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A study of growth of COVID-19 with super-spreaders using the modified SIR model including iceberg phenomenon

Gurpreet Singh Tuteja[a], Tapshi Lal[b,*]

[a]*Zakir Husain Delhi College, University of Delhi, Delhi, India*
[b]*Satyawati College, University of Delhi, Delhi, India*

Abstract

This paper explores the dynamics of COVID-19 transmission, particularly focusing on super-spreaders, through the lens of the SIR model. The model comprises six compartments representing susceptible, exposed, symptomatic infected, super-spreader, asymptomatic infected, and recovered individuals. Utilizing a set of non-linear, interdependent differential equations, we numerically solve for model parameters to examine the influence of super-spreaders on infection spread within the population. We calculate the basic reproduction number (R_0) and discuss the stability of disease-free equilibrium. Our findings underscore the significant role played by super-spreaders and asymptomatic individuals in disease dissemination. Drawing on the epidemiological concept of the iceberg phenomenon, we offer insights into super-spreader events (SSEs) in India and their ramifications.

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Keywords: SIR model, COVID-19, Super-spreaders, Super Spreading Event, Iceberg phenomenon in epidemiology, Basic reproduction number

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1. Introduction

Infectious disease dynamics and its control are complicated by variability in transmission. The coronavirus spreads in humans primarily through person-to-person contact via respiratory droplets generated by breathing, sneezing, coughing, etc., as well as contact [1, 2]. A super-spreader emits thrice the quanta of infection compared to a normal person [3]. Mathematical modelling studies have suggested that asymptomatic individuals might be major drivers [4, 5] for the growth of the COVID-19 pandemic. The probability of transmission of infection is found to increase by 72% and 185% for a normal person at distances of $0.5m$ and $1m$, respectively, from the source [3]. In Ningbo, Zhejiang, a case of a super-spreader was recorded who infected over twenty-eight people [6]. COVID-19 has already killed more persons than SARS and MERS combined. Both of these coronavirus infections were fuelled by SSEs [7]. For the control and management of the present COVID-19 pandemic, detecting and isolating super-spreaders, as well as understanding the reasons for their efficient transmission capacity, is crucial. A robust description of super spreading events as well as a framework for predicting their frequency to help frame ongoing debate is given in Ref [8].

In the early studies of host-pathogen interactions, it was believed that infected individuals in a population had similar chances of infecting others. Following that, most transmission

*Corresponding author Tel.: +91-981-064-9384;
Email address: tapshisingh@ymail.com (Tapshi Lal)

events were found to be regulated by a small percentage of individuals within any population, which became known as the 20/80 law [9]. The 80 − 20 "Pareto" rule has been proposed to describe this heterogeneity whereby 80% of transmission is accounted for by 20% of individuals, herein called super-spreaders [10]. The definition of super-spreaders should not be confined to a person and must include other variables like groups, policies, events or a setting that can shape a course of disease transmission [11]. Since a super-spreader is someone who sheds and transmits more viruses than others, it appears that environmental factors play a significant role in SSEs (Super Spreading Events). SSEs are divided into two categories: 'societal' and 'isolated' SSEs [12]. Religious teachers and religious gatherings made significant contributions to SSEs all over the world [11]. When academic institutions are closed, religious meetings should also be stopped. Mobilizing religious leaders should be included as a priority in an epidemic containment plan from the early stages [3].

In this study, the compartmental model has been used to study the transmission of contagious diseases [1, 13–15]. The compartment model is primarily divided into the following three distinct, mutually exclusive compartments: susceptible $S(t)$, infected/infectious $I(t)$ and recovered $R(t)$, at any time t, based on the epidemiological status of the population. Several modifications of the compartmental model were studied, such as SI (susceptible-infected), SIS (susceptible-infected-susceptible), SIRS (susceptible-infected-recovered-susceptible), and SEIRS (susceptible-exposed-infected-recovered-susceptible) [16–18]. The movement of the population from one compartment to another depends on the transmission rate [19]. Further, Ref. [20] considered a time-dependent SIR Model for COVID-19 with two types of infected persons: apparent and inapparent. The spread of the COVID-19 disease with a special focus on the transmissibility of super-spreader individuals was studied in Refs. [19, 21].

The purpose of this paper is to study the transmission of disease due to super-spreaders considering separate compartments for symptomatic infected, super-spreaders and asymptomatic infected. An insight into the iceberg phenomenon is also presented in the context of apparent and inapparent infection. The numerical solutions of the differential equations by varying model parameters are obtained by using the Wolfram Mathematica software and compared graphically also.

