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An analytical model for point source pollutants in an urban area with mesoscale wind and wet deposition

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Abstract

Industrialization has led to severe environmental degradation, posing substantial health risks. The primary pollutants originate from land, air, and water sources. Monitoring air pollution typically requires expensive equipment. To address this, scientists have created various models based on specific criteria to predict air pollution levels. This paper explores an analytical solution to the problem of air pollution caused by point sources that disperse contaminants into the atmosphere. Specifically, it investigates how the removal processes affect the concentration of key pollutants. We apply the methods of separation of variables and Fourier transformation to derive an analytical solution for the mathematical model.

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1. Introduction

The industrial revolution, while driving technological and societal progress, also generated vast quantities of pollutants that continue to pose serious risks to human health. Airborne contaminants infiltrate the body's respiratory and circulatory systems, damaging vital organs such as the brain, heart, and lungs. Beyond their immediate environmental impacts, air pollutants are also a major contributor to climate change, which exacerbates the threats to human health. This issue has become especially severe in urban areas, where populations are exposed to high levels of pollution, primarily due to traffic emissions. Industrial accidents can further escalate the situation by releasing toxic fogs into nearby communities. The dispersion of pollutants in the atmosphere is influenced by key meteorological factors, including air stability and wind speed.

Meteorological conditions play a critical role in the emission, transport, formation, and deposition of air pollutants. Many studies have highlighted the connection between weather patterns and pollution characteristics. Accurately predicting the transport and distribution of pollutants requires a thorough understanding of the pollution sources and surrounding geographical features, including the types, quantities, and conditions of emitted pollutants, exhaust gas conditions, stack height, and relevant meteorological factors.

Recent research has focused on developing analytical solutions to the advection-diffusion equation, which is central to

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understanding pollutant dispersion. Analytical solutions help describe the physical phenomena underlying pollution spread. Pasquil and Smith [1] emphasized that mathematical models incorporating all relevant factors simplify the examination of how different parameters affect pollutant concentrations. Hildemann and Lin [2] developed the atmospheric diffusion equation using eddy diffusivities and height-dependent wind speeds, while Essa *et al.* [3] derived a three-dimensional solution using Hankel transforms.

Other researchers have also provided significant contributions. Sharan and Modani [4] solved the advection-diffusion equation by modeling eddy diffusivity as a linear function of downwind distance and wind speed using a power law. Marrouf *et al.* [5] approached the equation similarly with power law representations for both eddy diffusivity and wind speed. Kumar and Sharan [6] derived an analytical solution for the two-dimensional advection-diffusion equation, integrating power and logarithmic wind velocity profiles with vertical turbulent eddy diffusivity.

Furthermore, studies like those by Dilley and Yen [7] examined mesoscale wind patterns' impact on pollutant dispersion, while Arora *et al.* [8] explored pollutant removal from line sources under varying wind profiles and diffusion coefficients. Buske [9] utilized the Laplace transform method to derive solutions for the two-dimensional convection-diffusion equation. Park *et al.* [10] demonstrated that the ground-level concentration ratio between finite area and point sources depends primarily on lateral eddy diffusivity, while wind speed and vertical eddy diffusivity play a lesser role.

Several studies have also focused on urban pollutant transport, including Lakshminarayanachari *et al.* [11], who analyzed primary pollutant transport considering chemical reactions and mesoscale airflow. Essa *et al.* [12] modeled Gaussian concentration in puff models, while Bhaskar *et al.* [13] used separable methods and Bessel functions to study pollution from line sources. These studies collectively show that mesoscale wind patterns and atmospheric conditions significantly affect pollutant concentrations.

