Published by
NIGERIAN SOCIETY OF PHYSICAL SCIENCES
Available online @ https://journal.nsps.org.ng/index.php/jnsps

J. Nig. Soc. Phys. Sci. 6 (2024) 2149

Journal of the Nigerian Society of Physical Sciences

Numerical computation of cut off wave number in polygonal wave guide by eight node finite element mesh generation approach

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Abstract

This study proposes a two-dimensional, eight-noded automated mesh generator for precise and efficient finite element analysis (FEA) in microwave applications. The suggested method for solving the Helmholtz problem employs an optimal domain discretization procedure. MAPLE-13 software's advanced automatic mesh generator was developed specifically for this work. To demonstrate the effectiveness of the approach, three distinct waveguide structures are analyzed, with the results compared to the best available analytical or numerical solutions. The findings indicate that the proposed method yields highly accurate and efficient finite element results, particularly for waveguide structures containing singularities. In microwave applications, this method can significantly enhance energy transmission efficiency.

DOI:10.46481/jnsps.2024.2149

Keywords: Numerical integration, 8-noded mesh, Quadrature method, waveguide, FEM.

Article History :
Received: 17 May 2024
Received in revised form: 23 September 2024
Accepted for publication: 07 October 2024
Published: 22 October 2024

Communicated by: B. J. Falaye

1. Introduction

Electromagnetic waves can be guided and transferred with minimal energy loss using hollow metal structures called waveguides. These structures are considered essential optical devices, widely employed in various electrical applications. Waveguides have been extensively studied for their ability to efficiently transport electromagnetic energy, as explained in Ref. [1], which provides a comprehensive discussion of optical and microwave waveguides, along with waveguide-based equipment. A key aspect of understanding complex waveguide systems is the analysis of waveguide modes. This is crucial in radio frequency and optical computations, especially in solving the waveguide eigenvalue problem, where the eigenvalue root of the eigenmodes yields the cutoff wavenumber for a particular waveguide shape. Waveguides can take on various shapes, such as ridge-shaped, L-shaped, circular, coaxial line, or other configurations. Recent research, such as the work by Bernabeu *et al.* [2], has demonstrated that microwave waveguides with well-rounded, curved geometries—rather than sharp edges—are effective in transporting electromagnetic energy. Therefore, determining a waveguide's cutoff frequencies in microwave applications, where powerful computational tools are required, is

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of critical importance. Computational electromagnetics (EM) plays a vital role in obtaining accurate approximations of these values, and the literature presents several numerical methods for calculating waveguide propagation modes, including finite element analysis (FEA) of square waveguides.

Increasing the complexity of the finite element method (FEM) model allows us to address more practical and useful questions. When solving problems in domains with complex geometries, the finite difference approach (FDA), the method of fundamental solutions with an excitation source (MFSES), and the method of moments (MOM), which are mesh-free methods, often face significant challenges. However, increasing the number of 8-noded quadrilateral elements in the domain leads to more accurate results, highlighting the strength of FEM as a promising method for waveguide eigenvalue problems.

Several researchers have proposed different methods to tackle eigenvalue problems in ridge waveguides. Full solutions to the ridge waveguide eigenvalue problem, using integral equation frameworks, have been applied to calculate transverse magnetic (TM) and transverse electric (TE) modes [3]-[10]. Recently, Tingting *et al.* [11] used a four-noded quadrilateral finite element method to determine the cutoff wavenumber in L-shaped and square waveguides. Microwave applications often involve curved geometries and sharp edges, which make traditional approaches computationally demanding for finite element analysis. Most existing software utilizes elements up to quadratic order, but Wang *et al.* [12] have developed a simple and efficient linear mesh generation technique. Additionally, finite element methods have been applied to a wide range of differential equations, as demonstrated by Xu *et al.* [13], offering practical solutions across various fields of technology.

