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Estimation of reliability characteristics of single-unit repairable system with preventive maintenance and server arrival time

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Abstract

The study seeks to determine the reliability features of the single unit repairable systems around the roles of the maximum operating time, preventive maintenance (PM), and the server arrival time. The main objective is to analyze trade-offs between scheduled maintenance outage and system availability. With that in mind, two Weibull-distributed stochastic models Model-1 with PM and Model-2 without PM are compared with Semi-Markovian and Regenerative Point methods to calculate such metrics as MTSF, availability and profit. In addition, we utilized Maximum likelihood estimation (MLE) and Bayesian models to estimate reliability characteristics with the validation of the solution by Markov Chain Monte Carlo (MCMC) simulations. The numerical analysis shows that in both models, a rise in the rate of failures will cause a reduction in MTSF, availability and profit. Critical comparison reveals that Model 2 (without PM) surprisingly generates higher availability and MTSF as compared to Model 1. Also, the numerical and graphical outcomes prove that the MLE findings are very close to the true availability of the system. The research provides a strong framework on which the precise boundary of planned maintenance exceeding downtime.

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1. Introduction

Reliability analysis plays a crucial role to ensure consistent performance and safety in modern engineering and industrial systems. Accurate modeling and estimation of reliability measurements becomes very crucial as automation in these systems increase. Based on available resources and requirements industries used multi-unit, two-unit and single unit systems. Single-unit repairable systems, widely used in transportation, manufacturing, and power generation. These system's often face anomalies that require either preventive maintenance or corrective repair. It is suggested that regular maintenance can play a significant role in the overall system availability and performance of these systems. In recent times, reliability investigations of industrial system's attract the attention of researcher. Several researchers investigated reliability measures of single unit systems by developing stochastic models under different set of assumptions. Kadyan [1] used the concept of preventive maintenance for a single unit system which emphasized on how planned preventative maintenance improves system availability and cost performance. Kadyan *et al.* [2] analysed the joint impact of maintenance plans and warranty conditions on availability and cost-effectiveness on single unit systems. Kumar and Singh [3] analysed a single-unit repairable system subject to degradation and preventive maintenance.

Kumar *et al.* [4] analysed a stochastic model by using the

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concept of regenerative point technique for a single-unit repairable system that focused on maximum operation and repair times as well as preventive maintenance. Kumar and Saini [5] developed a stochastic model to enhance the reliability of a repairable single-unit system under ideas of preventative maintenance and Weibull distributed random variables. Kishan and Jain [6, 7] used the classical and Bayesian estimation techniques to enhance the reliability of non-identical standby unit systems with repair, inspection and post-repair process. The Bayesian estimation in studies performed under squared error loss functions. The usefulness of Bayesian approaches is demonstrated through a simulation-based comparison, particularly when prior knowledge is included. Rodrigues *et al.* [8] evaluated the performance of the Bayesian and classical estimator of reliability measures in series system. Li *et al.* [9] provided the applications and benefits of Bayesian methods in analysing real time data. Singh and Gupta [10] focused on Bayesian reliability estimation of a 1-out-of-k load sharing system model. Lin *et. al.* [11] applied the classical and Bayesian semi-parametric techniques for estimating reliability of locomotive wheelsets under preventive maintenance strategy.

Basri *et al.* [12] provided comprehensive review on preventive maintenance planning approaches to achieve efficient maintenance system. Ahlawat *et al.* [13] evaluated the reliability of six component system which was modeled under non-series parallel configuration by using Weibull failure laws. Kumar *et al.* [14] emphasized on reliability and availability of repairable mechanical systems which examined utilizing models and simulation tools, with an emphasis on maintenance strategies to enhance system performance. Rasekhi *et al.* [15] developed and compared the Bayesian and classical inference methods for estimating the reliability of a multi-component stress-strength model under generalized logistic distribution. Kayal *et al.* [16] estimated the reliability of a multi-component stress-strength model by using Chen distribution through classical and Bayesian approaches.

Toroody *et al.* [17] addressed the reliability challenges in repairable systems through hierarchical Bayesian and classical approaches. Malik [8] provided the key reliability metrics of a repairable system by considering the concept of server's arrival using semi-Markovian process. Saini and Kumar [19] designed a stochastic model for a single unit system that is exposed to inspection and degradation while running under different environmental conditions. Li *et al.* [20] performed reliability analysis of multi-state system having common cause failures by using Bayesian network approach with fuzzy probability. Wang *et al.* [21] performed reliability analysis of a multi-component stress-strength model by using a bathtub shaped distribution.

