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Exploring vibrational resonance in biophysical systems with fractional-order damping

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Abstract

Vibrational Resonance (VR), which is characterised by the enhancement of the maximum response of a weakly driven system's output signal induced by a high-frequency (HF) periodic signal, was numerically examined in a bi-harmonically driven dimensionless model of an enzyme-substrate reaction with fractional-order damping, which models coherent oscillations in brain waves. The model incorporates a damping force that depends on a non-integer (fractional) order derivative rather than the typical first-order derivative in classical damping models. The output response was obtained by solving the model numerically using Grünwald-Letnikov's fractional derivatives definition. The response amplitude, computed from the Fourier spectrum of the output signal, was used to characterise VR. The effect of the fractional-order damping coefficient on the observed VR was considered for different damping strength coefficients. Single-peak resonances were observed. The fractional-order damping modulated the observed VR in a manner similar to the damping strength in an integer-order system, by reducing the high-frequency signal amplitude at which VR occurs. Increased brain wave activity from enzyme-substrate reaction may be due to inherent energy transfers from changes in the rate of decay, hence significant behavioural changes in brain wave activity could be linked to inherent changes in the decay rate of the excited enzymes, even when there is no change in the number of enzyme-substrate carriers. This study reveals the potential of fractional-order damping for enhancing biophysical system modelling, with implications for understanding brain wave activities.

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1. Introduction

Molecules, atoms, and ions are the building blocks of biological systems, and they function coherently and efficiently by exhibiting quantum effects, such as the absorption of radiation at specific wavelengths (e.g., photosynthesis in plants, UV-ray absorption in DNA, and vision in humans), the making and breaking of chemical bonds (e.g. enzyme-catalysed reactions and enzymatic cleavage), the conversion of chemical energy into mechanical motion (e.g. Adenosine Triphosphate (ATP) hydrolysis), and single-electron transfer through biological polymers (e.g. DNA, proteins, and iron-sulphur proteins)

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[1–3]. Biophysical models describe how billions of biological components function cohesively to produce macroscopic effects, which are fundamental to life processes, using the laws of physics. In physics, biophysical models provide simplified answers that focus on collective behaviour rather than the comprehensive microscopic descriptions considered in biology and chemistry. The laws of physics commonly applied are Newton's laws. The development of differential calculus facilitated the extension of Newton's equations to describe continuous changes in various physical quantities.

Differential equations describe how continuous systems evolve, and the behaviour of a system can be analysed by solving the modelled differential equation of motion [4]. Usually, a system's evolution is interpreted through a series of phenomena such as limit cycles, chaos, and resonance [5, 6]. Biophysical mechanics modelling accounts for memory effects during system evolution. Hence, such systems are modelled with fractional derivatives [7, 8].

Fractional-order differential equations (FDEs) contain arbitrary (non-integer) order derivatives. A system is said to be a fractional-order system if its dynamics can be modelled by a fractional differential equation [9]. Fractional calculus offers two key benefits, which account for the remarkable increase in the applicability of FDEs across various scientific and engineering domains [9–14]. First, it provides access to unlimited system memory and hereditary properties, which are highly useful in the modelling of natural systems. Secondly, the additional fractional-order (FO) parameters introduced into the system create more space and flexibility. Furthermore, it is often difficult to establish the dynamics between two different points when the system is modelled with an integer-order (IO) derivative. Consequently, FO systems reveal new, previously unobserved, and intriguing system behaviour, improving the physical modelling of real-life problems with greater reliability and precision than their IO counterparts [15].

FDEs have been adapted to model biological systems due to the need to incorporate long-range temporal memory effects, which are present in many life processes, into existing biological models [16]. Many intriguing results have been obtained from these studies. For instance, the electrical conductance of biological organisms' cell membranes is of fractional order [17]. The stability conditions, bifurcations, and chaos of fractional-order prey-predator systems are fractional-order dependent [18].

In epidemiology, the impact of infectious disease transmission within a community is established using a fractional operator. The result is important for decision-making on disease control [19]. Mansal and Sene (2020) deduced that the dynamics of the fractional-order fishery model depend critically on the harvesting rate [20]. The dynamics of the hepatitis E virus, modelled using a fractional derivative, are explained in terms of the existence and uniqueness of solutions to the model [21]. The mathematical modelling of tuberculosis infection dynamics, with a prototype solution highlighting the significance of the fractional operator, has been studied in [22]. For a more extensive review of fractional calculus and its applications to epidemic models in life sciences, we encourage readers to consult the recent review by Nisar et al. (2024) [19].

