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Extension of hesitant fuzzy weight geometric (HFWG)-VIKOR method under hesitant fuzzy information

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Abstract

The hesitant fuzzy set (HFS) is an innovative approach to decision-making under uncertainty. This work is primarily concerned with the HFS decision matrix's aggregated operation. The introduction of induced VIKOR procedures, various extensions of HFSs aggregation operator, and essential approaches for multi-criteria decision-making (MCDM) are presented. This technique uses the aggregation operator, the HFWG operator, to rank alternatives and identify the compromise solution closest to the ideal solution. In this research work, the hesitant fuzzy weight geometric-VIKOR (HFWG-VIKOR) model is a novel technique to achieve this. By combining the hesitant fuzzy elements, the HFWG aggregation operator creates aggregated values expressed as a single value. As per the scope of our research work, MCDM under hesitant fuzzy sets with the HFWG-VIKOR method has been used, and their result revealed the best alternative is to find out. These results indicate good potential for objectives. The multi-criteria problem is then solved using the combined HFWG-VIKOR technique, and the outcomes are presented in an easy-to-understand way owing to aggregation operators. The application of the HFWG-VIKOR technique is finally illustrated using a numerical example.

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1. Introduction

The fuzzy sets [1–10] were expanded upon by Torra and Narukawa [11] and Torra [12] to create the HFS, which is a range of values that are available to the membership. Additionally, they talked about the connection between intuitionistic fuzzy sets and HFS. A thorough study of hesitant fuzzy information aggregation techniques was conducted by Xia and Xu [13] in order to address decision-making challenges while maintaining anonymity. They suggested a number of operators in different scenarios. Xu and Xia [14, 15] provide a thorough distance measurement for hesitant fuzzy data. Furthermore, a variety of HFS distance measures were provided, depending on which relevant similarity measurements may be generated. Xia *et al.* [16] used quasi-arithmetic methods to build many sets of aggregating processes for HFSs. The HFS is an excellent tool for handling ambiguity. Determining the membership degree of an element becomes more difficult when there are multiple possible values for it, together with a margin of error or possibility distribution on the possibility values. Every criterion can be described as an HFS that is expressed in terms of decision-makers' viewpoints. In these kinds of situations, the reluctant fuzzy set is quite helpful in avoiding such problems. A con-

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siderable amount of studies are being done on the ideas and methods of MCDM in fuzzy, uncertain contexts. Therefore, we extend the concept of the VIKOR approach by adding the HFWG aggregation operator to address MCDM problems with hesitant fuzzy elements.

The single evaluations for each alternative are combined into a collective evaluation in the aggregation process so that the conditions represented in all the evaluations are summarized. The worldwide assessment of the options is changed into a ranking of the options throughout the exploitation phase. There are a number of ways to do this, the most widely used of which is to derive a score function using a ranking algorithm [17]. Wang and Liu [18] have examined numerous families of aggregation operators [19–28]. Yager [29] invented the Ordered Weighted Averaging (OWA) operator, which is the most widely used of them. According to Zadeh [30], a fuzzy linguistic quantifier that indicates the percentage of satisfied criteria necessary for a satisfactory result, a fuzzy majority can be employed during the aggregation stage. The OWA operator is widely used for this reason, among others [31]. This is achieved by employing the linguistic quantifier to calculate the weights related to the OWA operator. The two most common fuzzy integrals are the Sugeno integral and the Choquet integral [32, 33]. In general, there are three steps in the OWA operator-based aggregating method: It is commonly known that the OWA operator is a specific instance of the Choquet integral, and hence, utilizing the OWA operator does not necessitate assuming independence of criteria [34]. Step 1 involves rearranging the input arguments in decreasing order; Step 2 involves defining operator weights; and Step 3 involves aggregating the rearranged arguments using the OWA weights.

In order to handle a discrete decision problem with competing and non-commensurable criteria, the multi-criteria VIKOR approach was developed by Opricovic and Tzeng [35]. The fundamental concept was to identify the positive ideal solution (PIS) and negative ideal solution (NIS) initially and then choose the optimal plan by comparing all of the available possibilities. This technique rates and selects from a range of options to identify compromise solutions for a scenario where there are competing needs. This may help those making the decisions in the end. Here, a compromise is defined as an agreement established via reciprocal concessions, and the compromise solution is the workable path that approaches the ideal as closely as possible.

The VIKOR was developed based on a certain “closeness” measure to the PIS. For this reason, it functions well in situations when the decision-maker prioritizes money over risk. Sayadi *et al.* [36] created the VIKOR methodology for interval number choice-making challenges with the goal of establishing a decision maker’s optimism level and determining the larger VIKOR method ranking through interval number comparisons. The hierarchical MCDM model that Sanayei *et al.* [37] created to address supplier selection issues in supply chain systems is based on fuzzy sets theory and the VIKOR approach. The fuzzy VIKOR technique was utilized by Chen and Wang [38] to deliberately and logically choose the best choice and compromise solution for each selection criterion. The study’s findings provide a helpful framework for handling complex scenarios where decision-makers must consider a number of variables. Vahdani *et al*. [39] created the interval-valued fuzzy VIKOR technique to manage MCDM situations with mismatched criteria weights by utilizing interval-valued fuzzy set notions.