The paper is organized in the following sections: In Section 2, the modified SIR model for super-spreaders with model parameters is briefly described, along with the justification for the existence, uniqueness and positivity of the solution. In Section 3, the basic reproduction number is computed and stability at disease-free equilibrium is established. In Section 4, numerical solutions by varying model parameters are presented graphically. Finally, Section 5 has conclusions.

2. Formulation

In this study, we consider the following six compartments: susceptible $S(t)$, exposed $E(t)$, infected (asymptomatic $A(t)$ &

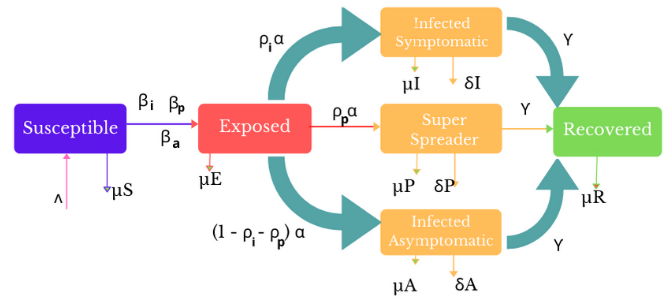

Figure 1. Super Spreader modified SIR model with vital dynamics

symptomatic $I(t)$, super-spreader $P(t)$ separately) and recovered $R(t)$ containing the population according to the epidemiological status. The birth rate Λ, natural death rate μ and disease-induced death rate δ are assumed to be constants. The model is given in Figure 1 which is represented by a set of following six mutually dependent non-linear ordinary differential equations:

$$\frac{dS}{dt} = \Lambda N - \frac{\beta_i S(t)I(t)}{N} - \frac{\beta_p S(t)P(t)}{N} - \frac{\beta_a S(t)A(t)}{N} - \mu S(t), \tag{1}$$

$$\frac{dE}{dt} = \frac{\beta_i S(t)I(t)}{N} + \frac{\beta_p S(t)P(t)}{N} + \frac{\beta_a S(t)A(t)}{N} - (\mu + \alpha)E(t), \tag{2}$$

$$\frac{dI}{dt} = \rho_i \alpha E(t) - (\mu + \delta + \gamma)I(t), \tag{3}$$

$$\frac{dP}{dt} = \rho_p \alpha E(t) - (\mu + \delta + \gamma)P(t), \tag{4}$$

$$\frac{dA}{dt} = (1 - \rho_i - \rho_p)\alpha E(t) - (\mu + \delta + \gamma)A(t), \tag{5}$$

$$\frac{dR}{dt} = \gamma(I(t) + P(t) + A(t)) - \mu R(t), \tag{6}$$

$$N(t) = S(t) + E(t) + I(t) + P(t) + A(t) + R(t). \tag{7}$$

$S(0) = S_0 > 0, E(0) = E_0 \geq 0, I(0) = I_0 \geq 0, P(0) = P_0 \geq 0, A(0) = A_0 \geq 0, R(0) = R_0 \geq 0.$

The parameter ΛN is the change in total population (the rate of recruitment or birth rate including migration is Λ). The probability of a susceptible individual being infected with the coronavirus is $\beta_i I/N$, where β_i is the effective contact rate with which the susceptible becomes symptomatically infected. β_a is the effective contact rate with which the susceptible becomes asymptotically infected and β_p is the effective contact rate with which the susceptible become a super-spreader. μS represents the number of natural deaths among susceptible population and μE, μI, μP, μA, μR is the number of natural deaths in the exposed, symptomatic infected, super-spreader, asymptomatic infected and recovered population, respectively. R is the total human population that has recovered from the infection. The exposed population moves from compartment E to the symptomatic infected, super-spreader or asymptomatic infected compartments with transmission rate α having probabilities ρ_i, ρ_p and 1-ρ_i-ρ_p, called as the probability of being symptomatic infected, probability of being a super-spreader and probability of being asymptomatic infected, respectively. The parameters β_i,β_p, β_a, μ, γ, δ and α are all positive constants.

Using $S/N = S'$, $E/N = E'$, $I/N = I'$, $P/N = P'$, $A/N = A'$, $R/N = R'$ and dropping dashes, we obtain the following dimensionless equations from equations [1-6]:

$$\frac{dS}{dt} = \Lambda - \beta_i S(t)I(t) - \beta_p S(t)P(t) - \beta_a S(t)A(t) - \mu S(t), \tag{8}$$

$$\frac{dE}{dt} = \beta_i S(t)I(t) + \beta_p S(t)P(t) + \beta_a S(t)A(t) - (\mu + \alpha)E(t), \tag{9}$$