Nirmaladevi *et al.* [14] used Fourier transform techniques to solve the three-dimensional atmospheric diffusion equation, examining mesoscale winds and removal mechanisms on pollutant distribution. Their findings indicated that pollutant concentrations decrease with increasing downwind, crosswind, and vertical distances, as well as with higher removal rates. Ravindranath *et al.* [15] employed Fourier's transform method to solve the Dobbins advection-diffusion equation, analyzing pollutant dispersion under stable and neutral atmospheric conditions, and found that pollutant concentrations decrease in the upper part of the mixing layer under such conditions.

Latha *et al.* [16] investigated pollutant dispersion from line sources in urban areas, using eddy diffusivity as a function of vertical height and large-scale wind. They employed separation of variables and Bessel functions in their approach. Similarly, Latha *et al.* [17] analyzed the dispersion of primary pollutants from point sources, concluding that dry and wet deposition processes reduce pollutant concentrations in urban areas. McNider and Pour-Biazar [18] focused on improving meteorological models, particularly in dealing with uncertainties in turbulent mixing in the nighttime boundary layer, emphasizing mesoscale phenomena like the nocturnal low-level jet and its impact on air quality.

Bhaskar *et al.* [19] developed an analytical model for air pollution from high-point sources in the presence of mesoscale winds, simplifying their approach using special functions and Bessel function orthogonality. Their findings indicated that mesoscale winds accelerate pollutant dispersion towards the center of urban heat islands, thus reducing atmospheric pollutant concentrations. Johnson [20] underscored the importance of atmospheric dispersion models in setting and controlling emission levels, particularly for improving air quality in developed countries. Ravindranath *et al.* [21] applied Fourier's method to a simplified advection-diffusion equation, showing that pollutant concentrations decrease with increased wind speed and reduced terrain roughness. Kafle *et al.* [22] conduct in-depth research into the mathematical foundations that support atmospheric dispersion modeling. The application of these ideas entails using a Gaussian plume model to solve the advection-diffusion problem.

In this research, we assume that eddy diffusivities are linearly dependent on downwind distance and use boundary conditions consistent with physical reality. By applying an analytical approach, we derive solutions to the advection-diffusion problem. Our findings show that pollutant concentrations peak near the source and at ground level, with additional removal systems leading to decreased pollution levels.

2. Methodology

2.1. Mathematical Model Formulation

The equations used to simulate the dispersion of air contaminants in the atmosphere are based on the gradient transport theory.

$$\begin{aligned}&\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}+w\frac{\partial c}{\partial z}\\&=\frac{\partial}{\partial x}\left(k_x\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial c}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial c}{\partial z}\right)\\&-(\propto+k_w)\,C.\end{aligned}\tag{1}$$

C represents the concentration of pollutants in the atmosphere. The variables u, v, and w represent the wind speeds in the x, y, and z directions, respectively, while k_x, k_y, and k_z denote the diffusivities in the x, y, and z directions. The variable α represents the natural removal rate of atmospheric pollutants.

In this problem, the point source is located at a height of h_s meters above ground level. The point source emits pollutants continuously and migrates horizontally, parallel to the large-scale airflow. The coordinates $x = 0$, $y = 0$ fix the point source at the origin. We examine the dispersion of pollutants at a downwind distance of $l = 5$ kilometers.

The following assumptions are made to construct the mathematical model:

1. The rate of discharge of pollutants from the point source into the atmosphere is constant.

2. The steady-state condition is assumed, i.e., $\frac{\partial C}{\partial t} = 0$.
3. The x-coordinate is aligned with the average wind speed (with $u = U$ and $v = 0$).
4. The horizontal wind dominates the pollutant diffusion.