Saxena *et al.* [22] analysed two-unit parallel system model under both classical and Bayesian frameworks incorporating the idea of working and rest time of the repairman. Gurler *et al.* [23] focused on optimization of imperfect preventive maintenance approach using reliability-based methods integrated with Bayesian estimation. Abbas [24] emphasized on estimating system reliability through classical and Bayesian techniques by using bi-variate geometric distribution for correlated count data. Bhat and Simon [25] focused on stochastic analysis of complex repairable system having conditions on number of repairs. Kumar [26] investigated live industrial refrigeration system and used the stochastic modelling approach to enhance the performance of the system. Saini [27] applied the Exponentiated Pareto distribution to estimates the reliability of multi component system by using lower record values and compared the classical and Bayesian estimates of the reliability measures.

Saini *et al.* [28] proposed classical and Bayesian estimation techniques for multi component stress-strength reliability based on power Lindley distribution under gradually first-failure censored samples. Saini [29] focused on classical and Bayesian estimation of multi stress strength reliability under progressive first failure censoring by using generalized inverted exponential distribution. Kumari *et al.* [30] estimated the multi component stress strength reliability under classical and Bayesian approach by using the Lindley distribution. Kadyan and Malik [31] performed stochastic analysis of a non-identical system having $(n + 1)$ units classified as type-1 and type-2 units which are subject to maximum repair time of type-1 unit. Gupta and Kumar [32] analyzed the effect of several failure types, such as component, hardware, software, common-cause, and human errors on the availability of a repairable machining system. Kumar and Shekar [33] applied Bayesian approach to model a two-unit repairable system having imperfect coverage and delayed detection. Failure, detection, repair, recovery, and reboot times are all modeled exponentially for analytical tractability. Chaudhary *et al.* [34] evaluated the reliability characteristics for Ready-mix cement plant by using the classical and Bayesian approaches. Yadav *et al.* [35] analyzed the Classical and Bayesian estimation of reliability characteristics for Logistic-exponential distribution. A reliability based thermal comfort model was proposed by Sargazi *et al.* [36], to investigate the predictive performance of the Urban interventions in hot and arid zones with the use of Monte Carlo simulation and the study discovered that the Universal Thermal Climate Index (UTCI) has better reliability and predictive ability than the Physiological Equivalent Temperature (PET) index in extreme conditions of heat stress. Park *et al.* [37] studied the channel-length dependency in IGZO TFTs and have found that shorter-channel based devices have better positive bias temperature stress (PBTS) reliability because defect passivation in both channel and gate dielectric is promoted by hydrogen diffusion out of source/drain regions.

However, most of the studies mentioned above focused primarily on obtaining different reliability characteristics, such as MTSF, profit, steady-state availability, and their classical and Bayesian estimates.

Although considerable research has been carried out in this area, the literature has given limited attention to maintenance optimization and statistical estimation. This article fills that important gap by providing a comparative analysis of systems with and without preventive maintenance under Maximum Operating Time and Server Arrival Time. It also evaluates the efficiency of these models using Maximum Likelihood Estimation (MLE) and Bayesian inference supported by Markov Chain Monte Carlo (MCMC) simulations.

To address this gap, the present study develops and analyzes a single-unit repairable system with and without preventive maintenance using the Weibull distribution for failure, preventive maintenance, server arrival, and repair times, with distinct scale parameters but similar shape parameters. The reliability characteristics of the system are evaluated using the regenerative point technique. These characteristics include steady-state transition probabilities and mean sojourn times, system reliability and MTSF, point-wise and steady-state availability, the expected busy period of the server in the time interval $(0, t)$, and the net expected profit generated by the system in the time interval $(0, t)$ and in the steady state.

Since life testing experiments require a long period of time, the parameters describing the system's or unit's reliability characteristics are treated as random variables. A Markov Chain Monte Carlo simulation study is carried out to demonstrate the effectiveness of the proposed estimation techniques under different scenarios, thereby supporting the theoretical results and providing useful insights into the system model under consideration. In the classical framework, the maximum likelihood estimates of the model parameters and reliability characteristics, together with their standard errors (SE) and confidence interval widths, are obtained. In the Bayesian framework, the widths of the highest posterior density (HPD) intervals and the posterior standard errors (PSE) of the Bayes estimates of the parameters and reliability characteristics are determined. Finally, a comparison is made to assess the performance of the MLE and Bayes estimators.

This paper is organized into six sections. Section 1 presents the introduction, Section 2 describes the model and notations, Section 3 gives the transition probabilities and mean sojourn times, Section 4 analyzes the reliability characteristics, Section 5 presents the classical and Bayesian estimation of parameters, Section 6 contains the simulation study, and the final section concludes the findings.

2. Description of the system model, system states, and notation

Here, two stochastic models proposed under various set of assumptions. Model-1 is with preventive maintenance and model-2 is without preventive maintenance, and each system consists of single unit.

In Model 1, the system initially begins functioning from state S_0, where the unit is in operative (or normal) mode. From state S_0, the unit can transition to either state S_1, where the unit has failed and is awaiting repair, or state S_2, where the unit reaches its maximum operative time and is waiting for preventive maintenance. A single repairman is available to service both failed units and units undergoing preventive maintenance.