Generally, FDEs have been reported to exhibit chaotic attractors [23, 24], hidden attractors [25], self-excited attractors [26], resonance, and other fascinating features of nonlinear differential equations, including vibrational resonance [27]. The resonance phenomenon is especially important because, due to their resonance, bodies and cells can transfer energy between domains, resulting in a state of interference and interaction that maintains the connectivity of the whole universe. The concept of VR in biophysical systems is important for understanding image processing techniques such as Magnetic Resonance Imaging (MRI) and Positron Emission Tomography (PET). Since resonance involves energy transfer, enzyme-substrate reaction modelling brain wave activity is interpreted in terms of increased processes, such as enzymatic combination-recombination processes, chemical reactions, transport processes, and decay rate of excited enzymes.

Vibrational Resonance (VR) is a deterministic phenomenon characterised by the enhancement of the response of a weak signal to an external high-frequency perturbation [27, 28]. It serves as a deterministic substitute for noise in stochastic processes [29–31]. Many of the systems considered in studies on VR are modelled as ODEs [32–38]. However, there is a significant body of literature analysing vibrational resonance in fractional-order systems. For instance, an overdamped fractional Duffing oscillator exhibits a double-resonance pattern due to the fractional-order derivative term [39].

VR has been examined in several fractional-order systems, including the Duffing system [40], the quintic oscillator [41], delayed systems [42], multistable systems [43], the Toda oscillator [44], and biophysical models such as the FitzHugh-Nagumo (FHN) neuron model [45] and a birhythmic biological system [46]. For a comprehensive review of the concept of VR–including its variants, the many systems in which it has been observed, the types of external excitations and the mechanisms initiating the resonance phenomenon, as well as its applications–we refer the reader to a recent review by Yang et al. (2024) [27].

VR has been shown to be significantly impacted by fractional order. For instance, when compared to the integer-order model, the fractional order FHN neuron exhibits robust electrical activity [45]. The fractional-order damping coefficient has been shown to contribute significantly to the observed VR. For example, Mbong et al. (2016) [41] showed that damping influences the quintic oscillator's high-frequency force amplitude at resonance.

Fractional-order models are known to produce more abundant dynamical behaviours than their traditional integer-order counterparts. While fractional-order systems offer enhanced modelling capabilities, their role in modulating vibrational resonance in biophysical systems remains underexplored, and only a few studies have examined the implications of fractional-order damping in biological contexts. Specifically, the effect of fractional-order damping on observed vibrational resonance in coherent oscillations in a biophysical system modelling an enzyme-substrate reaction with ferroelectric behaviour in brain wave models has not been reported. Understanding vibra-

tional resonance in enzyme-substrate reactions can provide insights into brain wave dynamics and other biological phenomena. Hence, this research examines the role of fractional-order damping in the VR phenomenon in a biophysical system modelling coherent oscillations in brain waves.

2. The model

The model considered in this research describes coherent chemical oscillations of the enzyme-substrate reaction with ferroelectric behavior in brain wave model. It employs macroscopic theories to describe the functional complexity of biological materials as proposed by Froehlich [1], based on the following assumptions:

- A simple unit of biological system consists of biomolecules surrounded by water
- The configuration of the biological setup can lead to existence of very high electric field that causes a strongly polarized membrane.
- When energized, the setup exists in high electrically polarized metastable state.
- The oscillating polarized membrane can set up an electric vibration.
- The longest of these longitudinal electric vibrations sets up a dipole vibration of the whole system. In the presence of metabolic energy, these electric vibrations are strongly and coherently excited.
- The long-range selective interaction, coupled with the existence of this polar metastable state, explains the high sensitivity of certain biological systems to magnetic and electric fields. The presence of coherent electric vibrations may be decisive for the EEG observed in the brain. [47–49].