That is, $u\frac{\partial C}{\partial x} \gg \frac{\partial}{\partial x}\left(k_x\frac{\partial C}{\partial x}\right)$. Considering the above assumptions, equation (1) becomes

$$u(x)\frac{\partial C}{\partial x} + w(z)\frac{\partial C}{\partial z} = k_y\frac{\partial^2 C}{\partial y^2} + k_z\frac{\partial^2 C}{\partial z^2} - (\alpha + k_w)C. \quad (2)$$

Here, x, y, and z are the coordinate axes. The speed of airflow, $U(x)$, in the x direction is found to vary with vertical distance from the ground level, and it is expressed as:

$$U(x) = U_0(1 - ax),$$

where U_0 represents the average wind speed. The velocity of air in the z direction is given by:

$$W(z) = U_0(az).$$

Typically, in atmospheric conditions, $K_y > K_z$. The boundary conditions for equation (2) are as follows:

(i) Pollutants are discharged from the elevated point source, with a concentration Q, located at $(0, 0, h_s)$. The concentration is given by

$$C(x,y,z) = \frac{Q\delta(y)\delta(z-h_s)}{U(x)} \quad \text{at} \quad x = 0,\ 0 \le h_s \le H. \quad (3)$$

where h_s is the stack height, H is the mixing height, and δ is the Dirac delta function.

(ii) The concentration of contaminants approaches zero as they move in the y direction away from the point source, i.e.,

$$C(x,y,z) = 0 \quad \text{as} \quad y \to \infty. \quad (4)$$

(iii) At the ground level ($z = h$), pollutants are reflected from the Earth's surface, such that

$$\frac{\partial C}{\partial z} = v_d C \quad \text{at} \quad z = h. \quad (5)$$

(iv) Pollutants are reflected at height H from the ground surface, given by

$$k_z\frac{\partial C}{\partial z} = w_s C \quad \text{at} \quad z = H. \quad (6)$$

Recent studies show that large-scale wind speeds are considered constant. We assume the horizontal mesoscale wind to flow in a vertical direction equal to $u = U_0$. We can apply the continuity equation to determine the vertical mesoscale airflow $w(z)$.

$$w_e = aU_0 z,$$
$$U(x) = u + u_e = U_0(1 - ax),$$

where a is constant.

2.2. Method of Solution

Using the following dimensionless parameter approach, we reduce the differential equation (2), which describes the diffusion of atmospheric pollutants and boundary constraints, to dimensionless to obtain the solution.

$$x^* = \frac{K_{Z_0}x}{U_0H^2}, y^* = \frac{y}{H}, Z^* = \frac{z}{H},$$
$$U^* = \frac{U}{U_0}, C^* = \frac{U_0H^2C}{Q},$$
$$\beta^* = \frac{K_Y}{K_{z_0}},\ \gamma^* = \frac{K_z}{K_{z_0}},\ \delta(y^*) = H\delta(y),$$
$$\alpha^* = \frac{U_0H^2\alpha}{K_{Z_0}},\ k^*{}_w = \frac{H^2k_w}{K_{Z_0}}.$$

On dropping asterisks (*) and using the boundary conditions (3) - (6), equation (2) can be made in the dimensional-less form as shown below:

$$(1 - ax)\frac{\partial C}{\partial x} + az\frac{\partial C}{\partial Z} = \beta\frac{\partial^2 C}{\partial y^2} + \gamma\frac{\partial^2 C}{\partial z^2} - (\propto + k_w)C. \quad (7)$$

$$C(x,y,z) = \frac{Q\delta(y)\delta(z-h_s)}{U(x)} \quad at \quad x = 0. \quad (8)$$

$$C = 0 \quad when \quad y \to \pm\infty. \quad (9)$$

$$\frac{\partial C}{\partial z} = NC \quad at\ z = 1. \quad (10)$$

$$\frac{\partial C}{\partial z} = MC \quad at\ z = h/H. \quad (11)$$

The solution to equation (7) is obtained by applying the Fourier transform technique, utilizing the boundary conditions (8) to (11). By taking the Fourier transform of equation (7) with respect to the variable "y", we obtain

$$(1 - ax)\frac{\partial \bar{C}}{\partial x} + (p^2\beta + \alpha + kw)\bar{C} = \gamma\frac{\partial^2\bar{C}}{\partial z^2} - az\frac{\partial\bar{C}}{\partial z}, \quad (12)$$