When the server arrives at state S_3, the unit undergoes preventive maintenance and becomes operational; similarly, at state S_4, a failed unit is repaired and becomes operational again. After repair, the unit is as good as new. S_0 is the only up-state, while S_1, S_2, S_3, and S_4 are the down-states. Note that in this single-unit system, all states are regenerative. The distributions for failure, maximum operative time, server arrival, preventive maintenance, and repair time are assumed to follow a Weibull distribution with an identical shape parameter q and distinct scale parameters $\gamma_1, \gamma_2, \mu_1, \gamma_{m1}$, and γ_{m2}.

Similarly, in Model 2 (without preventive maintenance), the system begins functioning from state S_0 in operative mode. From S_0, the system transitions to state S_1, where the unit has failed and is awaiting repair. A single repairman is available to repair the failed unit. When the server arrives at state S_2, the failed unit is repaired and becomes operational again. After repair, the unit is as good as new. In this model, S_0 is the only up-state, and S_1 and S_2 are down-states. The failure rate, server arrival, and repair time distributions follow a Weibull distribution with a common shape parameter q and different scale parameters γ_1, μ_1, and γ_{m2}. The state transition diagrams for both models are provided in Figures 1 and 2.

2.1. Notations

The notations used for development of stochastic and transition diagram are appended in Table-2.

2.2. Symbols for the state of the system

O:	The unit is operational and operating normally.
Fwr:	The unit is now in a failed mode and is awaiting repair.
Wpm:	Unit is waiting for preventive maintenance.
Fr:	The unit is in failed status and it's being repaired.
Pm:	The unit has undergone preventive maintenance.

Using these symbols and assumptions stated previously, the transition diagram of the system model along with all possible state and transition is shown in Figs. 1-2. Here we observe that state S_0 is the only up state and S_1, S_2, S_3, S_4 are the down states. Notes that in single unit system all states are regenerative states.

3. Transition probabilities and sojourn times:

The transition probabilities matrix of the above Markov chain state diagram with non-zero elements is

$$p_{ij} = \begin{bmatrix} p_{00} & p_{01} & p_{02} & p_{03} & p_{04} \\ p_{10} & p_{11} & p_{12} & p_{13} & p_{14} \\ p_{20} & p_{21} & p_{22} & p_{23} & p_{24} \\ p_{30} & p_{31} & p_{32} & p_{33} & p_{34} \\ p_{40} & p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix} \text{ TPM Model-1,} \quad (1)$$

$$p_{ij} = \begin{bmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{11} & p_{12} \\ p_{20} & p_{21} & p_{22} \end{bmatrix} \text{ TPM Model-2.} \quad (2)$$

3.0.1. For Model-1

To obtain p_{01}, the probability that system transit from state S_0 to S_1 during time interval $(0, \infty)$ as follows:

$$p_{01} = \int \text{(probability that the operating unit in state } S_0$$

fails during time $(t, t + \Delta t)$ and does not goes under preventive maintenance up to time t) dt.