In this study, we employ 2-D automated mesh generators in MAPLE to present a simple, accurate, and efficient approach for Galerkin finite element method (GFEM) calculations on L-shaped, square, and U-shaped waveguides. The higher-order automated mesh generators used in this method are straightforward and yield high-quality results. The cutoff frequencies for these waveguides, which exhibit singularities, are calculated using the proposed method. Our approach can efficiently handle a wide range of waveguide structures. Specifically, we employ the 8-noded mesh generation technique to implement FEM. The L-shaped, square, and U-shaped waveguides are discretized into a set of 8-noded quadrilaterals, followed by the application of optimal numerical integration techniques for these quadrilaterals. The accuracy of the results improves as the number of 8-noded quadrilaterals increases, and the results are compared with those obtained from existing methods.

Section 2 details the automated process for creating an 8-noded quadrilateral mesh and maps the mathematical formulation of the 8-noded quadrilateral element into a square region. Section 3 presents the finite element formulation, where the Helmholtz equation is mathematically formulated using an 8-noded finite element mesh generator and the proposed Galerkin weighted residual FEM. In Section 4, the proposed method is validated for various waveguide structures through three numerical examples, and the results are compared with the best available analytical or numerical solutions. A comprehensive discussion of the numerical results is provided in Section 5.

When compared to other numerical approaches, the results show that the proposed method, incorporating an automated mesh generator and the Galerkin weighted residual FEM, significantly enhances the accuracy and efficiency of the numerical method.

2. Mathematical formulation and creation of an eight noded quadrilateral mesh.

Mesh generation is a crucial first stage in many applications, including finite element modeling or finite element analysis. Physical phenomena or structures are broken down into more manageable components to speed up numerical calculations due to their computational efficiency and simplicity. Quadrilateral elements are frequently utilized in 2D simulations. In numerical simulations and finite element analysis, a type of finite element mesh known as an 8-noded quadrilateral mesh is employed. The term "8-noded" designates an element that has eight nodes, or vertices, per quadrilateral, which is defined as a four-sided polygonal shape. Typically, the "8-noded" quadrilateral element consists of four nodes at each corner and four additional nodes at the midpoints of the edges. Compared to lower-order elements, this node distribution allows for more accurate depiction of the geometry and deformation within the element.

A physical domain is discretized into quadrilateral elements, each with eight nodes, to create an 8-noded quadrilateral mesh, which is frequently used in numerical simulations for structural and mechanical analysis. The physical problem is determined by the field variable u. It is given by Figure 1. In finite

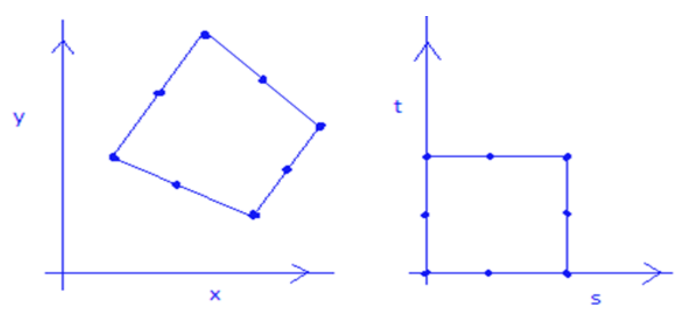

Figure 1. 8-noded rectangular element mapped to 8-noded standard square element

element analysis (FEA), a quadrilateral element is often defined by a set of nodal points, and the geometry within the element is interpolated based on the values assigned to these nodal points. Figure 1 illustrates how a random eight-noded convex quadrilateral element's (x, y) region is translated into a square (s, t) region. Interpolation functions, also known as shape functions, are used to determine the variations of the physical quantities within the element based on the values assigned to the eight nodal points and the values of s and t, both varying from -1 to 1. The shape functions for an eight-noded quadrilateral element are typically given as follows.