where $\bar{C} = \bar{C}(x,p,z)$ is the Fourier transform of c with respect to y and p. By considering the Fourier transform, the boundary conditions become

$$\bar{C}(x,y,z) = \frac{Q\delta(y)\delta(z-h_s)}{U(x)} \quad at \quad x = 0, \quad (13)$$

$$\bar{C} = 0 \quad when \quad y \to \pm\infty, \quad (14)$$

$$\frac{\partial\bar{C}}{\partial z} = N\bar{C} \quad at\ z = 1, \quad (15)$$

$$\partial\bar{C}/\partial z = M\bar{C} \quad at\ z = h/H. \quad (16)$$

When we solve equation (12) using the separation of variables approach, we assume the following trial solution.

$$\bar{c} = X(x)Z(z), \quad (17)$$

where $Z(z)$ is a function of z only and $X(x)$ is a function of only x. By substituting the equation (17) in equation (12), the differential equations are obtained

$$\frac{(1-ax)}{X}\frac{dX}{dx} + \left(p^2\beta + \lambda^2 + k_w + \alpha\right) = 0. \tag{18}$$

$$\gamma\frac{d^2z}{dz^2} - az\frac{dZ}{dz} + \lambda^2 Z = 0. \tag{19}$$

The term λ^2 represents the separation constant. The solutions of equation (18) & equation (19) are of the form

$$X = C_1(1-ax)^{\frac{p^2\beta+k_w+\lambda^2+\alpha}{\alpha}}. \tag{20}$$

$$Z = a_0 f(z) + a_1 g(z), \tag{21}$$

where a_0, a_1, and C_1 are constants and

$$f(z) = 1 - \frac{\lambda^2}{2!\gamma}z^2 - \frac{\lambda^2(2a-\lambda^2)}{4!\gamma^2}z^4 - \frac{\lambda^2(2a-\lambda^2)(3a-\lambda^2)z^6}{6!\gamma^3}$$

$$g(z) = z + \frac{(a-\lambda^2)}{3!\gamma}z^3 + \frac{(a-\lambda^2)(3a-\lambda^2)}{5!\gamma^2}z^5 + \frac{(a-\lambda^2)(5a-\lambda^2)(3a-\lambda^2)z^7}{7!\gamma^3}.$$

After substituting the values of $X(x)$ and $Z(z)$ from the equations (20) and (21) we get

$$\bar{C} = (1-ax)^{\frac{\left(p^2\beta+\alpha+k_w+\lambda^2\right)}{\alpha}}\left(a_0 f(z) + a_1 g(z)\right). \tag{22}$$

The term C_1 is considered to be 1 without loss of generality. After using the boundary conditions $\partial C/\partial z = NC$ at $z = 1$, we get

$$N = \frac{f(1)}{f'(1)} \tag{23}$$

$$\partial\bar{C}/\partial z = M\bar{C} \quad at\ z = h/H, \quad M = \frac{f(h/H)}{f'(h/H)}. \tag{24}$$

Again, using the boundary condition $\bar{C}(x,y,z) = \frac{\delta(z-h_s)}{(1-ax)}$ at x=0 and as well as applying

$$\int_0^1 \delta(z-h_s) f_n(z)\, dz = f_n(h_s),$$

and

$$\int_0^1 z^p f_m b(z) f_n(z)\, dz = 0.$$

For $m \neq n$ the solution can be written as

$$\bar{C} = (1-ax)^{\frac{\left(p^2\beta+\lambda^2+k_w+\alpha\right)}{\alpha}}\frac{f(h_s)}{p}f(z), \tag{25}$$

where

$$p = \int_0^1 f^2(z)dz.$$

After taking the inverse Fourier transform of the equation (25), we can write

$$C = 0.28209\sqrt{\frac{a}{\beta\log\left(\frac{1}{1-ax}\right)}}\,(1-ax)^{\frac{\left(\lambda^2+k_w+\alpha\right)}{\alpha}}\frac{f(h_s)}{p}.$$