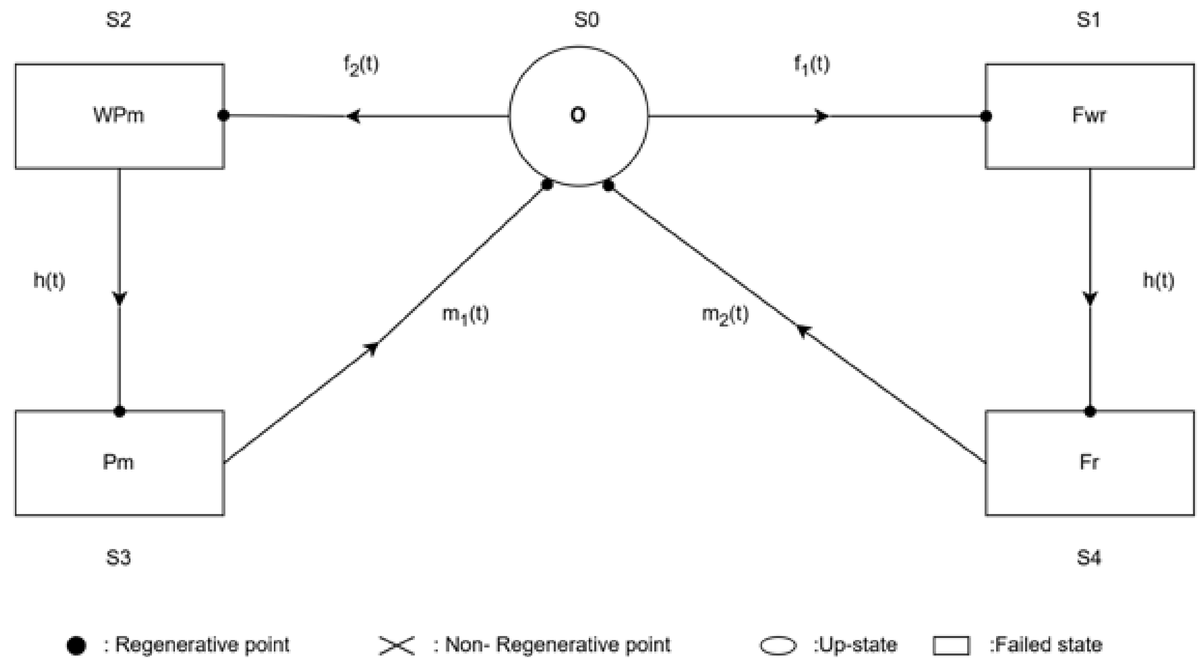

Figure 1. Transition diagram of system with preventive maintenance (Model-1).

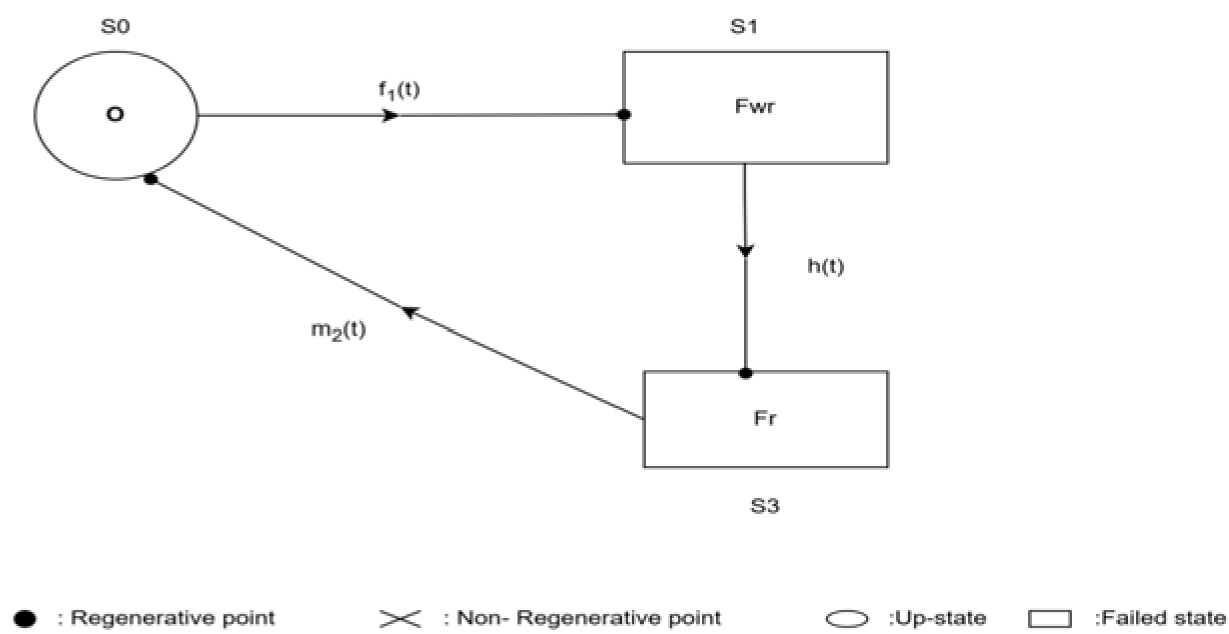

Figure 2. Transition diagram of system without preventive maintenance (Model-2).

Thus

$$p_{01} = \int_0^\infty \gamma_1 q t^{q-1} e^{-(\gamma_1 t^q)} \cdot e^{-(\gamma_2 t^q)} dt = \frac{\gamma_1}{\gamma_1 + \gamma_2}. \tag{3}$$

Similarly,

$$p_{02} = \int_0^\infty \gamma_2 q t^{q-1} e^{-(\gamma_1 t^q)} \cdot e^{-(\gamma_2 t^q)} dt = \frac{\gamma_2}{\gamma_1 + \gamma_2}, \tag{4}$$

where

$p_{14} = 1, p_{23} = 1, p_{30} = 1$ and $p_{40} = 1$; only one path certainty

Table 1. Probability density function of Random variables.