Using VIKOR technology, San Cristóbal [40] selected a renewable energy project that complied with the Spanish Government’s Renewable Energy Plan. Sarkar *et al.* [41] propose a hybrid approach utilizing dual hesitant q-rung orthopair fuzzy Frank power partitioned Heronian mean aggregation operators to address sustainable urban transport solutions, enhancing multi-criteria group decision-making efficacy. Adeel *et al.* [42] introduce a hesitant fuzzy N-soft ELECTRE-I approach for multi-attribute decision-making, integrating hesitant fuzzy set theory and N-soft sets to enhance decision robustness and accuracy. Akram *et al.* [43] introduce a hesitant fuzzy soft expert set model for multi-attribute group decision-making, enhancing decision reliability and flexibility through the integration of hesitation and fuzziness. Akram *et al.* [44] propose an integrated ELECTRE-I approach for risk evaluation using hesitant Pythagorean fuzzy sets, enhancing failure modes and effects analysis (FMEA) by effectively incorporating vague data. Akram *et al.* [45] develop an ELECTRE-II method using hesitant Pythagorean fuzzy sets for multi-criteria decision-making, enhancing decision accuracy and flexibility through diverse expert opinions and comprehensive outranking analysis. Akram *et al.* [46] introduce an enhanced VIKOR method for multi-criteria group decision-making using complex Fermatean fuzzy N-soft sets, improving decision accuracy by handling two-dimensional uncertain information and employing a weighted average operator. Akram *et al.* [47] introduce the complex spherical fuzzy VIKOR (CSF-VIKOR) approach for multi-criteria group decision-making, enhancing decision accuracy by handling two-dimensional ambiguous information and employing new aggregation operators. Akram *et al.* [48] developed spherical fuzzy set-based outranking techniques using ELECTRE for decision-making in the digitalization of Istanbul’s public transportation, focusing on environmental impact reduction and group decision-making. Akram *et al.* [49] extend the MARCOS method for MCGDM by integrating 2-tuple linguistic q-rung picture fuzzy sets, enhancing decision-making with qualitative data and Dombi-based aggregation operators.

1.1. Research gaps and motivations

Besides, many researchers have explored the extension of the fuzzy VIKOR method. However, there is a deficiency in the existing literature on the fuzzy VIKOR method. The extended VIKOR method under hesitant fuzzy sets may involve increased computational complexity, particularly when dealing with large decision problems or complex hesitant fuzzy information. Efficient algorithms are necessary to handle the added computational load. Determining the ideal and anti-ideal solutions in the context of hesitant fuzzy sets can be challenging due to the inherent uncertainty and hesitancy. Furthermore, there is a research gap in the extension of the VIKOR method under hesitant fuzzy sets. The fuzzy VIKOR technique may be more computationally complex when managing huge decision problems or complex situations with hesitant fuzzy information, especially when tackling complicated or major decision-making issues. Efficient algorithms are necessary to handle the added computational load. Defining these solutions in a way that effectively captures the decision-maker's preferences may introduce ambiguity. The purpose of this research is to incorporate the VIKOR method under hesitant fuzzy sets with an aggregation operator involving the hesitant fuzzy weighted geometric (HFWG) aggregation operator. The proposed model aggregates the hesitant fuzzy element and induces these values in the fuzzy VIKOR method. The proposed model aggregates the hesitant fuzzy element and induces these aggregated values in the fuzzy VIKOR method, which solves the problem of uncertainty in the traditional VIKOR method. The proposed model proves to be a more generalized framework to capture the problems in uncertainty. Hence, motivated by these qualities, we aim to develop this model.

The novelty of this work is that it introduced the Hesitant fuzzy weight geometric-VIKOR (HFWG-VIKOR) method with hesitant fuzzy set information. This proposed method first aggregates the hesitant fuzzy sets with a hesitant fuzzy weight geometric operator, and the aggregated values are then applied to the VIKOR method.

1.2. Structure of the study

The information in the form of hesitant fuzzy elements and related concepts are explained in Sections 2 and 3 of the VIKOR method, which also introduces the fundamentals of the technique. The problem of multiple criteria decision marking is explained in Section 4, along with the suggested procedures and concepts of the HFWG-VIKOR method. A numerical example demonstrates how the HFWG-VIKOR approach is applied in Section 5. The last section included the conclusion.

2. Preliminaries

2.1. Definition [50]

Suppose X be a fixed set. The fuzzy set in X is defined by membership function $B\,\mu(x) : X \rightarrow [0,1]$ is given by

$$B = \{(x, \mu(x)), x \in X\}, \tag{1}$$

where $\mu(x)$ is the degree of membership of x in B, and each pair $(x, \mu(x))$ is singleton.

2.2. Definition [11, 12]

Suppose X be the universal set, Then a hesitant fuzzy sets as B on X is defined by function $\rho_B(\tilde{x})$ that X returns to subset of $[0,1]$ is given by:

$$B = \{\langle \tilde{x}, \rho_B(\tilde{x})\rangle \mid \tilde{x} \in X\}, \tag{2}$$

where $\rho_B(\tilde{x}) \in [0,1]$ and denotes the membership degree of an element in subsets of X.

2.3. Definition [13]

Suppose $h_1, h_2, h_3, \ldots, h_n$ be the group of hesitant fuzzy elements (HFEs). HFWG is a mapping $H^n \rightarrow H$

$$\mathrm{HFWG}\,(h_1, h_2, \ldots, h_n) = U_{\gamma_1 \in h_1, \gamma_2 \in h_2, \ldots, \gamma_n \in h_n} \left\{ \prod_{\mathrm{j}=1}^{\mathrm{n}} \left(\gamma_j\right)^{w_j} \right\}, \tag{3}$$

where $w_j \in [0,1]$ is the weight vector of $h_i (i = 1, 2, 3, ..n)$ and $\sum_{j=1}^{n} w_j = 1$.

3. Mathematical procedure of VIKOR method under HFSs

Opricovic suggested a compromise ranking technique called VIKOR [51]. The weight stability intervals for the compromise solution, the compromise ranking list, and the compromise solution are the first three things established by the VIKOR approach. In order to help with ranking and selection, the positive-ideal solution and the negative-ideal solution are ascertained. The fundamental idea behind the VIKOR MCDM approach is to handle the ranking and selection of alternatives with many competing or non-commensurable requirements. The VIKOR method was additionally expanded to account for subjectivity and imprecise data in a fuzzy environment, as is typical of most MCDM methodologies and presented in Figure 1.