Certain chemical processes may be made possible by the selective transport of enzymes brought about by long-range interactions. The entire chemical oscillation results from the excitation and de-excitation of enzyme molecules from reactions with substrates, assuming the enzymes and substrates show long-range selective interactions that increase their influx. Each enzymatic transition from a weakly or non-polar ground state to a highly polar excited state chemically destroys a substrate molecule. Additionally, spontaneous de-excitation of enzymes from the excited state to the ground (or weakly polar) state takes place. Given the population of N excited and R unexcited enzymes, and S as the number of substrate molecules, then a biological reaction is described by

$$\begin{aligned} \frac{dN}{d\tau_0} &= vNRS - \xi N, \\ \frac{dS}{d\tau_0} &= \gamma S - vNRS, \\ \frac{dR}{d\tau_0} &= \xi N - vNR - \lambda(R - C), \end{aligned} \tag{1}$$

where ξ denotes the de-excitation decay rate of enzymes, v the nonlinear enzyme-substrate reaction strength, γ the range of attraction of the substrate particle due to the auto-catalytic reaction, C is the equilibrium concentration of the unexcited enzyme molecules when $N = S = 0$, that is, when neither the excited enzyme nor the substrate are present.

To simplify Equation (1), we use the adiabatic elimination of fast variable by supposes that the equilibrium of the unexcited enzyme concentration is reached fast, Equation (1) is reduced to the Lotka-Voltera equation of the form

$$\frac{dN}{d\tau_0} = vCNS - \xi N, \tag{2}$$

$$\frac{dS}{d\tau_0} = \gamma S - vCNS. \tag{3}$$

From equation (2) and (3), two steady states are obtained at $N_o, S_o = (0, 0)$ and $N_o S_o = \left(\frac{\gamma}{vC}, \frac{\xi}{vC}\right)$ and perturbing around this steady states give

$$\frac{d\varepsilon}{d\tau_0} = \gamma\eta - vCn\varepsilon, \tag{4}$$

$$\frac{d\eta}{d\tau_0} = -\xi\varepsilon - vCn\varepsilon, \tag{5}$$

where ε and η are respectively the excess concentration of activated enzymes and substrate molecules beyond their equilibrium value N_o and S_o. Given the electric resistance of the overall system due to oscillation as - σP and assuming that the macroscopic polarization P to be proportional to the time dependent number ε of excited enzymes molecules, then

$$\frac{d\varepsilon}{d\tau_0} = (k^2 e^{-\psi^2\varepsilon^2} - \sigma^2)\varepsilon. \tag{6}$$

Lastly accounting for the electric field F which interacts with polarization, then

$$\frac{d\varepsilon}{d\tau_0} = \gamma\eta + \left(k^2 e^{-\psi^2\varepsilon^2} - \sigma^2\right)\varepsilon + vCn\varepsilon + F(\tau_0). \tag{7}$$

Considering the series development in series of the function $e^{-\psi^2\varepsilon^2}$ at third order with a frequency of chemical oscillation as $\omega_0 = \sqrt{\xi\gamma}$, the following rescaling was applied.

$$\begin{aligned} &t = \omega_0\tau, \quad \omega_0^2 = \xi\gamma, \quad x = \Xi\varepsilon, \quad \Xi = \sqrt{\frac{3}{\kappa^2 - \sigma^2}}\kappa\psi, \quad , \\ &\mu = \frac{\kappa^2 - \sigma^2}{\omega_0}, \quad f(t) = \frac{\Xi}{\omega_0^2}\frac{d}{dt}F\left(\frac{t}{\omega_0}\right), \quad \alpha = \frac{5}{18\kappa^2}(\kappa^2 - \sigma^2), \\ &\beta = \frac{7}{162\kappa}(\kappa^2 - \sigma^2)^2. \end{aligned} \tag{8}$$

It was found that the biological system process was governed by the equation

$$\frac{d^2x}{dt^2} - \mu\left(1 - x^2 + \alpha x^4 - \beta x^6\right)\frac{dx}{dt} + x = f\cos\omega t, \tag{9}$$

where f and ω are amplitude and frequency of external excitation respectively and α, β and μ are positive parameters of nonlinearity [49].