	Model-1 with PM	Model-2 without PM
Failure rate	$f_1(t) = \gamma_1 q t^{q-1} e^{-(\gamma_1 t^q)}$	$f_1(t) = \gamma_1 q t^{q-1} e^{-(\gamma_1 t^q)}$
Maximum operational time	$f_2(t) = \gamma_2 q t^{q-1} e^{-(\gamma_2 t^q)}$	——
Server arrival rate	$h(t) = \mu_i q t^{q-1} e^{-(\mu_i t^q)}$	$h(t) = \mu_i q t^{q-1} e^{-(\mu_i t^q)}$
Preventive maintenance rate	$m_1(t) = \gamma_{m1} q t^{q-1} e^{-(\gamma_{m1} t^q)}$	——
Repair rate	$m_2(t) = \gamma_{m2} q t^{q-1} e^{-(\gamma_{m2} t^q)}$ where $t \geq 0$ and $p > 0$.	$m_2(t) = \gamma_{m2} q t^{q-1} e^{-(\gamma_{m2} t^q)}$

Table 2. Notations for development of state transition diagram.

E	Set states that are regenerative $(S_0, S_1, S_2, S_3, S_4)$
$\gamma_i, \mu_i, \gamma_{mi}$	Scale parameters of failure, maximum operative time, arrival of server, preventive maintenance, and repair rate
q	Shape parameter of failure, maximum operating time, server arrival, rate of repair, and preventative maintenance
$q_{ij}(\cdot)$ / $Q_{ij}(\cdot)$	Pdf and Cdf of one step or direct transition time from state $S_i \in E$ to $S_j \in E$
p_{ij}	Steady state transition probability from state S_i to S_j such that $p_{ij} = \lim_{t\to\infty} Q_{ij}(t)$
$W_i(t)$	Probability that system sojourns in state S_i up to time t
Φ_i	Mean sojourn time in state S_i i.e. $\Phi_i = \int_0^\infty P[T_i > t]dt$
$R_i(t)$	Reliability of the system at time t when system starts from $S_i \in E$
$A_i(t)$	Probability that the system will be operative at time t in state $S_i \in E$
$B_i(t)$	Probability that the repairman will be busy at time t in state $S_i \in E$
$\mu_{up}(t)$	Expected up time of the system during interval $(0, t)$ i.e $\mu_{up}(t) = \int_0^t A_0(u)du$
$\mu_b(t)$	Expected busy period of repairman during interval $(0, t)$ i.e $\mu_b(t) = \int_0^t B_0(u)du$
$P(t)$	Profit incurred by the system during interval $(0, t)$
*	Symbol for Laplace Transformation of a function i.e., $q^*_{ij} = \int_0^\infty e^{-st} q_{ij}(t)dt$
•	Regenerative State
×	Non-Regenerative state

; after server arrives and other elements of t.p.m will be zero.

It can be easily verified that

$$p_{01} + p_{02} = 1. \tag{5}$$

In model-2
$p_{01} = 1, p_{12} = 1$ and $p_{20} = 1$; only one path certainty after server arrives. And other elements of t.p.m will be zero. So t.p.m will be

$$p_{ij} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}. \tag{6}$$

3.1. Mean sojourn time

For model-1: Mean Sojourn Time (MST) Φ_i in state S_i denotes the average duration a system or component remains in a specific state prior to shifting to an alternative state. If T_i denotes the sojourn time in state S_i then MST can be calculated as:

$$\Phi_i = \int_0^\infty P[T_i > t]dt. \tag{7}$$

Then

$$\Phi_0 = \int [\text{Probability that the operating unit in state } S_0$$
$$\text{do not fail up to time } t \text{ and do not reach its maximum}$$
$$\text{operating time } t]\ dt,$$

$$\Phi_0 = \int_0^\infty e^{-(\gamma_1 t^q)} \cdot e^{-(\gamma_2 t^q)} dt = \frac{\Gamma(1+\frac{1}{p})}{(\gamma_1+\gamma_2)^{1/p}}. \tag{8}$$

Similarly

$$\begin{aligned}
\Phi_1 &= \int_0^\infty e^{-(\mu_i t^q)} dt = \frac{\Gamma(1+\frac{1}{p})}{(\mu_i)^{1/p}} \\
\Phi_2 &= \int_0^\infty e^{-(\mu_i t^q)} dt = \frac{\Gamma(1+\frac{1}{p})}{(\mu_i)^{1/p}} \\
\Phi_3 &= \int_0^\infty e^{-(\gamma_{m1} t^q)} dt = \frac{\Gamma(1+\frac{1}{p})}{(\gamma_{m1})^{1/p}} \\
\Phi_4 &= \int_0^\infty e^{-(\gamma_{m2} t^q)} dt = \frac{\Gamma(1+\frac{1}{p})}{(\gamma_{m2})^{1/p}}.
\end{aligned} \tag{9}$$