Step 1:

Find out the positive ideal solution (PIS) and negative ideal solution (NIS) as follows:

$$A^* = \left\{h_1^*, h_2^*, h_3^*, \ldots, h_n^*\right\},$$

where $h_i^* = \cup_i^m h_{ij}$.

$$A^- = \left\{h_1^-, h_2^-, h_3^-, \ldots, h_n^-\right\},$$

where $h_i^- = \bigcap_i^m h_{ij}, \quad j = 1, 2, 3, \ldots, m.$

Step 2:

Compute maximum group utility $S_i (i = 1, 2, 3, \ldots, m)$ as given below

$$S_i = \sum_{j=1}^{n} w_j \frac{\left\| h_j^* - h_{ij} \right\|}{\left\| h_j^* - h_j^- \right\|}.$$

Calculate regret measure $R_i (i = 1, 2, 3, \ldots, m.)$ as given below

$$R_i = \max_j (S_i) = w_j \frac{\left\| h_j^* - h_{ij} \right\|}{\left\| h_j^* - h_j^- \right\|}$$

3.1. Step 3:

Evaluate the index value Q_i as given below

$$Q_i = v \frac{S_i - S^-}{S^* - S^-} + (1 - v) \frac{R_i - R^-}{R^* - R^-},$$

and v is introduced weight in a decision-making procedure, such as voting by majority rule $v > 0.5$ or by consensus $v \approx 0.5$ or with veto $v < 0.5$, where $S^* = \min_i (S_i), S^- = \max_i (S_i), R^* = \min_i (R_i)$ and $R^- = \max_i (R_i)$ where $i = 1, 2, 3, \ldots, m$.

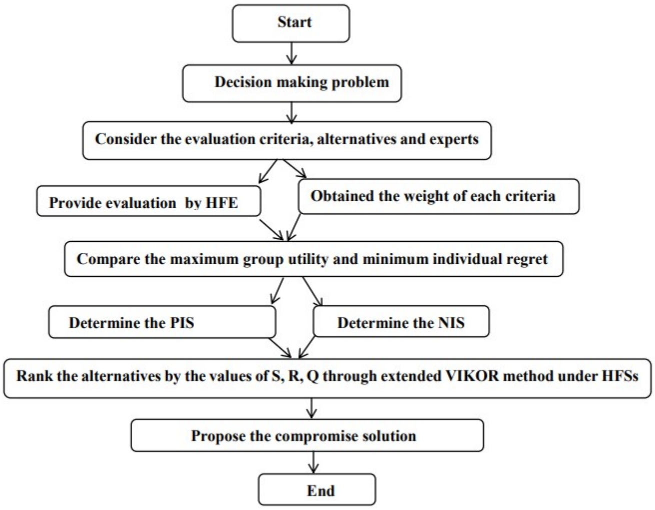

Figure 1. Mathematical step of VIKOR method under hesitant fuzzy sets.

Step 4:

Sort the alternatives into groups according to their S, R, and Q values, going from greatest to smallest.

Step 5:

The alternative (A_1) which is the first position through the lowest values of Q, as a compromise solution if the two conditions hold true as given below:

C1. "Acceptable advantage":

$$Q(A_2) - Q(A_1) \geq DQ_1,$$

where (A_2) is the alternative with 2^{nd} position in the grading list by $Q, DQ = \frac{1}{m-1}$. Where m denotes the possible alternatives.

C2 . "Acceptable stability in decision making":

An alternative (A_1) must be first ranked position from S or R. In case that one of the requirements is not met, the following compromise solution is suggested.

Alternative (A_1) and (A_2) if only C2 is not satisfied, or Alternatives (A_i) where $i = 1, 2, 3, \ldots, m$. if C1 is not satisfied; (A_m) is determined by the relation $(A_m) - Q(A_1) \leq DQ$ by maximum m.

4. Extension of hesitant fuzzy weight geometric operator-VIKOR method under hesitant fuzzy sets

The hesitant fuzzy weight geometric-VIKOR (HFWG-VIKOR) method with hesitant fuzzy set information. First, the hesitant fuzzy sets are aggregated with the hesitant fuzzy weight geometric operator. In the aggregated values, our positive ideal solution and negative ideal solution are then applied using the VIKOR method. The procedure of the hesitant fuzzy weight geometric-VIKOR method under hesitant fuzzy sets is given and represented in Figure 2.

Step 1:

Evaluate the hesitant fuzzy element through the aggregation operator using equation (3).

Step 2:

Find out a positive ideal solution (PIS) and negative ideal solution (NIS) of the aggregated values of the decision matrix as follows:

$$A^* = \left\{h_1^*, h_2^*, h_3^*, \ldots, h_n^*\right\},$$

where $h_i^* = \bigcup_i^m h_{ij}$

$$A^- = \left\{h_1^-, h_2^-, h_3^-, \ldots, h_n^-\right\},$$

where $h_i^- = \bigcap_i^m h_{ij}, \quad j = 1, 2, 3, \ldots, m.$

Step 3:

Compute maximum group utility S_i and individual regret measure R_i of the aggregated values of the decision matrix as given below

$$S_i = \sum_{j=1}^{n} w_j \frac{\left\| h_j^* - h_{ij} \right\|}{\left\| h_j^* - h_j^- \right\|}.$$

Calculate the regret measure as given below.

$$R_i = \max_j (S_i) = w_j \frac{\left\| h_j^* - h_{ij} \right\|}{\left\| h_j^* - h_j^- \right\|},$$

where $i = 1, 2, 3, \ldots, m$.

Step 4:

Evaluate the index values Q_i as given below

$$Q_i = v \frac{S_i - S^-}{S^* - S^-} + (1 - v) \frac{R_i - R^-}{R^* - R^-},$$

and v is introduced weight in a decision-making procedure, such as voting by majority rule $v > 0.5$ or by consensus $v \approx 0.5$ or with veto $v < 0.5$. where $S^* = \min_i (S_i), S^- = \max_i (S_i), R^* = \min_i (R_i)$ and $R^- = \max_i (R_i)\, i = 1, 2, 3, \ldots, m$.

Step 5:

Sort the alternatives into groups according to their S, R, and Q values, going from greatest to smallest.

Step 6:

The alternative (A_1) which is the first position through the lowest values of Q, as a compromise solution if the given two cases are satisfied:

C1. "Acceptable advantage":

$Q(A_2) - Q(A_1) \geq DQ$

Where (A_2) is the alternative with 2^{nd} position in the grading list by $Q, DQ = \frac{1}{m-1}$. Where m denotes the possible alternatives.

C2."Acceptable stability in decision-making":

Also, it is necessary that the alternative (A_1) is the first position ranked through the values of S or R.