$$\frac{dI}{dt} = \rho_i \alpha E(t) - (\mu + \delta + \gamma)I(t), \tag{10}$$

$$\frac{dP}{dt} = \rho_p \alpha E(t) - (\mu + \delta + \gamma)P(t), \tag{11}$$

$$\frac{dA}{dt} = (1 - \rho_i - \rho_p)\alpha E(t) - (\mu + \delta + \gamma)A(t), \tag{12}$$

$$\frac{dR}{dt} = \gamma(I(t) + P(t) + A(t)) - \mu R(t), \tag{13}$$

where,

$$1 = S(t) + E(t) + I(t) + P(t) + A(t) + R(t). \tag{14}$$

$S(0) = S_0 > 0, E(0) = E_0 \geq 0, I(0) = I_0 \geq 0, P(0) = P_0 \geq 0, A(0) = A_0 \geq 0, R(0) = R_0 \geq 0$.

2.1. Existence, Uniqueness and Positivity of Solution

Using Lipchitz condition to verify the existence and uniqueness of the solution [22] for the model equations (8)–(13):

$$Q_1 = \Lambda - \beta_i S(t)I(t) - \beta_p S(t)P(t) - \beta_a S(t)A(t) - \mu S(t), \tag{15}$$

$$Q_2 = \beta_i S(t)I(t) + \beta_p S(t)P(t) + \beta_a S(t)A(t) - (\mu + \alpha)E(t), \tag{16}$$

$$Q_3 = \rho_i \alpha E(t) - (\mu + \delta + \gamma)I(t), \tag{17}$$

$$Q_4 = \rho_p \alpha E(t) - (\mu + \delta + \gamma)P(t), \tag{18}$$

$$Q_5 = (1 - \rho_i - \rho_p)\alpha E(t) - (\mu + \delta + \gamma)A(t), \tag{19}$$

$$Q_6 = \gamma(I(t) + P(t) + A(t)) - \mu R(t). \tag{20}$$

Let Ω denote the region,
$|t - t_0| \leq \delta$, $\|x - x_0\| \leq \alpha$, where $x = (x_1, x_2, \ldots x_n)$, $x_0 = (x_{10}, x_{20}, \ldots x_{n0})$.
Also, suppose that $a(t, x)$ satisfies the Lipschitz condition:
$\|a(t, x_1) - a(t, x_2)\| \leq k\|x_1 - x_2\|$,
whenever the pairs $(t, x_1), (t, x_2)$ belong to Ω, where k is a positive constant. Then, there is a positive constant $\delta \geq 0$, such that there exists a unique and continuous vector solution $x(t)$ of the system in the interval $|t - t_0| < \delta$. The condition is satisfied by the requirement that $\frac{\partial a_i}{\partial x_j}, i, j = 1, 2, 3, \ldots n$, be continuous and bounded in Ω. Considering the model equations (15)-(20), we are interested in the region $0 \leq \alpha \leq R$.
Let Ω denote the region $0 \leq \alpha \leq R$, then equations (15)-(20) will have a unique solution if $\frac{\partial a_i}{\partial x_j}, i, j = 1, 2, 3, \ldots 6$ are continuous and bounded in Ω.
For Q_1 :

$$\left|\frac{\partial Q_1}{\partial S}\right| = |-\beta_i I(t) - \beta_p P(t) - \beta_a A(t) - \mu| < \infty,$$

$$\left|\frac{\partial Q_1}{\partial E}\right| = 0 < \infty, \left|\frac{\partial Q_1}{\partial I}\right| = |-\beta_i S(t)| < \infty,$$

$$\left|\frac{\partial Q_1}{\partial P}\right| = |-\beta_p S(t)| < \infty,$$

$$\left|\frac{\partial Q_1}{\partial A}\right| = |-\beta_a S(t)| < \infty, \left|\frac{\partial Q_1}{\partial R}\right| = 0 < \infty.$$

These partial derivatives exist, are continuous and are bounded. Similarly, for Q_2, Q_3, Q_4, Q_5 and Q_6. Hence, the model has a unique solution. The positivity of the solution can be shown easily.