$$f(z)\ exp\left(\frac{ay^2}{4\beta\log(1-ax)}\right). \tag{26}$$

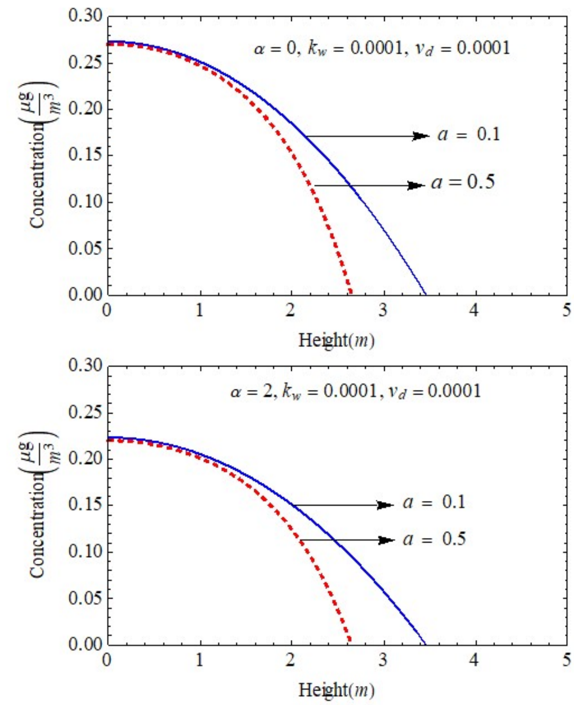

Figure 1. Concentration versus Height for various values of the removal rate

3. Results and Discussion

Considering the removal mechanism, we have developed an analytical mathematical model to investigate the impacts of primary pollutant concentrations in both the horizontal and vertical directions. Appropriate boundary conditions are incorporated to represent the point source in the city region. Contaminants are expected to undergo removal mechanisms through both dry and wet deposition processes. Convection-diffusion equations with initial and boundary conditions are solved using the method of separation of variables, Fourier transform, and series solutions.

Figure 1 illustrates how the concentration varies with height for different values of the removal rate. It is evident that as the removal rate increases, the concentration of contaminants decreases. The graph shows the pollutant concentration curve with respect to vertical distance ($0 \leq z \leq 5$) for values of $a = 0.1$ and $a = 0.5$ at crosswind distance $y = 0$. The graphs are plotted for both height and concentration in the absence and presence of mesoscale wind. It is clear that as α increases, the concentration first rises, reaches a maximum at a certain height, and then decreases as the value of a increases, eventually tending toward zero. This behavior is consistent with the natural dispersion of pollution. Figure 2 shows the variation of air impurity concentration with distance, both in the presence and absence of a removal rate. We observe that as the removal rate α increases, the concentration decreases along the horizontal distance ($0 \leq x \leq 5$). The graph of air impurity concentration with respect to downwind distance for values of $\alpha = 0$ and $\alpha = 2$ is analyzed. We observe that the concentration declines after reaching a maximum at a specific distance and eventually