$$\left.\begin{aligned} A_1(s,t) &= 0.25(1-s)(-s-t-1)(-t+1) \\ A_2(s,t) &= 0.25(s+1)(-1-t+s)(-t+1) \\ A_3(s,t) &= 0.25(1+s)(1+t)(-1+s+t)(1+t) \\ A_4(s,t) &= 0.25(-s+1)(-s+t-1)(t+1) \\ A_5(s,t) &= 0.5(1-t)\left(1-s^2\right) \\ A_6(s,t) &= 0.5\left(1-t^2\right)(1+s) \\ A_7(s,t) &= 0.5(1+t)\left(1-s^2\right) \\ A_8(s,t) &= 0.5\left(1-t^2\right)(1-s) \end{aligned}\right\} \tag{1}$$

$$\left.\begin{aligned} x &= \sum_{k=1}^{8} x_k A_k(s,t) \\ y &= \sum_{k=1}^{8} y_k A_k(s,t) \end{aligned}\right\} \tag{2}$$

$$\left.\begin{aligned} \frac{\partial x}{\partial s} &= \sum_{k=1}^{8} \frac{\partial A_k}{\partial s} x_k, \quad \frac{\partial x}{\partial t} = \sum_{k=1}^{8} \frac{\partial A_k}{\partial t} x_k \\ \frac{\partial y}{\partial s} &= \sum_{k=1}^{8} \frac{\partial A_k}{\partial s} y_k, \quad \frac{\partial y}{\partial t} = \sum_{k=1}^{8} \frac{\partial A_k}{\partial t} y_k \end{aligned}\right\} \tag{3}$$

and

$$\left. J = \frac{\partial(x,y)}{\partial(s,t)} = \frac{\partial x}{\partial s}\frac{\partial y}{\partial t} - \frac{\partial y}{\partial s}\frac{\partial x}{\partial t} \right\} \tag{4}$$

3. Mesh generation by finite element method

An 8-noded quadrilateral mesh is a type of mesh that uses quadrilateral elements, each defined by eight nodes. This type of mesh is commonly used in finite element analysis for solving various engineering and scientific problems. Figure 2 illustrates the discretization of the L-, U-, and square-shaped domains for an eight-noded quadratic order. Tables 1-3 list the solutions obtained using the finite element method (FEM) at each coordinate. Here are the key characteristics and steps involved in generating an 8-noded quadrilateral mesh:

- Each element has four sides and eight nodes.
- The nodes are typically numbered consecutively around the element in a consistent manner.
- The shape functions for interpolation within the element are more complex compared to lower-order elements, providing higher accuracy.
- Identify and define the geometry of the domain for which the mesh is required. This could be a complex shape defined by mathematical equations or obtained from experimental data.