By adding a fast periodic siganl, $g\cos\Omega t$ with amplitude g and frequency Ω ($\Omega \gg \omega$) to the system (Equation 9), and factoring in the memory capabilities of contributing properties, the biophysical system is modelled as a bi-harmonically driven FO system of the form

$$\begin{aligned} &\frac{d^{\lambda_2}x}{dt^{\lambda_2}} - \mu(1 - x^2 + \alpha x^4 - \beta x^6)\frac{d^{\lambda_1}x}{dt^{\lambda_1}} + x \\ &= g\cos\Omega t + f\cos\omega t, \end{aligned} \tag{10}$$

where $y = \frac{d^{\lambda_1}x}{dt^{\lambda_1}}$ and λ_1 and λ_2 are the non-integer orders of the differential equation. To obtain the output signal, Equation (10) is first expressed in form of a coupled lower order differential equation of the form

$$\begin{aligned} \frac{d^{\lambda_1}x}{dt^{\lambda_1}} &= y, \\ \frac{d^{\lambda_2}y}{dt^{\lambda_2}} &= \mu(1 - x^2 + \alpha x^4 - \beta x^6)y - x + g\cos\Omega t + f\cos\omega t. \end{aligned} \tag{11}$$

3. Numerical simulation of VR

To obtain the output signal, which corresponds to the solution of Equation (10), we apply an approximation method using discretisation of the corresponding fractional-order differentiator, which typically requires information about the previous state of the system, a condition known as the *memory effect*. Here, we apply the *short memory principle* to the Grünwald-Letnikov definition of the fractional derivative, so as to reduce the computational cost and control the accuracy of the numerical solution of the system [50].

The Grünwald-Letnikov definition of the fractional derivative is a simpler definition compared to other well-known definitions, such as the Riemann-Liouville and Caputo definitions. However, for a wide class of functions, the three definitions are equivalent under certain conditions (zero initial condition and lower terminal $a = 0$) [50, 51]. It is employed because of its simplicity in the discretisation of fractional-order operators, overcoming the difficulties of evaluating an integral and a gamma function embedded in the Caputo definition, as well as the rigorous practical implementation issues due to the difficulty in assigning some physical meaning to the initial conditions associated with the Riemann-Liouville definition.

The short memory principle is effective for time-dependent dynamical systems. The technique is an approximation, which assumes that, at large time t, the effect of the historical memory becomes negligible and can be truncated at a certain time interval, instead of considering all past values. As such, the infinite sum in the Grünwald-Letnikov definition of the fractional derivative is approximated by truncating the series to a finite number L, representing the "memory length" of the system. Consequently, the Grünwald-Letnikov binomial coefficients, which correspond to the values of the function around the initial condition (say $t = 0$), have a negligible contribution to the present state (say, the solution at time t). The approximate numerical solution is solely dependent on the system's recent history $[t - L, t]$, with the derivative computed over a moving low limit.

Generally, the technique guarantees that for $f(a) = 0$, the Riemann-Liouville, Caputo and Grünwald-Letnikov definitions of fractional derivative are equivalent for most functions [51]. By applying the short memory principle, the explicit numerical approximation of the Grünwald-Letnikov definition of the fractional derivative of order λ ($0 < \lambda < 1$) for a function $f(t)$ at the points $t_k = kh$, where $k = 1, 2, \ldots$, is given by

$$\begin{aligned} {}_L\mathcal{D}^{\lambda}_{t_k} &\approx \lim_{h\to 0}\frac{1}{h^{\lambda}}\sum_{j=0}^{[N(t)]}(-1)^j\binom{\lambda}{j}f(t_{k-j}) \\ &= \lim_{h\to 0}\frac{1}{h^{\lambda}}\sum_{j=0}^{[N(t)]}c_j^{(\lambda)}f(t_{k-j}), \end{aligned} \tag{12}$$

where h is the integration step size. The relation $N(t) = \min(\frac{t_k - L}{h}, \frac{L}{h})$ helps to eliminate the dependence on initial conditions $t = 0$ as normally required for systems with memory. $c_j^{(\lambda)}$ ($j = 0, 1, 2, \ldots, k$) is the binomial coefficient generalized for fractional order, which is computed using the recursive relation

$$c_0^{(\lambda)} = 1, \quad c_j^{(\lambda)} = \left(1 - \frac{1+\lambda}{j}\right)c_{j-1}^{(\lambda)}.$$

The discretisation of the system (Equation (11)) is achieved by using the relation for the explicit approximation of fractional derivatives (Equation (12)) in Equation (10), such that Equation (11) can be written in the discretised form as:

$$\begin{aligned} x(t_k) &= (y(t_{k-1}))h^{\lambda_1} - \sum_{j=1}^{N-1}c_j^{(\lambda_1)}x(t_{k-j}), \\ y(t_k) &= (-x(t_k) + \mu(1 - x(t_k)^2 + \alpha x(t_k)^4 - \beta x(t_k)^6)y(t_{k-1}) \\ &+ g\cos(\Omega h t_k))h^{\lambda_2} + f\cos(\omega h t_k))h^{\lambda_2} - \sum_{j=1}^{N-1}c_j^{(\lambda_2)}y(t_{k-j}). \end{aligned} \tag{13}$$

Resonance is typically described using the response factor Q, also referred to as the response amplitude. It provides insight into how an amplified output results from the parameter modulation of a high-frequency signal coupled to a weakly driven nonlinear system.