Also $\Phi_0 = m_{01} + m_{02}$, $\Phi_1 = m_{14}$, $\Phi_2 = m_{23}$, $\Phi_2 = m_{23}$, $\Phi_3 = m_{30}$ and $\Phi_4 = m_{40}$.
Similarly for model-2:

$$\begin{aligned}
\Phi_0 &= \int_0^\infty e^{-(\gamma_1 t^q)} dt = \frac{\Gamma(1+\frac{1}{p})}{(\gamma_1)^{1/p}} \\
\Phi_1 &= \int_0^\infty e^{-(\mu_i t^q)} dt = \frac{\Gamma(1+\frac{1}{p})}{(\mu_i)^{1/p}} \\
\Phi_2 &= \int_0^\infty e^{-(\gamma_{m2} t^q)} dt = \frac{\Gamma(1+\frac{1}{p})}{(\gamma_{m2})^{1/p}}.
\end{aligned} \tag{10}$$

4. Analysis of characteristics

4.1. Reliability and MTSF

Let the random variable T_i denotes the time to system failure i.e. time taken by system to reach any of the failed state, when the system starts initially from regenerative state $S_i \in E$, then reliability of the system is given by

$$R_i(t) = P(T_i > t). \tag{11}$$

To determine the reliability of the system, we consider the failed state as the absorbing states. In probabilistic argument we have the following recursive relation:

$$R_0(t) = Z_0(t), \tag{12}$$

where

$$Z_0(t) = e^{-(\gamma_1+\gamma_2)t^p} \quad \text{(Model-1)} \tag{13}$$
$$Z_0(t) = e^{-(\gamma_1)t^p} \quad \text{(Model-2)}. \tag{14}$$

On taking Laplace transformation of equation (M.1.3) and simplifying for $R_0^*(S)$, we get

$$R_0^*(S) = Z_0^*(S). \tag{15}$$

On taking Inverse Laplace of (15), allow us to determine the system's reliability when it commences at S_0.
MTSF (Mean time to system failure) On starting the system from state S_0, the MTSF of the system (for both model-1 and model-2) is determined by:

$$E(T_0) = \int_0^\infty R_0(t)dt = \lim_{s\to 0} R_0^*(S) = \Phi_0. \tag{16}$$

4.2. Availability

The probability that a system will be operational and functional at any random epoch is referred to as availability. It indicates how often a system is available when it is needed. When the system first begins in the regenerative state S_i, let $A_i(t)$ be the probability that it is operational at epoch t. There are recursive relations for model-1 in $A_i(t)$ based on fundamental probabilistic arguments:

$$\begin{aligned}
A_0(t) &= Z_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t) \\
A_1(t) &= q_{14}(t)©A_4(t) \\
A_2(t) &= q_{23}(t)©A_3(t) \\
A_3(t) &= q_{30}(t)©A_0(t) \\
A_4(t) &= q_{40}(t)©A_0(t).
\end{aligned} \tag{17}$$

Recursive relations for model-2:

$$\begin{aligned}
A_0(t) &= Z_0(t) + q_{01}(t)©A_1(t) \\
A_1(t) &= q_{12}(t)©A_2(t) \\
A_2(t) &= q_{20}(t)©A_0(t),
\end{aligned} \tag{18}$$

where

$$Z_0(t) = e^{-(\gamma_1+\gamma_2)t^p} \quad \text{(Model-1)}$$
$$Z_0(t) = e^{-(\gamma_1)t^p} \quad \text{(Model-2)}. \tag{19}$$

Taking Laplace transformation of equations (17), (18) and simplifying for $A_0^*(s)$ by using cramer's rule we get

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)}, \tag{20}$$

where $N_1(s) = Z_0^*(s)$ and $D_1(s) = 1 - q_{01}^* q_{14}^* q_{40}^* - q_{02}^* q_{23}^* q_{30}^*$ (Model-1)

Table 3. Numerical values of MTSF for fixed $\gamma_{m2} = 0.5, q = 1.0$ and varying γ_1.

Model-1										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
MTSF	90.9091	47.6190	32.2581	24.3902	19.6078	16.3934	14.0845	12.3457	10.9890	9.9010
MTSF_MLE	90.5895	47.4781	32.1811	24.4747	19.6003	16.3659	14.0810	12.353	10.9754	9.9084
SE.MLE	6.1695	3.4034	2.3886	1.7025	1.3983	1.2149	1.0530	0.9270	0.8134	0.7297
Width.CI	24.1846	13.3412	9.3633	6.6739	5.4812	4.7625	4.1278	3.6338	3.1886	2.8604
MTSF_BAY										
ES	91.0147	47.7176	32.3505	24.604	19.7062	16.4541	14.1579	12.4201	11.0362	9.9628
PSE	6.1946	3.4183	2.4026	1.7104	1.4059	1.2222	1.0585	0.9319	0.8177	0.7339
Width.HPD	22.8688	13.1971	9.1516	6.5808	5.4199	4.7545	4.2064	3.6227	3.2319	2.9444
Model-2										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
MTSF	100.0000	50.0000	33.3333	25.0000	20.0000	16.6667	14.2857	12.5000	11.1111	10.0000
MTSF_MLE	100.0135	49.7783	33.2851	24.8905	19.9205	16.6698	14.2768	12.5477	11.1223	9.9877
SE.MLE	7.2789	3.6153	2.4180	1.8243	1.5185	1.2511	1.0201	0.9452	0.8367	0.7625
Width.CI	28.5331	14.1721	9.4786	7.1514	5.9526	4.9042	3.9987	3.7051	3.2799	2.9892
MTSF_BAY										
ES	100.5714	50.0551	33.4699	25.0294	20.0327	16.7636	14.3564	12.6177	11.1845	10.0431
PSE	7.3227	3.6364	2.4307	1.8345	1.5273	1.2581	1.0251	0.9513	0.8414	0.7667
Width.HPD	28.5477	14.3585	9.1097	7.0338	5.9955	4.9401	3.9507	3.7121	3.2563	2.9479