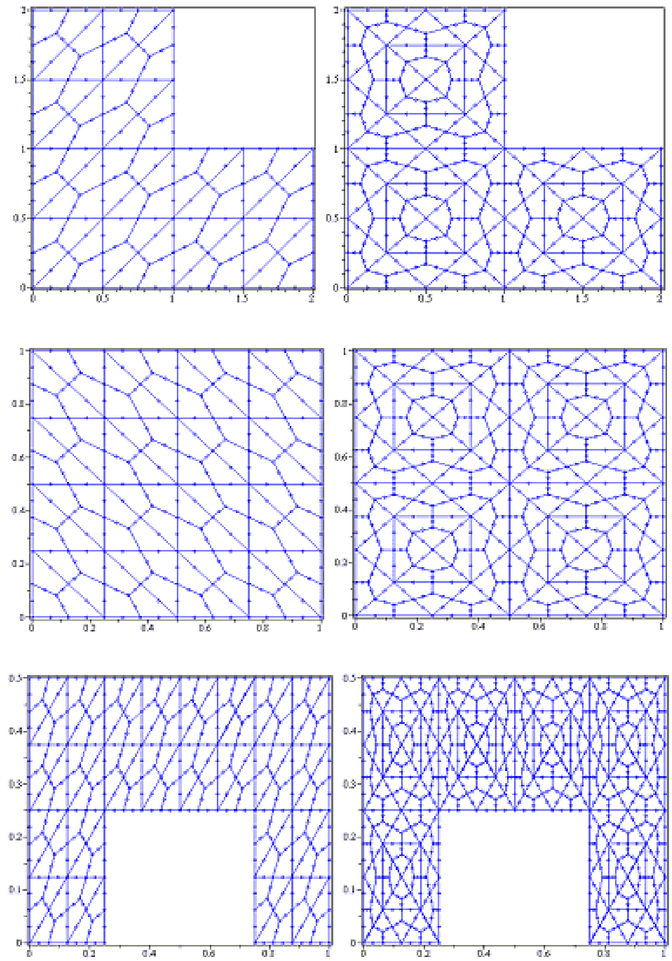

Figure 2. L-, U-, and square-shaped waveguides discretized into 8-noded quadrilateral meshes.

- Generate nodes within the domain. These nodes represent the discrete points where the numerical solution will be approximated. Ensure proper distribution of nodes to capture the geometry accurately.
- Define the connectivity between nodes to form quadrilateral elements. For an 8-noded quadrilateral element, there will be a set of eight nodes that define each element. The order of nodes is crucial for correct interpolation.
- Implement shape functions for interpolation within each 8-noded quadrilateral element. These shape functions determine how the solution varies within the element based on the values at its nodes.
- Check the quality of the generated mesh by examining metrics such as element aspect ratios, skewness, and other quality measures. A good-quality mesh contributes to the accuracy and stability of the numerical solution.
- If the mesh is intended for solving partial differential equations or other physical simulations, integrate it with appropriate solvers or simulation codes.

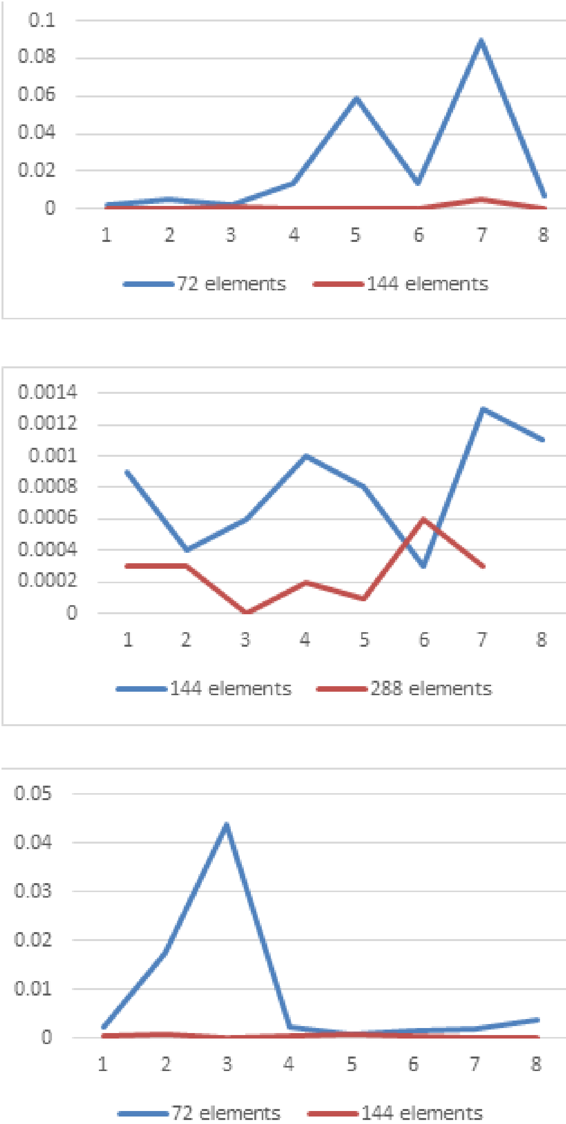

Figure 3. Relative error of Sarkar [10], Tingting [11], and the present method in L-, U-, and square-shaped waveguides.

- Apply boundary conditions to the nodes on the domain boundary. This step is crucial for solving problems with well-defined boundary constraints.