The Fourier spectrum of the output signal is typically used to calculate the response amplitude Q. This is because any periodic function can be approximated by the sum of its Fourier components. The response amplitude Q is therefore expressed in terms of the Fourier sine component Q_s and cosine component Q_c as

$$\begin{aligned} Q_s &= \frac{2}{nT}\int_0^{nT}x(t)\sin\omega t dt \\ Q_c &= \frac{2}{nT}\int_0^{nT}x(t)\cos\omega t dt, \end{aligned} \tag{14}$$

so that the amplitude, A and phase shift Φ are then computed from Equation (14) a

$$\begin{aligned} A &= \sqrt{Q_s^2 + Q_c^2} \\ \Phi &= \tan^{-1}\left(\frac{Q_s}{Q_c}\right), \end{aligned} \tag{15}$$

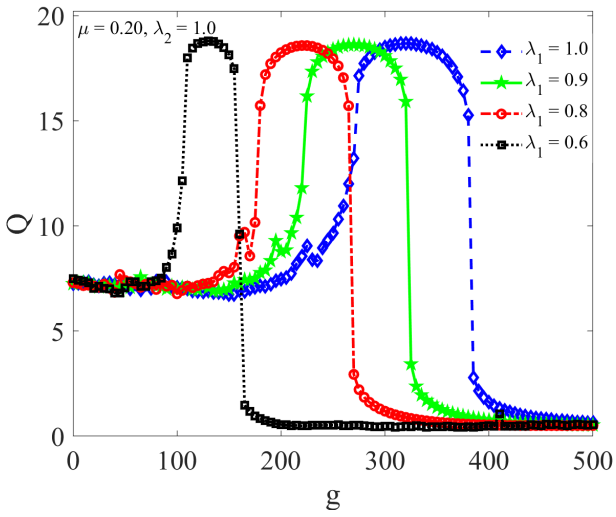

Figure 1. Resonance Curves (Response amplitude Q versus amplitude of the HF signal, g) for different values of arbitrary real fractional orders, $\lambda_1 = [1.0, 0.9, 0.8, 0.6]$. Other system parameters are set as: $\lambda_2 = 1.0$, $\mu = 0.20$, $\alpha = 0.5$, $\beta = 0.02$, $\omega = 0.2$, $\Omega = 50\omega$ and $f = 1.0$.

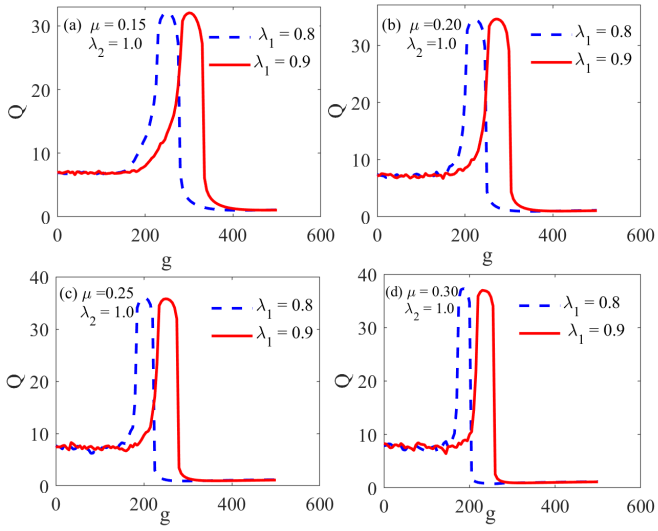

Figure 2. Resonance Curves (Response amplitude, Q versus amplitude of the HF signal, g) for two values of arbitrary fractional orders, $\lambda_1 = [0.9, 0.8]$ for (a) $\mu = 0.15$; (b) $\mu = 0.20$; (c) $\mu = 0.25$; and (d) $\mu = 0.30$. Other system parameters are set as: $\lambda_2 = 1.0$, $\alpha = 0.5$, $\beta = 0.02$, $\omega = 0.2$, $\Omega = 50\omega$ and $f = 1.0$.