2.2. Positivity of Solution

We show that the model equations (8–13) are biologically and epidemiologically meaningful i.e. problem is well-posed, as the solutions of all the stated variables are non-negative, assuming $E(t)$ (exposed population) is non-negative with time t.
If $S(0) > 0, E(0) \geq 0, I(0) \geq 0, P(0) \geq 0, A(0) \geq 0$ and $R(0) \geq 0$, then the solution regions $S(t), I(t), P(t), A(t)$ and $R(t)$ of the system of equations (8-13) are always non-negative.
Theorem: Positivity of infected symptomatic population: Consider differential equation (10):

$$\frac{dI}{dt} = \rho_i \alpha E(t) - (\mu + \delta + \gamma)I(t) \geq -(\mu + \delta + \gamma)I(t),$$

$\rho_i \geq 0$ is the probability of being symptomatic infected, $\alpha > 0$ is the transmission rate from the exposed compartment to infected compartment, and $E(t)$ also being positive, we can write as:

$$\frac{dI}{I} = -(\mu + \delta + \gamma)dt.$$

On integrating, the solution is:

$$I = I_0 e^{-\int_0^t (\mu+\delta+\gamma)dt}.$$

It is clear from the solution that $I(t)$ is positive since $I_0 = I(0) > 0$ and the exponential function is always positive. Similarly, we can prove that $P(t)$, $R(t)$, $S(t)$ and $A(t)$ are positive for time t(as in Ref. [23]).

3. Disease Dynamics

The model equations (8-13) are biologically and epidemiologically meaningful and well-posed, as the solutions of all the stated variables are bounded. Also,

$$\frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dP}{dt} + \frac{dA}{dt} + \frac{dR}{dt} = 0.$$

$$\omega = [(S^*, E^*, I^*, P^*, A^*, R^*) \in R^{6+} : S^*, E^*, I^*, P^*, A^*, R^* \leq \frac{\Lambda}{\mu}].$$

From above, the Disease-Free Equilibrium (DFE) and the Endemic Equilibrium will be obtained.

$$\begin{aligned} \Lambda - \beta_i S(t)I(t) - \beta_p S(t)P(t) - \beta_a S(t)A(t) - \\ \mu S(t) = 0, \\ \beta_i S(t)I(t) + \beta_p S(t)P(t) + \beta_a S(t)A(t) \\ -(\mu + \alpha)E(t) = 0, \\ \rho_i \alpha E(t) - (\mu + \delta + \gamma)I(t) = 0, \\ \rho_p \alpha E(t) - (\mu + \delta + \gamma)P(t) = 0, \\ (1 - \rho_i - \rho_p)\alpha E(t) - (\mu + \delta + \gamma)A(t) = 0, \\ \gamma(I(t) + P(t) + A(t)) - \mu R(t) = 0. \end{aligned}$$

Using the Jacobian Matrix method, we obtain characteristic polynomial:

$$\frac{(\lambda + \mu)^2(\gamma + \delta + \lambda + \mu)^2}{\mu} D,$$

where, $D = (\mu(\lambda + \mu)(\gamma + \delta + \lambda + \mu) + \alpha(\gamma\mu + \delta\mu + \lambda\mu + \mu^2 - \beta_i \Lambda \rho_i - \beta_p \Lambda \rho_p + \beta_a \Lambda(-1 + \rho_i + \rho_p)))$.
The eigenvalues are:

$$\begin{aligned} \lambda_1 &= -\mu, \lambda_2 = -\mu, \lambda_3 = -\gamma - \delta - \mu, \\ \lambda_4 &= -\gamma - \delta - \mu, \\ \lambda_5 &= \frac{1}{2\sqrt{\mu}}(-A - \sqrt{B}), \\ \lambda_6 &= \frac{1}{2\sqrt{\mu}}(-A + \sqrt{B}), \end{aligned}$$

where,

$$\begin{aligned} A &= -\alpha\sqrt{\mu} - \gamma\sqrt{\mu} - \delta\sqrt{\mu} - 2\mu^{3/2}, \\ B &= \alpha^2\mu + (\gamma + \delta)^2\mu \\ &\quad - 2\alpha(\gamma\mu + \delta\mu - 2\beta_i \Lambda \rho_i - \quad 2\beta_p \Lambda \rho_p \\ &\quad - 2\beta_a \Lambda(1 - \rho_i - \rho_p)). \end{aligned}$$

For a system to be asymptotically stable, all the eigenvalues must be real and negative. The radical part of B is greater than zero, being real:

$$\begin{aligned} B = 4\alpha\Lambda(\beta_i \rho_i + \beta_p \rho_p + \beta_a \Lambda(1 - \rho_i - \rho_p)) \\ + \mu(\alpha - (\gamma + \delta))^2 > 0 \end{aligned}$$

Also, being $\lambda_6 < 0$, therefore, on solving we get,

$$R_0 = \frac{\Lambda(\beta_i \rho_i + \beta_p \rho_p + \beta_a(1 - \rho_i - \rho_p)}{(\mu + \alpha)(\gamma + \delta + \mu)\mu} < 1,$$