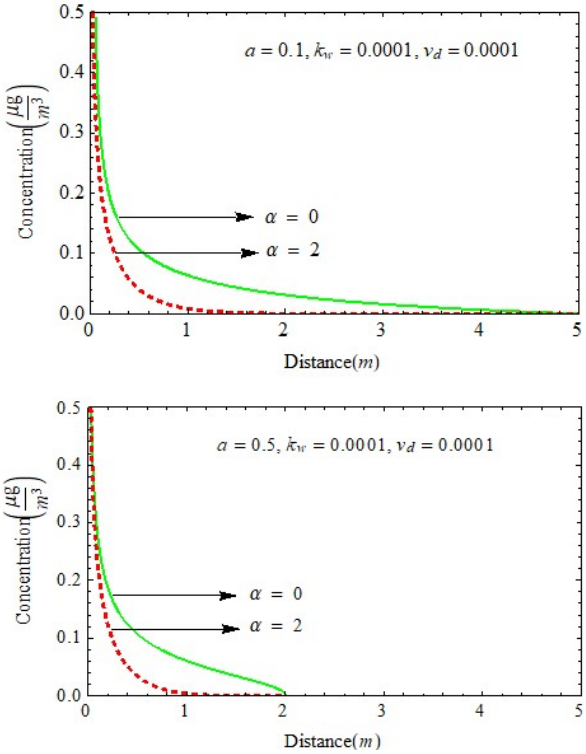

Figure 2. Concentration versus Distance for various values of α

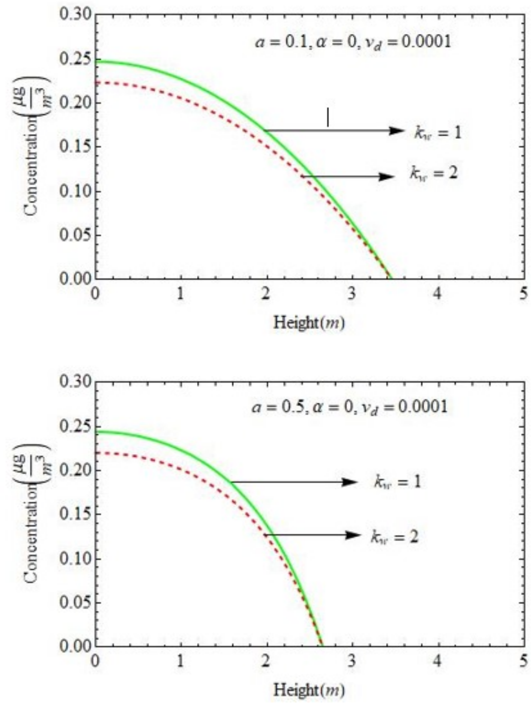

Figure 3. Concentration versus Height for various values of k_w

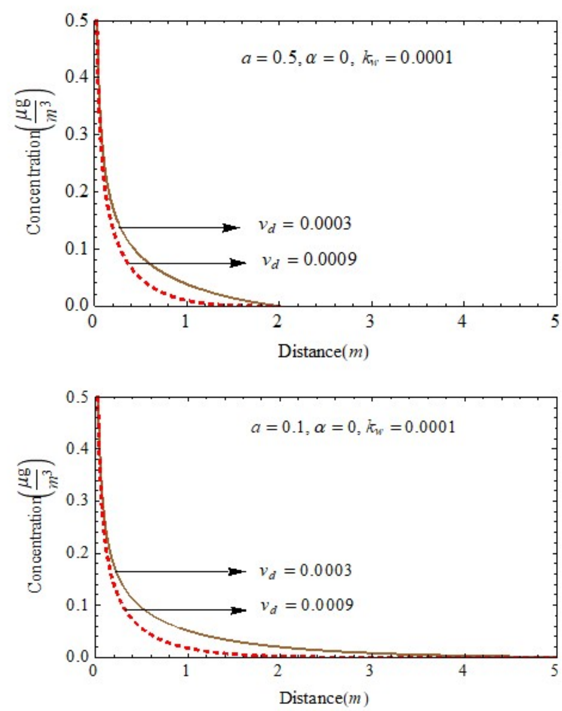

Figure 4. Concentration versus Distance for various values of v_d

reaches zero as the distance increases.

Figure 3 illustrates the concentration variation with height for different values of wet deposition k_w. It is observed that as z increases, the concentration initially rises, attains a maximum at certain heights, and then decreases as z continues to increase, eventually tending toward zero. The concentration approaches zero more quickly when $a = 0.5$ compared to $a = 0.1$.