4. Finite element formulation procedure for solving Helmholtz equation

The proposed finite element approach employing transformations with the mesh generators provides the mathematical expression of the Helmholtz equation as given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \omega_c^2 u = 0. \tag{5}$$

The finite element method is utilized in MAPLE-13 to solve this problem using the family of eight-noded quadrilateral elements. For the TM modes, the wave amplitude is zero at the boundary, whereas for the TE modes, the normal derivative is zero, following the subsequent process:

- First, use the automated mesh generators to create an eight-noded quadrilateral mesh over the two-dimensional waveguide structure.
- Utilizing the Galerkin weighted residual finite element technique, the element geometry must be expressed in terms of the Lagrange shape function in order to derive the finite element equation:

$$[K + L]\, 8 \times 8 * U8 \times 1 = [0]_{8\times 1} \tag{6}$$

$$K_{i,j} = \iint_{\Omega} \left(\frac{\partial A_i}{\partial u}\frac{\partial A_j}{\partial u} + \frac{\partial A_i}{\partial v}\frac{\partial A_j}{\partial v}\right) dxdy = K_{u,u} + K_{v,v} \tag{7}$$

$$\begin{aligned} K_{u,u} &= \int_{-1}^{1}\int_{-1}^{1} \left(\frac{\partial v}{\partial \eta}\frac{\partial A_i}{\partial \xi} + \frac{\partial v}{\partial \xi}\frac{\partial A_i}{\partial \eta}\right) \\ &\quad * \left(\frac{\partial v}{\partial \eta}\frac{\partial A_j}{\partial \xi} + \frac{\partial v}{\partial \xi}\frac{\partial A_j}{\partial \eta}\right)\frac{1}{J} d\xi d\eta \end{aligned}$$

$$\begin{aligned} K_{v,v} &= \int_{-1}^{1}\int_{-1}^{1} \left(\frac{\partial u}{\partial \eta}\frac{\partial A_i}{\partial \xi} + \frac{\partial u}{\partial \xi}\frac{\partial A_i}{\partial \eta}\right) \\ &\quad * \left(\frac{\partial u}{\partial \eta}\frac{\partial A_j}{\partial \xi} + \frac{\partial u}{\partial \xi}\frac{\partial A_j}{\partial \eta}\right)\frac{1}{J} d\eta d\xi \end{aligned}$$

$$L_{I,J} = \iint_{\Omega} \omega_c^2 A_i\, A_j\, dxdy = \int_{-1}^{1}\int_{-1}^{1} \omega_c^2 A_i\,(\xi,\eta)\, A_j\,(\xi,\eta) J d\eta d\xi.$$

To obtain the global matrix equation, assemble the element equations so that the effects of each element are considered for the entire region based on the global node numbering:

$$[K + L]_{N_P \times N_P} * U_{N_P \times 1} = [0]_{N_P \times 1}\,, \tag{8}$$

where N_P is the overall node count. Eq. (3) becomes an eigenvalue problem, and the wavenumbers ω_C are calculated using the formula $\omega_C = \sqrt{\text{eigenvalue}}$.

Eq. (8) yields ω_C, representing the TE modes' ability to evaluate the Helmholtz equation, reduced to an eigenvalue problem with m algebraic equations after applying boundary conditions to determine the TM mode.

$$[K + L]_{m\times m} * U_{m\times 1} = [0]_{m\times 1}\,. \tag{9}$$

To determine the TM mode, calculate the eigenvalues, and the cutoff wavenumber is determined by obtaining the smallest wavenumber possible. This approach uses an efficient 8-noded mesh generator, quadrature method, and a program implemented for the FEM technique to reduce computational time and common errors in FEM analysis. It reduces numerical and discretization errors in solving FEM equations. Therefore, various energy engineering applications, including microwave engineering, can be solved with an effective and precise numerical solution using the proposed technique.

Table 1. TM and TE mode cutoff wavenumbers over an L-shaped waveguide structure.