so that the response factor Q is thus computed from

$$Q = \frac{A}{f} = \frac{\sqrt{Q_s^2 + Q_c^2}}{f}. \tag{16}$$

To generate the numerical results, Equation (13) was solved with zero initial conditions ($x(0) = y(0) = 0$). This choice is usually sufficient for the numerical integration of FDEs. The damping coefficients (α, β) are chosen within regimes that ensure the system remains in periodic oscillation, as resonance cannot be explored in a chaotic state. Additionally, the non-integer orders are constrained ($\lambda_2 = 1, \lambda_1 \leq 1$) to investigate the effect of fractional-order damping coefficients.

4. Results and discussion

The possibility of controlling and inducing nonlinear resonance phenomena through system components such as HF signal parameters, LF signal parameters, and damping coefficients has been established in the integer-order model of brain waves [52]. In this research, the major advantage of fractional-order systems is utilised to control the observed resonances. This is achieved by varying the order of the damping from an integer value to a fractional value. Figure 1 illustrates the effect of fractional-order damping $\lambda_1 = [1, 0.9, 0.8, 0.6]$ on the resonant state at $\mu = 0.2$. It can be observed that λ_1 reduces the value of the high frequency at which the maximum amplitude occurs, g_{VR}. This observation remains consistent for two fractional-order states, $\lambda_1 = 0.9$ and $\lambda_1 = 0.8$, across four different values of the damping strength μ, as shown in Figure 2(a)-(d) for $\mu = 0.15, 0.2, 0.25$, and 0.3, respectively.

The controlling effect of fractional-order damping on the maximum response amplitude, $Q_{\max}$, and the HF amplitude corresponding to VR, g_{VR}, is also observed in Figure 3, which depicts the variation of response amplitude Q with HF amplitude g for four values of the fractional-order damping coefficient, $\lambda_1 = [1, 0.9, 0.8, 0.6]$, at a constant damping coefficient, $\beta = 0.03$. Clearly, the observed effect of the fractional-order damping coefficient remains consistent for different values of damping nonlinearity, as shown in Figure 4(a)-(d) for $\beta = 0.018, 0.021, 0.025$, and 0.03, respectively.

The fractional-order damping coefficient, λ_1, significantly lowers g_{VR} and enhances the observed VR by slightly increasing the maximum response amplitude, $Q_{\max}$, for each of the four values of β considered. The role of the fractional-order coefficient remains unchanged when varying the value of the other damping coefficient, α, as shown in Figures 5 and 6. Figure 5 illustrates the dependence of the response amplitude on the HF amplitude, g, for different values of the fractional-order coefficient, $\lambda_1 = [1, 0.9, 0.8, 0.6]$.

The maximum response occurs at a lower HF amplitude, g, with the possibility of enhancement through the reduction of the fractional-order damping coefficient. This trend is also observed for four values of the nonlinear coefficient, α, as shown in Figure 6(a) for $\alpha = 0.20$; Figure 6(b) for $\alpha = 0.30$; Figure 6(c) for $\alpha = 0.40$; and Figure 6(d) for $\alpha = 0.50$. Additionally, there is a clear enhancement as α increases from 0.2 to 0.4, as reflected in the maximum response amplitude in Figures 6(a)-(d). The enhancement observed with increasing α has been previously discussed in Ref. [52].

In these results, the effect of fractional-order damping in controlling the vibrational resonant amplitude, g_{VR}, is consistent across all system parameters considered. Additionally, the possibility of enhancement and suppression through the modulation of the fractional order can be confirmed for different parameters. Therefore, for appropriate values of system param-