Figure 4 displays the variation of concentration with distance for different values of α. The graph shows the concentration curve with respect to downwind distance ($0 \leq x \leq 5$). We observe that the concentration approaches zero faster when $a = 0.5$ compared to $a = 0.1$. The concentration initially increases with increasing x, reaches a maximum at a certain distance, and then decreases, approaching zero along the horizontal direction.

4. Conclusion

This study demonstrates the use of mathematical models to evaluate the impact of air pollution levels, specifically through governing equations that assess air quality and emissions from various sources. Our analytical model successfully captures the concentration of pollutants from specific locations, focusing on both dry and wet deposition processes. The analysis highlights the importance of removal processes in reducing the concentration of primary pollutants, revealing that pollutant concentrations diminish with increasing distance from the source and height above ground, primarily due to diffusion and removal mechanisms.

The mesoscale model used in this study replicates key atmospheric phenomena, such as transient storms, weather fronts, and wind flow dynamics, making it particularly suitable for evaluating the effects of wind on pollutant dispersion. These models, which incorporate interactions between wind and terrain, provide insights into pollution patterns relevant to urban environments. The findings emphasize the necessity of removal processes to effectively reduce pollutant concentrations throughout the city, except in areas close to point sources.

This research underscores the importance of implementing effective air quality management strategies, such as eco-friendly transportation, sustainable construction practices, and stricter emission regulations. Moreover, public awareness campaigns and community involvement are crucial in fostering environmental responsibility. By building partnerships among government agencies, industries, and communities, cities can develop more comprehensive solutions to combat air pollution. Ultimately, this collaborative approach is essential for creating sustainable urban ecosystems that benefit both current and future generations.

References

[1] F. Pasquil & F. B. Smith, *Atmospheric diffusion (3rd Edition)*, Ellis Horwood Ltd., Chichester, England, 1983, pp 437. https://doi.org/10.1002/qj.49711046416.

[2] J. S. Lin & L. M. Hildemann, "Analytical solutions of the atmospheric diffusion equation with multiple sources and height-dependent wind speed and eddy diffusivities", Atmospheric Environment **30** (1996) 239. https://doi.org/10.1016/1352-310(95)00287-9

[3] K. S. M. Essa, A. S. Shalaby, M. A. E. Ibrahim *et al.*, "Analytical solutions of the advection-diffusion equation with variable vertical eddy diffusivity and wind speed using Hankel transform", Pure Appl. Geophys. **177** (2020) 4545. https://doi.org/10.1007/s00024-020-02496-y.

[4] M. Sharan & M. Modani, "An analytical study for the dispersion of pollutants in a finite Layer under Low Wind Conditions"', Pure appl geophy, 162,(2005) 1861, https://doi.org/10.1007/s00024-005-2696-5.

[5] A. A. Marrouf, K. S. M. Essa, M. S. El-Otaify1, A. S. Mohamed & G. Ismail, "The influence of eddy diffusivity variation on the atmospheric diffusion equation", Open Journal of Air Pollution **04** (2015) 109. http://dx.doi.org/10.4236/ojap.2015.43011.

[6] K. Pramod & S. Maithili, "An analytical model for dispersion of pollutants from a continuous source in the atmospheric boundary layer", Proc. R. Soc. A. **466** (2010) 383. https://doi.org/10.1098/rspa.2009.0394.

[7] J. F. Dilley & K. T. Yen, "Effect of a mesoscale type wind on the pollutant distribution from a line source", Atmospheric Environment **5** (1971) 843. https://doi.org/10.1016/0004-981(71)90014-X.

[8] U. Arora, S. Gakkhar & R. S. Gupta, "Removal model suitable for air pollutants emitted from an elevated source", Applied Mathematical Modelling **15** (1991) 386. https://doi.org/10.1016/0307-4X(91)90065-W.

