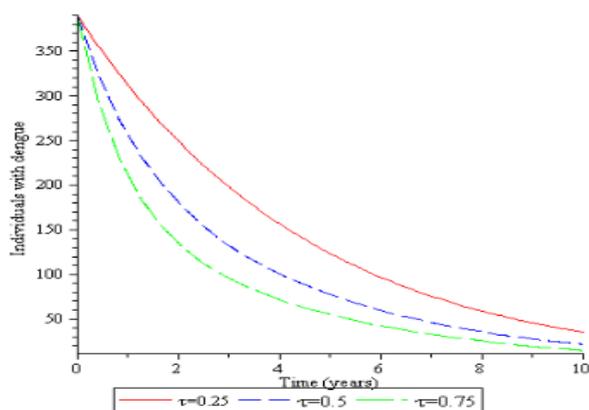


Figure 5: The Effect of Different Treatment Rates on Human Population with Dengue Using DTM

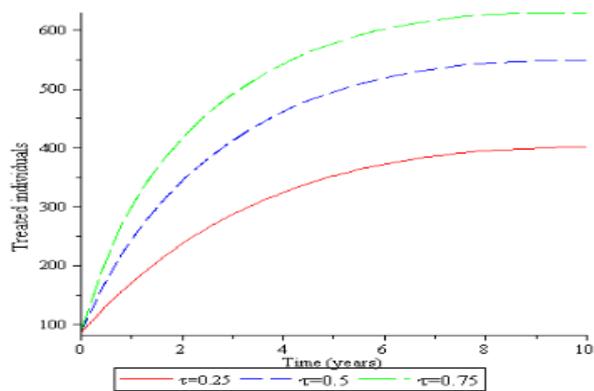


Figure 6: The Effect of Different Treatment Rates on Treated Human Population Using DTM

human population increases sharply to a point and then decreases. Figure 5 indicates that increasing the treatment rate of individuals in the infected population leads to a reduction in the number of infected individuals which is due to progression into the treated population while Figure 6 shows that increasing the treatment rate of individuals in the infected population leads to a corresponding increase in the treated population.

4. Conclusion

We formulated a compartmental model to investigate the dynamics of dengue fever in a population with treatment as a control measure. Differential Transform Method (DTM) was employed to obtain the series solution of the model. Numerical simulations were carried out to determine the long term behavior of Dengue fever model which shows that treatment plays a vital role in reducing the disease burden in a population. The results of the simulations were displayed graphically.

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