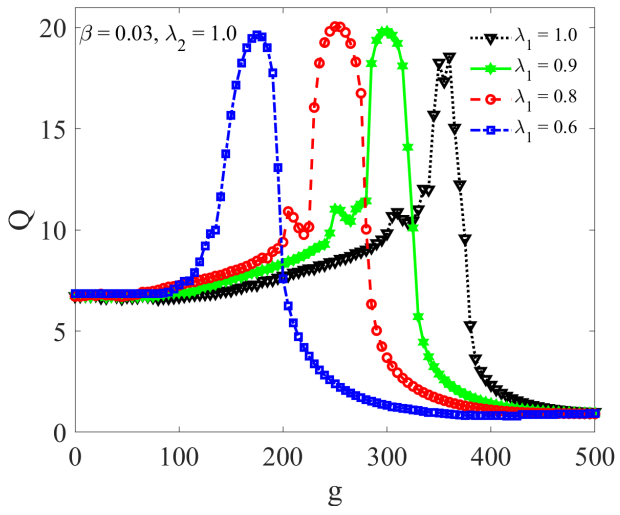

Figure 3. Resonance Curves (Response amplitude Q versus amplitude of the HF signal, g) for different values of arbitrary real fractional orders, $\lambda_1 = [1.0, 0.9, 0.8, 0.6]$. Other system parameters are set as: $\lambda_2 = 1.0$, $\mu = 0.2$, $\alpha = 0.5$, $\beta = 0.03$, $\omega = 0.2$, $\Omega = 50\omega$ and $f = 1.0$.

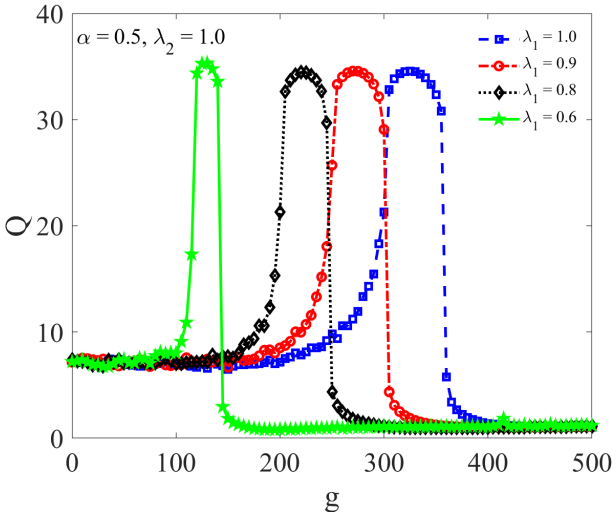

Figure 5. Response amplitude Q versus HF signal amplitude, g) for different fractional orders, $\lambda_1 = [1.0, 0.9, 0.8, 0.6]$. Other system parameters are set as: $\lambda_2 = 1.0$, $\alpha = 0.5$, $\mu = 0.2$, $\beta = 0.02$, $\omega = 0.2$, $\Omega = 50\omega$ and $f = 2.0$.

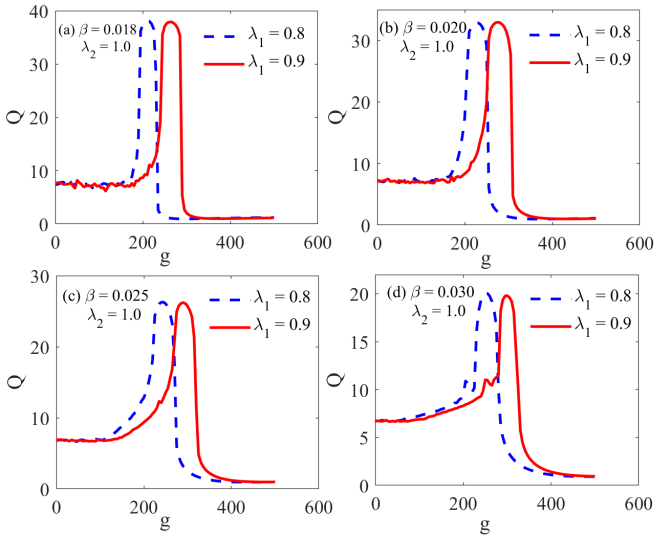

Figure 4. Resonance Curves (Response amplitude, Q versus amplitude of the HF signal, g) for two values of arbitrary real fractional orders, $\lambda_1 = [0.9, 0.8]$ for (a) $\beta = 0.018$; (b) $\beta = 0.020$; (c) $\beta = 0.025$; and (d) $\beta = 0.03$. Other system parameters are set as: $\lambda_2 = 1.0$, $\mu = 0.2$, $\alpha = 0.5$, $\omega = 0.2$, $\Omega = 50\omega$ and $f = 1.0$.