Table 4. Numerical values of MTSF for fixed $\gamma_{m2} = 0.5, q = 0.5$ and varying γ_1.

Model-1										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
MTSF	16528.9300	4535.1470	2081.1660	1189.7680	768.9350	537.4899	396.7467	304.8316	241.5167	196.0592
MTSF_MLE	16489.0400	4531.5070	2082.6580	1203.8220	772.2531	538.6345	398.7644	306.9122	242.2439	197.418
SE.MLE	2258.7490	647.5477	310.0413	167.4032	109.7664	79.9782	59.6958	46.2708	36.0404	29.1120
Width.CI	8854.2950	2538.3870	1215.3620	656.2206	430.2843	313.5146	234.0074	181.3816	141.2785	114.1191
MTSF_BAY										
ES	91.4049	52.9579	37.8318	29.7647	24.4349	20.8201	18.1472	16.1278	14.4796	13.1662
PSE	5.6900	3.4841	2.5633	1.9423	1.6365	1.4477	1.2912	1.1651	1.0281	0.9238
Width.HPD	21.4939	13.3238	9.7480	7.4709	6.1474	5.6957	5.0045	4.3756	3.9504	3.6612
Model-2										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
MTSF	20000.0000	5000.0000	2222.2220	1250.0000	800.0000	555.5556	408.1633	312.5000	246.9136	200.0000
MTSF_MLE	20111.3800	4981.895	2227.485	1245.731	798.2662	558.8958	409.735	316.6766	248.8124	200.6697
SE.MLE	2923.8370	728.7895	323.5417	183.0994	121.6175	84.0046	58.667	47.9592	37.1932	30.6824
Width.CI	11461.4400	2856.8550	1268.2830	717.7495	476.7405	329.2981	229.9745	188.0001	145.7973	120.2751
MTSF_BAY										
ES	20450.6600	5065.7660	2264.9300	1266.7500	811.8216	568.3798	416.6458	322.0206	253.0174	204.0447
PSE	2974.2890	741.1284	328.8530	186.1917	123.6972	85.4309	59.6128	48.8134	37.8197	31.1936
Width.HPD	11475.3700	2910.7820	1236.1620	704.8249	480.3704	334.4650	227.1198	186.0431	147.5186	121.2578

$N_1(s) = Z_0^*(s)$ and $D_1(s) = 1 - q_{01}^* q_{12}^* q_{20}^*$ (Model-2).

Inverse Laplace Transformation of equation (20) will give us the system's availability for known parameter values. The system's long-term steady state probability of being operational is determined by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) = \lim_{s\to 0} \frac{sN_1(s)}{D_1(s)} = \frac{N_1(s)}{D_1'(s)}, \quad (21)$$

where $N_1(s) = Z_0^*(s) = \Phi_0$ and $D_1'(s) = \Phi_0 + p_{01}\Phi_1 + p_{01}\Phi_4 + p_{02}\Phi_2 + p_{02}\Phi_3$ (for model-1)
and $N_1(s) = Z_0^*(s) = \Phi_0$ and $D_1'(s) = \Phi_0 p_{12}p_{20} + p_{01}p_{20}\Phi_1 + p_{01}p_{12}\Phi_2$ (for model-2).

The expected-up time of the system during (0,t) is given by:

$$\mu_{up}(t) = \int_0^t A_0(u)du, \quad (22)$$

so that

$$\mu_{up}^*(s) = \frac{A_0^*(s)}{s}. \quad (23)$$

Table 5. Numerical values of MTSF for fixed $\gamma_{m2} = 0.5, q = 2.0$ and varying γ_1.