Computed in [11]		Present method		Present method	
TE	TM	No. of eight noded elements = 72		No. of eight noded elements=144	
		CPU Time=1.5		CPU Time=2.3	
		TE	TM	TE	TM
1.913	4.891	1.9113	4.8321	1.9133	4.8912
2.961	6.139	2.9660	6.1253	2.9610	6.1387
4.945	6.997	4.9433	6.9073	4.9456	6.9920
5.315	8.557	5.3015	8.5503	5.3154	8.5565

Table 2. TM and TE mode cutoff wavenumbers over a U-shaped waveguide structure.

Computed in [10]		Present method		Present method	
TE	TM	No. of eight noded elements=144		No. of eight noded elements=288	
		CPU Time=1.6		CPU Time=2.8	
		TE	TM	TE	TM
2.2688	12.2338	2.2679	12.2330	2.2685	12.2337
5.0149	12.4106	5.0145	12.4103	5.0146	12.4100
6.6289	14.2152	6.6283	14.2139	6.6289	14.2155
7.7097	15.8221	7.7087	15.8232	7.7099	15.8227

Table 3. TM and TE mode cutoff wavenumbers over a square shaped waveguide structure.

Computed in [11]		Present method		Present method	
TE	TM	No. of eight noded elements = 72		No. of eight noded elements=144	
		CPU Time=1.8		CPU Time=2.5	
		TE	TM	TE	TM
1.571	2.221	1.5688	2.2203	1.5713	2.2216
2.221	3.512	2.2037	3.5106	2.2218	3.5124
3.142	4.442	3.0983	4.4438	3.1422	4.4421
3.512	4.967	3.5096	4.9631	3.5125	4.9670

5. Numerical Examples

5.1. L-shaped, U-shaped, and square waveguides

Maxwell's equations are the PDEs that govern electromagnetic radiation. They reduce to the Helmholtz equation for the propagation of electromagnetic waves in long waveguides. The cutoff wavenumbers for a variety of waveguide configurations can be found using this equation. These cutoff wavenumbers have been obtained using various techniques in the literature. A quick and efficient method to achieve optimal solutions to the problem is presented. This method is used to solve a variety of electromagnetic problems for any waveguide and also normalizes the CPU time for each analysis. The proposed method for the L-, U-, and square-shaped waveguides is implemented in the MAPLE program to calculate the first four cutoff wavenumbers. The waveguide structures found in Sarkar *et al.* [10] and Tingting *et al.* [11] are examined. The structured 8-noded quadrilateral mesh for the L-, U-, and square-shaped waveguides is depicted in Figure 2. A singularity exists at one of the sharp edges of this structure. Tables 1-3 present the calculated cutoff wavenumbers for both the transverse and longitudinal modes in the L-, U-, and square-shaped waveguides, utilizing the proposed structured automated mesh generator consisting of eight-noded quadratic elements.

For electromagnetic problems, 8-noded quadratic elements perform better than 3-noded triangular elements, according to the numerical results from Tables 1-3. It is apparent that the computing time is significantly reduced by using the 8-noded automatic meshing approach. The error estimates for both approaches are displayed in Figure 3. The most effective and straightforward method for computing eigenvalues is provided by the proposed finite element methodology. It also offers a productive way to determine the TE and TM modes for any kind of waveguide configuration. Because there is minimal energy loss, this makes it one of the best approaches for efficient energy transmission in microwave-based applications. Therefore, this approach can be effectively used to address a range of energy, electromagnetic, and microwave-related problems.

6. Conclusions.

This work presented a FEM technique using an automated 8-noded mesh generator as a straightforward, effective, and precise numerical solution procedure for microwave applications. Tables 1-3 show the numerical results for three distinct waveguides used to compute the eigenvalues and, consequently, the TE and TM modes, demonstrating the applicability of the proposed method. It is evident that the method is easy to use and efficiently utilizes computer resources. It provides the most accurate estimate for determining the dominant TM and TE modes' cutoff wavenumbers for various waveguide geometries and topologies containing singularities. Therefore, this methodology can be effectively applied to numerous microwave applications as well as various other energy-related problems.

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