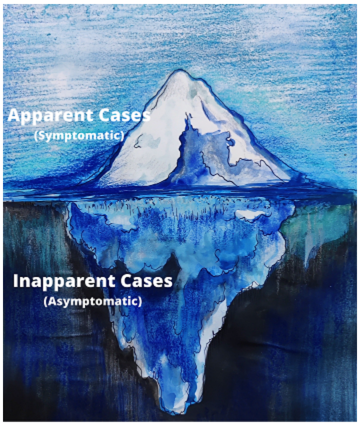

Figure 2. Iceberg phenomenon in epidemiology

is the basic reproduction number R_0. The basic reproduction number (BRN) is an important threshold number in epidemiology. It is defined as the number of secondary infections caused by one primary infection in an entirely susceptible population. It is widely known that, in many epidemiological models, $R_0 > 1$ indicates an epidemic case in which the infection progresses and grows away from zero infective [24]. The mobility of these super-spreaders can impact the estimates of R_0 from early time's dynamics [25], we have shown that R_0 is the function of β_p.

On solving equations (8-13), we obtain two equilibrium points, the DFE is governed by:
$(S^*, E^*, I^*, P^*, A^*, R^*) = (\Lambda/\mu, 0, 0, 0, 0, 0)$,

$$S^* = \frac{\Lambda}{R_0\mu}, \tag{21}$$

$$E^* = \frac{(-1 + R_0)\Lambda}{R_0(\alpha + \mu)}, \tag{22}$$

$$I^* = \frac{(-1 + R_0)\alpha\Lambda\rho_i}{R_0(\alpha + \mu)(\gamma + \delta + \mu)}, \tag{23}$$

$$P^* = \frac{(-1 + R_0)\alpha\Lambda\rho_p}{R_0(\alpha + \mu)(\gamma + \delta + \mu)}, \tag{24}$$

$$A^* = \frac{(-1 + R_0)\alpha\Lambda(1 - \rho_i - \rho_p)}{R_0(\alpha + \mu)(\gamma + \delta + \mu)}, \tag{25}$$

$$R^* = \frac{(-1 + R_0)\alpha\gamma\Lambda}{R_0\mu(\alpha + \mu)(\gamma + \delta + \mu)}. \tag{26}$$

Thus, DFE is asymptotically stable for $R_0 < 1$.

4. Numerical Investigation

In this section, we study the growth of the disease by varying model parameters and also study the iceberg phenomenon in this contagious disease. Infections underneath are not detectable, while those above are visible. However, the extent of the depth of disease beneath is difficult to predict [26]. The iceberg phenomenon in any disease gives the idea of the growth of disease from its sub-clinical stages to apparent stages in the population. The visible part of the iceberg above the surface represents apparent disease in the population while

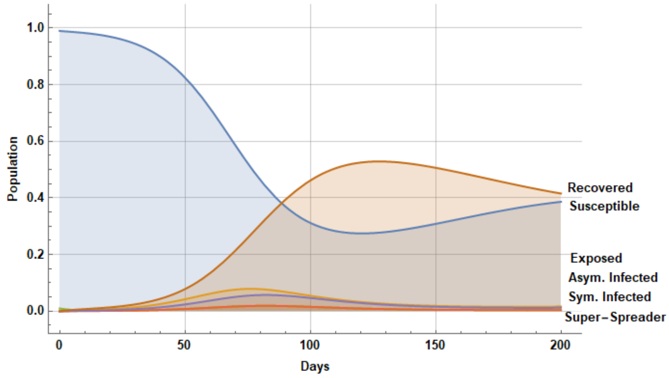

Figure 3. Daily Susceptible, Exposed, Infected (Asymptomatic and Symptomatic), Super-Spreader, Recovered ($\beta_i = 0.1, \beta_p = 0.6, \beta_a = 0.3, \rho_i = 0.2, \rho_p = 0.2$)

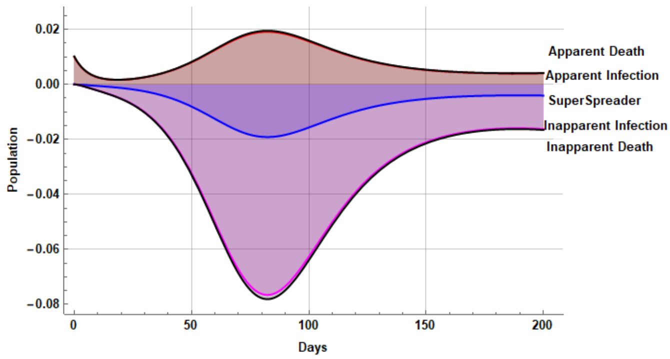

Figure 4. Iceberg phenomenon($\beta_i = 0.1, \beta_p = 0.6, \beta_a = 0.3, \rho_i = 0.2, \rho_p = 0.2$)