In case that one of the requirements is not met, the following compromise solution is suggested.

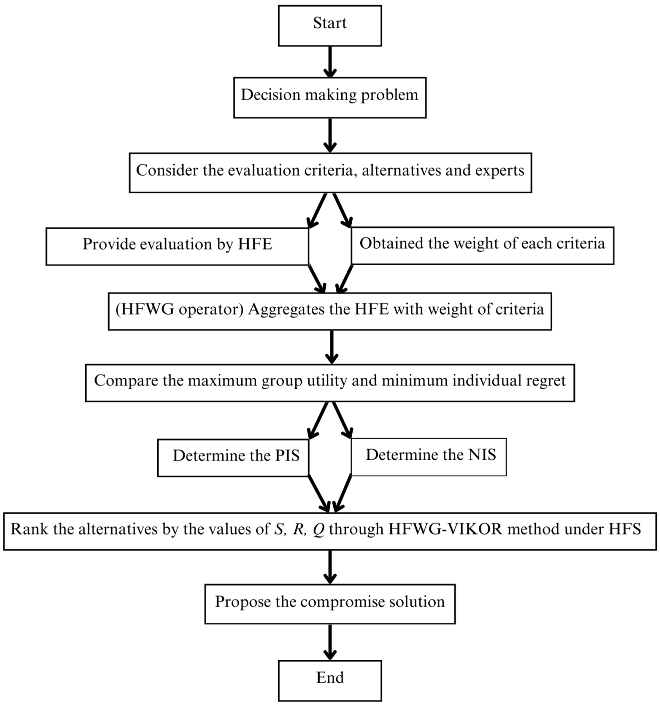

Figure 2. Steps of $HFWG-VIKOR$ method under hesitant fuzzy sets.

i Alternative (A_1) and (A_2) if only C 2 is not satisfied, or

ii Alternatives (A_i) where $i = 1, 2, 3, \ldots, m$. if C 1 is not satisfied; (A_m) is determined by the relation $Q(A_m) - Q(A_1) \leq DQ$ by maximum m.

5. Numerical example

Suppose someone rented a vehicle, namely A, B, C and D, on the basis of multiple criteria with criteria weight $W = (0.11, 0.21, 0.29, 0.39)$ as given below

κ_1 : Taxation κ_2 : Rental period κ_3 : Rental and other charges κ_4 : Use of the rental vehicle and represented in Table 1.

VIKOR method under HFSs

Step 1:

The PIS and NIS are as given below:

$$A^* = [0.8, 0.9, 0.9, 0.9]$$
$$A^- = [0.1, 0.4, 0.5, 0.3].$$

Step 2:

Calculated S_i as given below

$$S_i = \sum_{j=1}^{n} w_j \frac{\left\| h_j^* - h_{ij} \right\|}{\left\| h_j^* - h_j^- \right\|}$$

$$S_1 = 0.4677, S_2 = 0.3725, S_3 = 0.3688, S_4 = 0.3927.$$

Calculated regret measure R_i as given below:

$$R_i = \max_j (S_i) = w_j \frac{\left\| h_j^* - h_{ij} \right\|}{\left\| h_j^* - h_j^- \right\|}$$

$$R_1 = 0.1812, R_2 = 0.145, R_3 = 0.1631, R_4 = 0.195.$$

Step 3:

Evaluated Q_i as given below where $i = 1, 2, 3, 4$.

$$Q_i = v \frac{S_i - S^-}{S^* - S^-} + (1 - v) \frac{R_i - R^-}{R^* - R^-},$$

and $v = 0.5$ where $S^* = 0.3688, S^- = 0.4677, R^* = 0.145, R^- = 0.195$.

$Q_1 = 0.138, Q_2 = 0.9812, Q_3 = 0.819, Q_4 = 0.3791$.

Step 4:

Sort the alternatives into groups according to their S, R, and Q values, going from greatest to smallest. Table 2 lists the outcomes for the three grades.

Step 5:

The alternative (A_1) which is the first position through the lowest values of Q, as a compromise solution if the two factors persist as given below:

$$Q(A_2) - Q(A_1) \geq DQ = 0.3791 - 0.138 = 0.241 \ngeq 0.333,$$

so not satisfied.

C2. Also, the alternative (A_1) is not the first ranked position from S or R. In case one of the requirements is not met, the following compromise solution is suggested.

$$Q(A_m) - Q(A_1) \leq DQ \text{ Satisfied.}$$

Extension of hesitant fuzzy weight averaging operator-VIKOR (HFWG-VIKOR) method

Step 1:

Evaluate the hesitant fuzzy element through the equation (3):

$$h_{11} = 0.2441, h_{12} = 0.8002, h_{13} = 0.6879, h_{14} = 0.8230$$
$$h_{21} = 0.3489, h_{22} = 0.7546, h_{23} = 0.7536, h_{24} = 0.8540$$
$$h_{31} = 0.3875, h_{32} = 0.8230, h_{33} = 0.7203, h_{34} = 0.8540$$
$$h_{41} = 0.5462, h_{42} = 0.8754, h_{43} = 0.7890, h_{44} = 0.6983.$$

It can be written in matrix form

$$\begin{bmatrix} 0.2441 & 0.8002 & 0.6879 & 0.8230 \\ 0.3489 & 0.7546 & 0.7536 & 0.8540 \\ 0.3875 & 0.8230 & 0.7203 & 0.8540 \\ 0.5462 & 0.8754 & 0.7890 & 0.6983 \end{bmatrix}.$$

Table 1. Hesitant fuzzy decision matrix.

Alternatives	κ_1	κ_2	κ_3	κ_4
A	$(0.1, 0.2, 0.3)$	$(0.5, 0.7, 0.9)$	$(0.5, 0.6, 0.7, 0.8)$	$(0.5, 0.8, 0.9)$
B	$(0.2, 0.3, 0.4)$	$(0.4, 0.7, 0.8, 0.9)$	$(0.6, 0.7, 0.8)$	$(0.7, 0.8, 0.9)$
C	$(0.3, 0.4)$	$(0.5, 0.8, 0.9)$	$(0.5, 0.6, 0.7, 0.9)$	$(0.7, 0.8, 0.9)$
D	$(0.2, 0.4, 0.6, 0.8)$	$(0.7, 0.9)$	$(0.6, 0.7, 0.8, 0.9)$	$(0.3, 0.7, 0.8)$

Table 2. Ranking the alternatives.