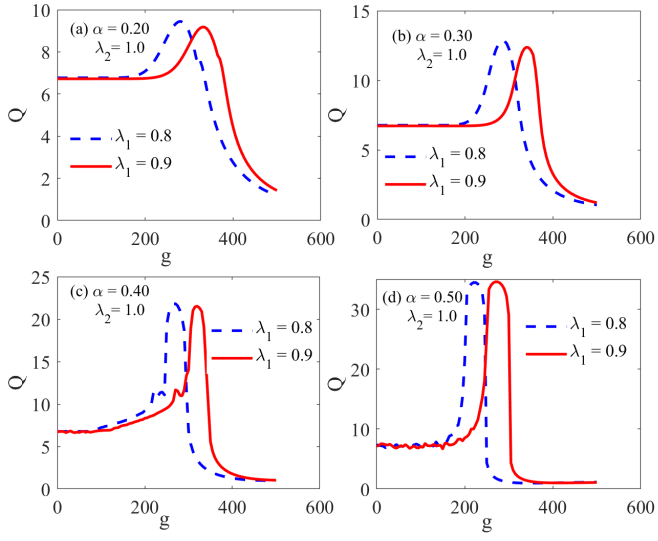

Figure 6. Resonance Curves (Response amplitude, Q versus amplitude of the HF signal, g) for two values of arbitrary fractional orders, $\lambda_1 = [0.9, 0.8]$ for (a) $\alpha = 0.2$; (b) $\alpha = 0.3$; (c) $\alpha = 0.4$; and (d) $\alpha = 0.5$. Other system parameters are set as: $\lambda_2 = 1.0$, $\mu = 0.2$, $\beta = 0.02$, $\omega = 0.2$, $\Omega = 50\omega$ and $f = 2.0$.

eters, the fractional order can be adjusted to enhance the system response.

Generally, in the fractional-order damped biophysical system, only single resonance peaks were observed for all values of fractional-order damping with $\lambda_1 \leq 1$, as shown in Figures 1 – 6. The non-integer order has the capacity to reduce the value of the HF amplitude at which VR occurs in the integer-order system.

Fractional-order damping is related to the decay rate of excited enzyme molecules, which affects enzyme-substrate interactions in brain wave activity in real biological systems. At resonance, the enzyme-substrate reaction is amplified, leading to increased brain wave activity. Consequently, modulating the fractional-order damping regulates energy transfer during molecular interactions by influencing enzyme excitation processes.

5. Conclusion

In this research, the role of fractional-order damping in observed vibrational resonances was studied using numerical methods. The response amplitude Q was used to characterise VR, as it defines an amplification factor that indicates the influence of the HF signal on the oscillator. The fractional-order damping modifies both the maximum peak of the response curves, Q_{max}, and the response amplitude at which VR occurs, g_{VR}, thereby effectively controlling the single-peak resonance behaviour of the system. Compared with vibrational resonance in ordinary integer-order systems, the new results obtained in the fractional-order system confirm the contributory role of fractional-order damping.

This research concludes that fractional-order damping plays a contributory role in the observed resonances. At resonance, the enzyme-substrate reaction is amplified, leading to increased brain wave activity. Increased brain wave activity resulting from the enzyme-substrate reaction may be linked to inherent energy transfers due to changes in the rate of decay induced by fractional-order modulation at resonance. Consequently, significant behavioural changes in brain wave activity could be observed due to inherent variations in the rate of enzyme-substrate decay, even in the absence of changes in the number of enzyme-substrate carriers. Fractional derivatives can be utilised to enhance the performance of population models.

This research demonstrated the capability of fractional calculus to capture the nonlocal and frequency-dependent inherent behaviour of biophysical systems, which has key applications in physiological signal analysis. Specifically, the findings provided enhanced insights into the underlying dynamics of the FDE model of the enzyme-substrate reaction in brain wave signals, explaining regions of enhanced resonance and control that are not available within the classical integer-order approach. The achieved control can be leveraged to improve stability and robustness, particularly in applications requiring modulation of resonance responses in cardiology and neuroscience, such as heart rate variability, EEG signals, and other bio-signals. This approach could also be extended to study other biophysical systems or improve medical imaging techniques.

Experimental studies of VR have been conducted in other physical systems, particularly in electronic circuits. Consequently, experimental studies implementing the techniques of vibrational resonance in a fractional-order biophysical system, based on the dielectric properties of biological materials and the action of enzymes, should be carried out to complement the theoretical framework established in this study.

Data availability

No data was used for the research described in the article.

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