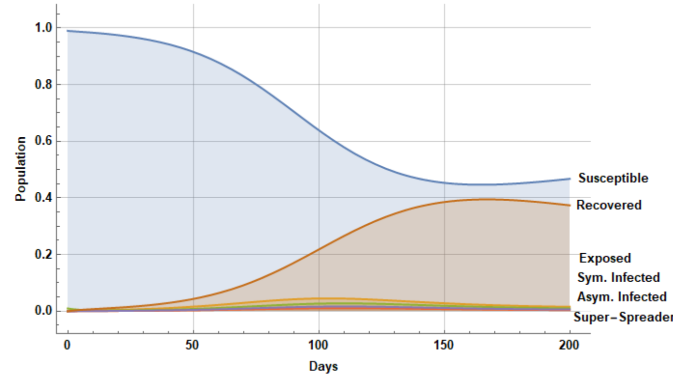

Figure 5. Daily Susceptible, Exposed, Infected (Asymptomatic and Symptomatic), Super-Spreader, Recovered($\beta_i = 0.1, \beta_p = 0.6, \beta_a = 0.3, \rho_i = 0.5, \rho_p = 0.2$)

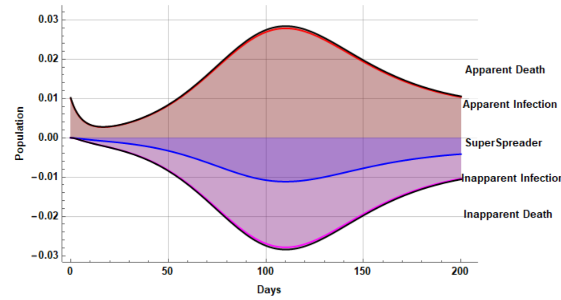

Figure 6. Iceberg phenomenon($\beta_i = 0.1, \beta_p = 0.6, \beta_a = 0.3, \rho_i = 0.5, \rho_p = 0.2$)

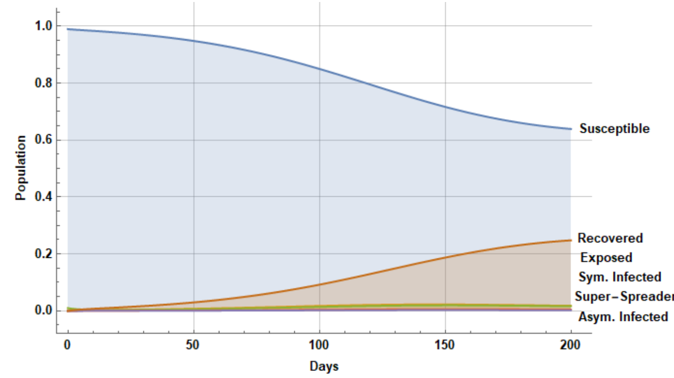

Figure 7. Daily Susceptible, Exposed, Infected (Asymptomatic and Symptomatic), Super-Spreader, Recovered ($\beta_i = 0.1, \beta_p = 0.6, \beta_a = 0.3, \rho_i = 0.7, \rho_p = 0.2$)

the largely hidden part is invisible in the population (See Figure 2) and is a matter of concern as it can bias the control measures. Majorly, it plays an important role in estimating the apparent (sub-clinical) and inapparent cases (presence of infection without the occurrence of recognizable clinical signs or symptoms). The objective is to control the sub-clinical cases and identify the inapparent cases and consequently to control the superspreaders and asymptomatic infected. The ratio of symptomatic infected with the sum of super-spreader and asymptomatic infected should be greater than M, being any large positive number. The higher the value of the ratio, the apparently infected population will be more than the inapparently infected population. Initially (disease-free equilibrium), the ratios (23), (24) and (25) give:

$$\frac{I^*}{P^* + A^*} > M$$
$$\implies \frac{\rho_i}{1 - \rho_i} > M$$

Therefore, the larger is the probability of symptomatic infected, the inapparent part of the infected population will be smaller and the disease can be controlled i.e., the Covid Appropriate Behaviour (CAB) including quarantine in the exposed population be imposed to avoid the iceberg phenomenon. The dimensionless equations (8)-(14) are solved using the Wolfram Mathematica built-in function NDSolve (RK4 Method). Keeping the model parameters constant $\beta_i = 0.1, \beta_p = 0.6, \beta_a = 0.3, \rho_p = 0.2$, we plot a pair of graphs depicting model variables and icebergs including apparent infection (symptomatic), inapparent infection (asymptomatic), deaths (both apparent and inapparent) and super-spreader (inapparent) for the probability of symptomatic infected $\rho_i = 0.2, 0.5$ and 0.7 (See Figures 3-8). The increase in ρ_i increases the spread of apparent infection in the population and reduces the super-spreaders. The apparent cases become more than the inapparent cases in the population, which will help the clinicians to control the disease rapidly, i.e. the large numbers of tests daily will enable the identification of