Alternatives	S	R	Q	Ranking
A	0.4677	0.1812	0.138	1
B	0.3725	0.145	0.9812	4
C	0.3688	0.1631	0.819	3
D	0.3927	0.195	0.3791	2

Step 2:

The PIS and NIS are given below:

$$A^* = [0.5462, 0.8754, 0.7890, 0.8540]$$
$$A^- = [0.2441, 0.7546, 0.6879, 0.6983].$$

Step 3:

Calculated S_i and R_i values as given below:

$$S_i = \sum_{j=1}^{n} w_j \frac{\left\| h_j^* - h_{ij} \right\|}{\left\| h_j^* - h_j^- \right\|}$$

$$S_1 = 0.6079, S_2 = 0.3833, S_3 = 0.3453, S_4 = 0.39.$$

Calculated regret measure:

$$R_i = \max_j (S_i) = w_j \frac{\left\| h_j^* - h_{ij} \right\|}{\left\| h_j^* - h_j^- \right\|}$$

$$R_1 = 0.29, R_2 = 0.21, R_3 = 0.1970, R_4 = 0.39.$$

Step 4:

Evaluated Q_i as given below where $i = 1, 2, 3, 4$.

$$Q_i = v \frac{S_i - S^-}{S^* - S^-} + (1 - v) \frac{R_i - R^-}{R^* - R^-},$$

and $v = 0.5$ where $S^* = 0.3435, S^- = 0.6079, R^* = 0.1970$, and $R^- = 0.39$.

$Q_1 = 0.9818, Q_2 = 0.106, Q_3 = 0.000, Q_4 = 0.5851.$

Step 5:

Sort the alternatives into groups according to their S, R, and Q values, going from greatest to smallest. The outcome in three grades is listed in Table 3.

Table 3. Ranking the alternatives.

Alternatives	S	R	Q	Ranking
A	0.6079	0.29	0.9818	4
B	0.3833	0.21	0.106	2
C	0.3453	0.1970	0.000	1
D	0.390	0.39	0.5851	3

Table 4. Hesitant fuzzy decision matrix.

Alternatives	c_1	c_2	c_3
A_1	$(0.2, 0.3, 0.4)$	$(0.3, 0.4, 0.5)$	$(0.8, 0.9)$
A_2	$(0.1, 0.6, 0.7)$	$(0.7, 0.8, 0.9)$	$(0.3, 0.4, 0.5)$
A_3	$(0.4, 0.8)$	$(0.1, 0.3, 0.4)$	$(0.2, 0.3)$
A_4	$(0.1, 0.4, 0.5)$	$(0.1, 0.2, 0.5)$	$(0.4, 0.5)$
A_5	$(0.5, 0.7, 0.9)$	$(0.2, 0.3, 0.4)$	$(0.7, 0.8)$

Step 6:

The alternative (A_1) which is the first position through the lowest values of Q, as a compromise solution if the two factors persist as given below:

C1. $Q(A_2) - Q(A_1) \geq DQ = 0.106 - 0.000 \ngeq 0.333$.

C2. Also, alternative (A_1) is not the first ranked position from S or R.

$$Q(A_m) - Q(A_1) \leq DQ \text{ Satisfied.}$$

5.1. Numerical example

Suppose someone is buying a new motorcycle, namely A_1, A_2, A_3, A_4 and A_5 on the basis of multiple criteria c_1 : Fuel efficiency, c_2 : Spare parts and accessories and c_3 : Price of the motorcycle with criteria weight $W = (0.23, 0.33, 0.44)$ represented in Table 4.

VIKOR method under HFSs

Step 1:

The PIS and NIS are as given below:

$$A^* = [0.9, 0.9, 0.9]$$
$$A^- = [0.1, 0.1, 0.2].$$

Step 2:

Calculated S_i as given below:

Table 5. Ranking the alternatives.

Alternatives	S	R	Q	Ranking
A_1	0.3956	0.2062	1.000	5
A_2	0.5367	0.3142	0.5373	3
A_3	0.7559	0.4085	0.000	1
A_4	0.7069	0.2828	0.3788	2
A_5	0.399	0.2475	0.8932	4

$$S_i = \sum_{j=1}^{n} w_j \frac{\left\| h_j^* - h_{ij} \right\|}{\left\| h_j^* - h_j^- \right\|}$$

$S_1 = 0.3956, S_2 = 0.5367, S_3 = 0.7559, S_4 = 0.7069, S_5 = 0.399.$

Calculated regret measure R_i as given below:

$$R_i = \max_j (S_i) = w_j \frac{\left\| h_j^* - h_{ij} \right\|}{\left\| h_j^* - h_j^- \right\|}$$

$R_1 = 0.2062, R_2 = 0.3142, R_3 = 0.4085, R_4 = 0.2828, R_5 = 0.2475.$

Step 3:

Evaluated Q_i as given below where $i = 1, 2, 3, 4, 5$.

$$Q_i = v\frac{S_i - S^-}{S^* - S^-} + (1-v)\frac{R_i - R^-}{R^* - R^-},$$

and $v = 0.5$ where $S^* = 0.3956, S^- = 0.7559, R^* = 0.2062, R^- = 0.4085$.

$Q_1 = 1.00, Q_2 = 0.5373, Q_3 = 0.000, Q_4 = 0.3788, Q_5 = 8932.$

Step 4:

Sort the alternatives into groups according to their S, R, and Q values, going from greatest to smallest. Table 5 lists the outcomes for the three grades.

Step 5:

The alternative (A_1) which is the first position through the lowest values of Q, as a compromise solution if the two factors persist as given below:

$$Q(A_2) - Q(A_1) \geq DQ = 0.3788 - 0.000 = 0.3788 \geq 0.25,$$

so satisfied.