[9] D. Buske, "An analytical solution of the advection-diffusion equation considering non-local turbulence closure. Environ Fluid Mech **7** (2007) 43–54. https://doi.org/10.1007/s10652-006-9012-5.

[10] Y. S. Park & J. J. Baik, "Analytical solution of the advection-diffusion equation for a ground-level finite area source", Atmospheric Environment **42** (2008) 9063. https://doi.org/10.1016/j.atmosenv.2008.09.019.

[11] K. Lakshminarayanachari, K. L. Sudheer Pai, M. S. Prasad & C. Pandurangappa, "A two-dimensional numerical model of primary pollutant emitted from an urban area source with mesoscale wind, dry deposition and chemical reaction", Atmospheric Pollution Research **4** (2013) 106. https://doi.org/10.5094/APR.2013.011.

[12] E. Khaled, S. Sawsan, W. Ali & E. N. Mostafa, "Studying the effect of the instantaneous and continuous pollutants sources on the advection-diffusion equation and its Applications", Iraqi Journal of Science **65** (2024) 1117. https://doi.org/10.24996/ijs.2024.65.2.41.

[13] C. Bhaskar, K. Lakshminarayanachari & C. Pandurangappa, "Analytical approach for Air pollution Emitted from Line source with Chemical reaction and wet deposition", International Journal of Research in Advent Technology **7** (2019) 268. https://doi.org/10.32622/ijrat.752019214.

[14] P. V. Nirmaladevi, K.Lakshminarayanachari & C. Pandurangappa, "Three-Dimensional Analytical Model for the Dispersion of Air Pollutants Emitted from Elevated Point Source with Mesoscale Wind and Removal Mechanism", J. Comput. Theor. Nanosci. 15 (2018) 3287. https://doi.org/10.1166/jctn.2018.7612.

[15] K. Ravindranath, K.Lakshminarayanachari & C. Pandurangappa "Time dependent analytical diffusion model of air pollutant in a protected zone due to point source with wet deposition", J. Comput. Theor. Nanosci. **15** (2018) 3344. https://doi.org/10.1166/jctn.2018.7624.

[16] Latha Rachachari, Lakshminarayanachari Kodandachari, Bhaskar Cholaraju, Two-dimensional analytical model for an air pollutants emitted from line source with mesoscale wind. AIP Conf. Proc. **2649** (2023) 030043. https://doi.org/10.1063/5.0117157.

[17] L. Rachachari, L. Kodandachari & B. Cholaraju, "Three-dimensional analytical model for point source with removal mechanisms dry and wet deposition", AIP Conf. Proc. 2649 (2023) 030032. https://doi.org/10.1063/5.0117156.

[18] R. T. McNider, & A. Pour-Biazar, "Meteorological modeling relevant to mesoscale and regional air quality applications: a review", Journal of the Air & Waste Management Association **7** (2019) 43. https://doi.org/10.1080/10962247.2019.1694602.

[19] C. Bhaskar, K. Lakshminarayanachari & C. Pandurangappa, "Analytical approach for air pollution emitted from an elevated point source with mesoscale wind", Journal of Computational and Theoretical Nanoscience **15** (2018) 3553. https://doi.org/10.1166/jctn.2018.7662.

[20] J. B. Johnson, "An Introduction to atmospheric pollutant dispersion modelling", Environ. Sci. Proc. **19** (2022) 18. https://doi.org/10.3390/ecas2022-12826.

[21] R. Kunchitiga & L. Kodandachari, "Mathematical time dependent diffusion model of air pollution with settling and wet deposition of larger particles of pollutants due to area sources in a protected zone", AIP Conf. Proc. 2649 (2023) 030046. https://doi.org/10.1063/5.0146712.

[22] J. Kafle, K. P. Adhikari, E. P. Poudel & R. R. Pant, "Mathematical modeling of pollutants dispersion in the atmosphere", Journal of Nepal Mathematical Society **7** (2024) 61. https://doi.org/10.3126/jnms.v7i1.67487.