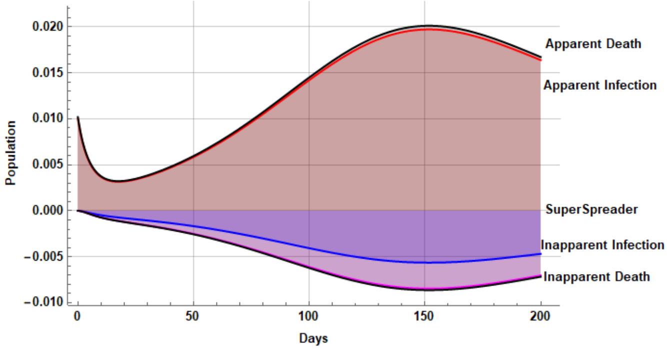

Figure 8. Iceberg phenomenon in epidemiology ($\beta_i = 0.1, \beta_p = 0.6, \beta_a = 0.3, \rho_i = 0.7, \rho_p = 0.2$)

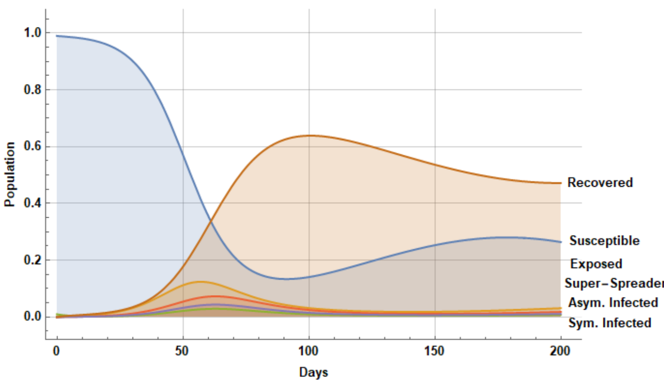

Figure 9. Daily Susceptible, Exposed, Infected (Asymptomatic and Symptomatic), Super-Spreader, Recovered ($\beta_i = 0.1, \beta_p = 0.6, \beta_a = 0.2, \rho_i = 0.2, \rho_p = 0.5$)

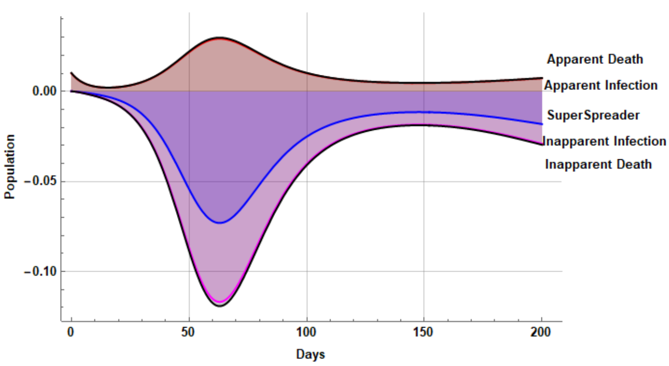

Figure 10. Iceberg Phenomenon in Epidemiology ($\beta_i = 0.1, \beta_p = 0.6, \beta_a = 0.2, \rho_i = 0.2, \rho_p = 0.5$)

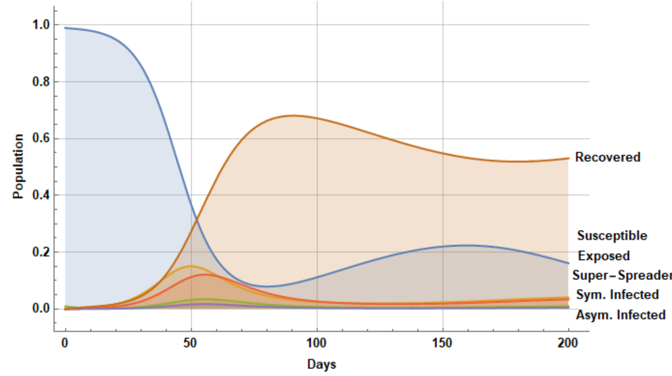

Figure 11. Daily Susceptible, Exposed, Infected (Asymptomatic and Symptomatic), Super-Spreader, Recovered ($\beta_i = 0.1, \beta_p = 0.6, \beta_a = 0.2, \rho_i = 0.2, \rho_p = 0.7$)