Extension of hesitant fuzzy weight averaging operator- VIKOR (HFWG-VIKOR) method

Step 1:

Evaluate the hesitant fuzzy element through the equation (3):

$$h_{11} = 0.3101, h_{12} = 0.4013, h_{13} = 0.8759$$
$$h_{21} = 0.4252, h_{22} = 0.8170, h_{23} = 0.4130$$
$$h_{31} = 0.5426, h_{32} = 0.2644, h_{33} = 0.2732$$
$$h_{41} = 0.3207, h_{42} = 0.2552, h_{43} = 0.4749$$
$$h_{51} = 0.7236, h_{52} = 0.3101, h_{53} = 0.7758$$

It can be written in matrix form

$$\begin{bmatrix} 0.3101 & 0.4013 & 0.8759 \\ 0.4252 & 0.8170 & 0.4130 \\ 0.5426 & 0.2644 & 0.2732 \\ 0.3207 & 0.2552 & 0.4749 \\ 0.7236 & 0.3101 & 0.7758 \end{bmatrix}$$

Step 2:

The PIS and NIS are given below:

$$A^* = [0.7236, 0.8170, 0.8759]$$
$$A^- = [0.3101, 0.2552, 0.2732]$$

Step 3:

Calculated S_i and R_i values as given below

$$S_i = \sum_{j=1}^{n} w_j \frac{\left\| h_j^* - h_{ij} \right\|}{\left\| h_j^* - h_j^- \right\|}$$

$S_1 = 0.4741, S_2 = 0.5035, S_3 = 0.8645, S_4 = 0.8468, S_5 = 3707.$

Calculated regret measure

$$R_i = \max_j (S_i) = w_j \frac{\left\| h_j^* - h_{ij} \right\|}{\left\| h_j^* - h_j^- \right\|}$$

$R_1 = 0.2441, R_2 = 0.3376, R_3 = 0.44, R_4 = 0.33, R_5 = 0.2977.$

Step 4:

Evaluated Q_i as given below where $i = 1, 2, 3, 4, 5$.

$Q_i = v\frac{S_i - S^-}{S^* - S^-} + (1-v)\frac{R_i - R^-}{R^* - R^-}$ and $v = 0.5$ where $S^* = 0.3707, S^- = 0.8645, R^* = 0.2441,$ and $R^- = 0.44$.

$Q_1 = 0.8949, Q_2 = 0.6268, Q_3 = 0.000, Q_4 = 0.2986, Q_5 = 0.1368.$

Step 5:

Sort the alternatives into groups according to their S, R, and Q values, going from greatest to smallest. Table 6 lists the outcome in three grades.

Step 6:

The alternative (A_1) which is the first position through the lowest values of Q, as a compromise solution if the two factors persist as given below:

C1. $Q(A_2) - Q(A_1) \geq DQ = 0.1368 - 0.000 \geq 0.25.$

C2. Also alternative (A_1) is not the first ranked position from S or R.

$$Q(A_m) - Q(A_1) \leq DQ \text{ Satisfied.}$$

Table 6. Ranking the alternatives.

Alternatives	S	R	Q	Ranking
A_1	0.4741	0.2441	0.8949	5
A_2	0.5035	0.3376	0.6268	4
A_3	0.8645	0.44	0.000	1
A_4	0.8468	0.33	0.2986	3
A_5	0.3707	0.2977	1368	2

5.2. *Discussion and analysis*

In this section, we discuss solving the numerical analysis on the extension of the VIKOR method under HFSs and the hesitant fuzzy weight geometric operator- VIKOR method. In an extension of the VIKOR method under HFSs, we first find the positive ideal solution and negative ideal solution and also use hesitant normalized hamming distance while finding the maximum group utility S_i, minimum individual regret R_i and index value Q_i but in the proposed approach as hesitant fuzzy weight geometric operator-VIKOR method (HFWG-VIKOR), there is no need to apply these steps in the fuzzy VIKOR method, just applying aggregation operator which aggregates the multiple values to single and the method under hesitant fuzzy sets looks like a traditional form of VIKOR method and solves the hesitant fuzzy data very easily. As the numerical examples were solved in the extension of the VIKOR method under hesitant fuzzy sets and proposed approach. In the VIKOR method under hesitant fuzzy sets there is some difficulty in finding positive ideal solution and negative ideal solution, and also normalized haming distance formula also applied for finding maximum group utility S_i and minimal individual regret R_i are used but as compared to the proposed approach aggregate the hesitant fuzzy sets and then find out the positive ideal solution and negative ideal solution, maximum group utility and minimal individual regret and there is no need to apply normalize haming distance but the data are in hesitant fuzzy sets and solved it like in the traditional VIKOR method.

6. Conclusion

When there are competing criteria for a set of options, the VIKOR technique was created as an MCDM model to establish the order of choice. The decision-makers may accept the negotiated agreement because it increases the majority's overall profit and reduces the opponent's personal regret. In this study, we evaluated the use of induced aggregation operators in the VIKOR method under hesitant fuzzy information and formed an integrated HFWG-VIKOR model to solve multi-criteria problems with differing and non-commensurable criteria, specifically taking into account the complicated subjective character of the decision-maker. The comprehensive proposed method accounts for the complex subjective nature of the decision maker and solves the issues in increased computational complexity, particularly with large decision problems or complex, hesitant fuzzy information. This study also evaluated the VIKOR techniques with the help of aggregation operators under hesitant fuzzy information. The HFWG-VIKOR technique helps the decision-maker choose the best meets their needs by giving them a complete grasp of the process. The numerical example was solved through the proposed method, and the issues in the extended VIKOR method were solved under hesitant fuzzy information. The HFWG-VIKOR technique helps the decision-maker choose the best meets their needs by giving them a complete grasp of the process. In a decision-making situation involving choosing a new integrated method, we have examined numerical examples where we can observe the findings made using the (HFWG)-VIKOR method.

References

[1] L. A. Zadeh, "Fuzzy sets as a basis for a theory of possibility", Fuzzy sets and systems **1** (1978) 3. https://www.sciencedirect.com/science/article/abs/pii/0165011478900295.

[2] K. T. Atanassov, "More on intuitionistic fuzzy sets", Fuzzy sets and systems **33** (1989) 37. https://doi.org/10.1016/0165-0114(89)90215-7.

[3] K. T. Atanassov & K. T. Atanassov, *Intuitionistic fuzzy sets*, Springer, Physica-Verlag HD, 1999, pp. 1–137. https://doi.org/10.1007/978-3-7908-1870-3_1.

[4] E. Szmidt & J. Kacprzyk, "Distances between intuitionistic fuzzy sets", Fuzzy sets and systems **114** (2000) 505. https://www.sciencedirect.com/science/article/abs/pii/S0165011498002449.

[5] F. Chiclana, F. Herrera & E. Herrera-Viedma, "Integrating multiplicative preference relations in a multipurpose decision-making model based on fuzzy preference relations", Fuzzy sets and systems **122** (2001) 277.https://doi.org/10.1016/S0165-0114(00)00004-X.

[6] P. Grzegorzewski, "Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric", Fuzzy sets and systems **148** (2004) 319. https://doi.org/10.1016/j.fss.2003.08.005.

[7] H. Mitchell, "A correlation coefficient for intuitionistic fuzzy sets", International Journal of Intelligent Systems **19** (2004) 483. https://doi.org/10.1002/int.20004.

[8] Z. Xu & R. R. Yager, "Some geometric aggregation operators based on intuitionistic fuzzy sets", International Journal of General Systems **35** (2006) 417. https://doi.org/10.1080/03081070600574353.

[9] G. Wei, X. Zhao & R. Lin, "Some induced aggregating operators with fuzzy number intuitionistic fuzzy information and their applications to group decision making", International Journal of Computational Intelligence Systems **3** (2010) 84. https://doi.org/10.1080/18756891.2010.9727679.

[10] Z. Pei & L. Zheng, "A novel approach to multi-attribute decision making based on intuitionistic fuzzy sets", Expert Systems with Applications **39** (2012) 2560. https://doi.org/10.1016/j.eswa.2011.08.108.

[11] V. Torra & Y. Narukawa, *On hesitant fuzzy sets and decision*, IEEE international conference on fuzzy systems, 2009, pp. 1378–1382. https://doi.org/10.1109/fuzzy.2009.5276884.

[12] V. Torra, "Hesitant fuzzy sets", International Journal of Intelligent Systems **25** (2010) 529. https://doi.org/10.1002/int.20418.

[13] M. Xia & Z. Xu, "Hesitant fuzzy information aggregation in decision making", International Journal of Approximate Reasoning **52** (2011) 395. https://doi.org/10.1016/j.ijar.2010.09.002.

[14] Z. Xu & M. Xia, "Distance and similarity measures for hesitant fuzzy sets", Information sciences **181** (2011) 2128. https://doi.org/10.1016/j.ins.2011.01.028.

[15] Z. Xu & M. Xia, "On distance and correlation measures of hesitant fuzzy information", International Journal of Intelligent Systems **26** (2011) 410. https://doi.org/10.1002/int.20474.

[16] M. Xia, Z. Xu & N. Chen, "Some hesitant fuzzy aggregation operators with their application in group decision making", Group Decision and Negotiation **22** (2013) 259. https://doi.org/10.1007/s10726-011-9261-7.

[17] F. Chiclana, F. Herrera & E. Herrera-Viedma, "Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations", Fuzzy sets and Systems **97** (1998) 33. https://doi.org/10.1016/S0165-0114(96)00339-9.

[18] W. Wang & X. Liu, "Some hesitant fuzzy geometric operators and their application to multiple attribute group decision making", Technological

and Economic Development of Economy **20** (2014) 371. https://doi.org/10.3846/20294913.2013.877094.

[19] H. Zhu, J. Zhao & Y. Xu, "2-dimension linguistic computational model with 2-tuples for multi-attribute group decision making", Knowledge-Based Systems **103** (2016) 132. https://doi.org/10.1016/j.knosys.2016.04.006.

[20] F. Jin, Z. Ni, H. Chen & Y. Li, "Approaches to group decision making with intuitionistic fuzzy preference relations based on multiplicative consistency", Knowledge-Based Systems **97** (2016) 48. https://doi.org/10.1016/j.knosys.2016.01.017.

[21] S. Zeng, J. M. Merigo, D. Palacios-Marques, H. Jin & F. Gu, "Intuitionistic fuzzy induced ordered weighted averaging distance operator and its application to decision making", Journal of Intelligent & Fuzzy Systems **32** (2017) 11. https://content.iospress.com/articles/journal-of-intelligent-and-fuzzy-systems/ifs141219.

[22] S. Zeng & Y. Xiao, "TOPSIS method for intuitionistic fuzzy multiple-criteria decision making and its application to investment selection", Kybernetes **45** (2016) 282. https://www.emerald.com/insight/content/doi/10.1108/K-04-2015-0093/full/html.

[23] Z. Shouzhen, "An extension of OWAD operator and its application to uncertain multiple-attribute group decision-making", Cybernetics and Systems **47** (2016) 363. http://dx.doi.org/10.1080/01969722.2016.1182362.

[24] J. Qin, X. Liu & W. Pedrycz, "An extended VIKOR method based on prospect theory for multiple attribute decision making under interval type-2 fuzzy environment", Knowledge-Based Systems **86** (2015) 116. https://doi.org/10.1016/j.knosys.2015.05.025.

[25] S. Zeng, W. Su & C. Zhang, "Intuitionistic fuzzy generalized probabilistic ordered weighted averaging operator and its application to group decision making", Technological and Economic Development of Economy **22** (2016) 177. https://doi.org/10.3846/20294913.2014.984253.

[26] L. Dymova, P. Sevastjanov & A. Tikhonenko, "An interval type-2 fuzzy extension of the TOPSIS method using alpha cuts", Knowledge-Based Systems **83** (2015) 116. https://doi.org/10.1016/j.knosys.2015.03.014.

[27] C. Zhu, L. Zhu & X. Zhang, "Linguistic hesitant fuzzy power aggregation operators and their applications in multiple attribute decision-making", Information Sciences **367** (2016) 809. https://doi.org/10.1016/j.ins.2016.07.011.