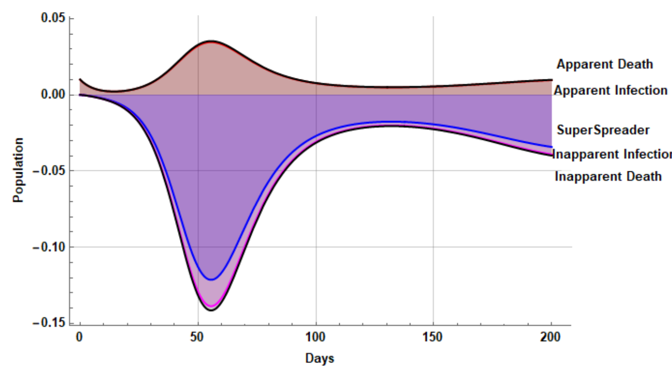

Figure 12. Iceberg Phenomenon in Epidemiology ($\beta_i = 0.1, \beta_p = 0.6, \beta_a = 0.2, \rho_i = 0.2, \rho_p = 0.7$)

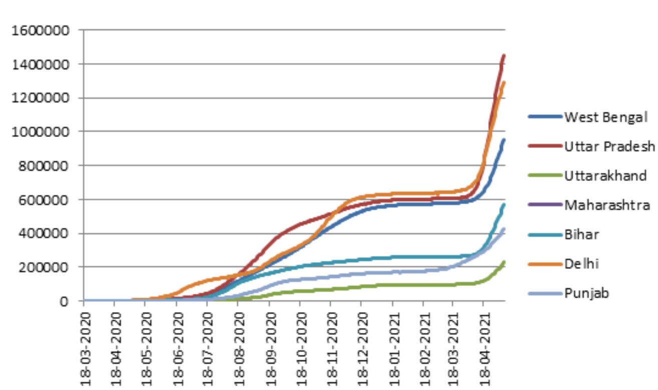

Figure 13. Situation of COVID19 in different States of India

symptomatic population and will reduce the number of asymptomatic and super-spreader cases.

Keeping other parameters constant, we consider $\rho_p = 0.2, 0.5$ and 0.7 (Figures 3, 4, 9-12). It is found that more is the value of ρ_p, the super-spreaders achieve a higher peak within a short interval of time. Consequently, there is a rise in the number of asymptomatic infected cases. Also, the susceptible population reduces rapidly with increasing probability of super-spreader ρ_p with a large number of inapparent population in a short interval of time. This justifies that SSEs cause a sudden jump in the inapparent cases and as a result, along with apparent cases, the number of infected cases rise in a short interval of time.

It is important to note that religious teachers and religious gatherings contributed a lot to SSEs across the world; some of the well-known SSEs are listed in Ref. [27]. In the recent past, at the onset of the second wave of COVID-19 in India, the major SSEs [28–32] and even the public gatherings in narrow lanes resulted in super spreaders [33]. The following graphs indicate that there was a steep rise in the number of infected cases in different states of India including Uttar Pradesh, Uttarakhand, Bihar, West Bengal, Delhi, and Punjab (Fig. 13). Thus, combining the effect of SSEs, India has recorded more than 300,000 cases of COVID-19 per day since April 21, up from 100,000 per day on April 04, 2021. TThese numbers eclipse India's previous highest number of new cases reported in a single day, at 97,860 cases on September 16, 2020 [34].

5. Conclusions

This study introduces a modified SIR model to investigate the propagation of COVID-19, with a specific focus on super-spreaders. Both super-spreaders and the asymptomatic population are individually addressed, acknowledging their crucial roles in disease transmission. The compartmental model consists of six compartments: susceptible, exposed, infected (comprising symptomatic, super-spreader, and asymptomatic individuals), and recovered individuals. Through the numerical solution of six interdependent ordinary differential equations, solutions at disease-free equilibrium are obtained. Furthermore, the basic reproduction number (R_0) is calculated using the characteristic polynomial. Notably, R_0 exhibits linear dependence on the sum of transmission and contact rates among symptomatic, super-spreader, and asymptomatic infected individuals. Stability analysis reveals that disease-free equilibrium is asymptotically stable when $R_0 < 1$. Numerical investigations with varied model parameter values underscore the significant impact of super-spreaders and asymptomatic individuals on disease spread. As the likelihood of symptomatic infections increases, the number of apparent cases surpasses that of inapparent cases, suggesting clinical manageability. To achieve this, periodic diagnostic testing of the exposed population is imperative. Conversely, heightened probabilities of super-spreader events result in a rapid rise in inapparent infections, contributing substantially to the epidemiological iceberg phenomenon. Consequently, super-spreader events facilitate widespread infection dissemination within the population over a short timeframe.

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