[28] J. J. Peng, J. Q. Wang, L. J. Yang & X. H. Chen, "An extension of ELECTRE to multi-criteria decision-making problems with multi-hesitant fuzzy sets", Information Sciences **307** (2015) 113. https://www.sciencedirect.com/science/article/abs/pii/S0020025515001309.

[29] R. R. Yager, "On ordered weighted averaging aggregation operators in multicriteria decision-making", IEEE Transactions on systems, Man, and Cybernetics **18** (1988) 183. https://doi.org/10.1109/21.87068.

[30] L. A. Zadeh, "A computational approach to fuzzy quantifiers in natural languages", Computers & Mathematics with Applications **9** (1983) 149. https://doi.org/10.1016/B978-0-08-030253-9.50016-0.

[31] R. R. Yager, "Quantifier guided aggregation using OWA operators", International Journal of Intelligent Systems **11** (1996) 49. https://onlinelibrary.wiley.com/doi/abs/10.1002/(SICI)1098-111X(199601)11:1%3C49::AID-INT3%3E3.0.CO;2-Z.

[32] T. Murofushi & M. Sugeno, "An interpretation of fuzzy measures and the Choquet integral as an integral with respect to a fuzzy measure", Fuzzy sets and Systems **29** (1989) 201. https://doi.org/10.1016/0165-0114(89)90194-2.

[33] M. Grabisch & C. Labreuche, "A decade of application of the Choquet and Sugeno integrals in multi-criteria decision aid", Annals of Operations Research **175** (2010) 247. https://doi.org/10.1007/s10479-009-0655-8.

[34] C. Tan & X. Chen, "Intuitionistic fuzzy Choquet integral operator for multi-criteria decision making", Expert Systems with Applications **37** (2010) 149. https://doi.org/10.1016/j.eswa.2009.05.005.

[35] S. Opricovic & G.-H. Tzeng, "Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS", European Journal of Operational Research **156** (2004) 445. https://doi.org/10.1016/S0377-2217(03)00020-1.

[36] M. K. Sayadi, M. Heydari & K. Shahanaghi, "Extension of VIKOR method for decision making problem with interval numbers", Applied mathematical modelling **33** (2009) 2257. https://doi.org/10.1016/j.apm.2008.06.002.

[37] A. Sanayei, S. F. Mousavi & A. Yazdankhah, "Group decision making process for supplier selection with VIKOR under fuzzy environment", Expert systems with applications **37** (2010) 24. https://doi.org/10.1016/j.eswa.2009.04.063.

[38] L. Y. Chen & T. C. Wang, "Optimizing partners choice in IS/IT outsourcing projects: The strategic decision of fuzzy VIKOR", International Journal of production economics **120** (2009) 233. https://doi.org/10.1016/j.ijpe.2008.07.022.

[39] B. Vahdani, H. Hadipour, J. S. Sadaghiani & M. Amiri, "Extension of VIKOR method based on interval-valued fuzzy sets", The International Journal of Advanced Manufacturing Technology **47** (2010) 1231. https://doi.org/10.1007/s00170-009-2241-2.

[40] J. R. San Cristóbal, "Multi-criteria decision-making in the selection of a renewable energy project in spain: The Vikor method", Renewable Energy **36** (2011) 498. https://doi.org/10.1016/j.renene.2010.07.031.

[41] A. Sarkar, S. Moslem, D. Esztergár-Kiss, M. Akram, L. Jin & T. Senapati, "A hybrid approach based on dual hesitant q-rung orthopair fuzzy Frank power partitioned Heronian mean aggregation operators for estimating sustainable urban transport solutions", Engineering Applications of Artificial Intelligence **124** (2023) 106505. https://doi.org/10.1016/j.engappai.2023.106505.

[42] A. Adeel, M. Akram & N. Çağman, "Decision-making analysis based on hesitant fuzzy N-soft ELECTRE-I approach", Soft Computing **26** (2022) 11849. https://doi.org/10.1007/s00500-022-06981-5.

[43] M. Akram, G. Ali & J. C. R. Alcantud, "A new method of multi-attribute group decision making based on hesitant fuzzy soft expert information", Expert Systems **40** (2023) e13357. https://doi.org/10.1111/exsy.13357.

[44] M. Akram, A. Luqman & J. C. R. Alcantud, "An integrated ELECTRE-I approach for risk evaluation with hesitant Pythagorean fuzzy information", Expert Systems with Applications **200** (2022) 116945. https://doi.org/10.1016/j.eswa.2022.116945.

[45] M. Akram, A. Luqman & C. Kahraman, "Hesitant Pythagorean fuzzy ELECTRE-II method for multi-criteria decision-making problems", Applied Soft Computing **108** (2021) 107479. https://doi.org/10.1016/j.asoc.2021.107479.

[46] M. Akram, G. Muhiuddin & G. Santos-García, "An enhanced VIKOR method for multi-criteria group decision-making with complex Fermatean fuzzy sets", Mathematical Biosciences and Engineering **19** (2022) 7201. http://dx.doi.org/10.3934/mbe.2022340.

[47] M. Akram, C. Kahraman & K. Zahid, "Group decision-making based on complex spherical fuzzy VIKOR approach", Knowledge-Based Systems **216** (2021) 106793. https://doi.org/10.1016/j.knosys.2021.106793.

[48] M. Akram, K. Zahid & C. Kahraman, "Integrated outranking techniques based on spherical fuzzy information for the digitalization of transportation system", Applied Soft Computing **134** (2023) 109992. https://doi.org/10.1016/j.asoc.2023.109992.

[49] M. Akram, A. Khan, A. Luqman, T. Senapati & D. Pamucar "An extended MARCOS method for MCGDM under 2-tuple linguistic q-rung picture fuzzy environment", Engineering Applications of Artificial Intelligence **120** (2023) 105892. https://doi.org/10.1016/j.engappai.2023.105892.

[50] L. A. Zadeh, "Fuzzy sets", Information and control **8** (1965) 338. https://doi.org/10.1016/S0019-9958(65)90241-X.

[51] S. Opricovic, "Fuzzy VIKOR with an application to water resources planning", Expert Systems with Applications **38** (2011) 12983. https://doi.org/10.1016/j.eswa.2011.04.097.