Model-1										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
MTSF	8.4498	6.1155	5.0334	4.3768	3.9243	3.5882	3.326	3.1139	2.9378	2.7886
MTSF_MLE	8.4301	6.1026	5.0240	4.3817	3.9210	3.5827	3.3232	3.1126	2.9340	2.7877
SE.MLE	0.2865	0.2193	0.1864	0.1525	0.1402	0.1331	0.1243	0.1166	0.1086	0.1027
Width.CI	1.1230	0.8595	0.7307	0.5979	0.5498	0.5218	0.4874	0.4573	0.4258	0.4025
MTSF_BAY					12.552					
ES	124.4103	62.2717	41.6386	31.4315	25.0387	20.8082	17.9038	15.6460	13.9018	8
PSE	10.2206	5.1746	3.6095	2.5129	1.9879	1.7722	1.5219	1.2865	1.1625	1.0714
Width.HPD	39.0395	19.9170	14.1836	9.8129	7.9420	6.7346	5.9226	4.9243	4.6544	4.2379
Model-2										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
MTSF	8.8623	6.2666	5.1166	4.4311	3.9633	3.618	3.3496	3.1333	2.9541	2.8025
MTSF_MLE	8.8570	6.2486	5.1096	4.4185	3.9526	3.6158	3.3464	3.1370	2.9535	2.7987
SE.MLE	0.3228	0.2265	0.1858	0.1619	0.1509	0.1357	0.1195	0.1179	0.1116	0.1069
Width.CI	1.2652	0.8877	0.7282	0.6345	0.5914	0.5321	0.4686	0.4624	0.4374	0.4189
MTSF_BAY										
ES	8.8755	6.2615	5.1202	4.4277	3.9609	3.6234	3.3534	3.1436	2.9596	2.8045
PSE	0.3236	0.2270	0.1861	0.1622	0.1512	0.1360	0.1197	0.1183	0.1118	0.1071
Width.HPD	1.2660	0.8932	0.6961	0.6233	0.5925	0.5346	0.4639	0.4670	0.4308	0.4092

Table 6. Numerical value of Availability for fixed $\gamma_{m2} = 0.5, q = 1.0$ and varying γ_1.

Model-1										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
AVAILABILITY	0.9630	0.9302	0.8995	0.8708	0.8438	0.8185	0.7947	0.7722	0.7509	0.7308
AV_MLE	0.9628	0.9297	0.8993	0.8708	0.8433	0.8179	0.7935	0.7714	0.7495	0.7300
SE	0.0033	0.0059	0.0083	0.0100	0.0118	0.0136	0.0152	0.0162	0.0172	0.0180
Width.CI	0.0130	0.0233	0.0327	0.0394	0.0461	0.0535	0.0597	0.0634	0.0673	0.0707
AV_BAYES	0.9626	0.9294	0.8989	0.8703	0.8427	0.8173	0.7928	0.7708	0.7488	0.7293
PSE	0.0033	0.0060	0.0084	0.0101	0.0118	0.0137	0.0152	0.0162	0.0172	0.0180
Width.HPD	0.0126	0.0226	0.0312	0.0377	0.0459	0.0534	0.0592	0.0628	0.0652	0.0721
Model-2										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
AVAILABILITY	0.9743	0.9503	0.9279	0.9070	0.8873	0.8689	0.8515	0.8351	0.8195	0.8049
AV_MLE	0.9741	0.9498	0.9276	0.9067	0.8869	0.8681	0.8513	0.8355	0.8191	0.8044
SE	0.0027	0.0048	0.007	0.0086	0.0104	0.0115	0.0121	0.0137	0.0154	0.0160
Width.CI	0.0104	0.0187	0.0276	0.0337	0.0408	0.0451	0.0476	0.0537	0.0605	0.0628
AV_BAYES	0.9739	0.9495	0.9271	0.9061	0.8862	0.8674	0.8506	0.8347	0.8183	0.8036
PSE	0.0027	0.0048	0.0071	0.0086	0.0104	0.0116	0.0122	0.0137	0.0154	0.0160
Width.HPD	0.0103	0.0185	0.0270	0.0332	0.0409	0.0448	0.0459	0.0527	0.0592	0.0608

4.3. Busy period analysis

The Busy Period, as applied in the theory of reliability, particularly in regenerative models of reliability, is the expected duration for which the repair facility (or service facility) will be continuously busy in performing system maintenance, either as repair or preventive maintenance. Let $B_i(t)$ be the probability that the server will be busy at epoch t when initially its operation starts from regenerative state S_i. By simple probabilistic arguments, we have the following recursive relations in $B_i(t)$ for model-1:

$$\begin{aligned} B_0(t) &= q_{01}(t)©B_1(t) + q_{02}(t)©B_2(t) \\ B_1(t) &= q_{14}(t)©B_4(t) \\ B_2(t) &= q_{23}(t)©B_3(t) \end{aligned} \tag{24}$$

Table 7. Numerical values of Availability for fixed $\gamma_{m2} = 0.5, q = 0.5$ and varying γ_1.

Model-1

γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
AVAILABILITY	0.9992	0.9971	0.9937	0.9889	0.9829	0.9756	0.9672	0.9577	0.9472	0.9357
AV_MLE	0.9992	0.9970	0.9935	0.9888	0.9825	0.9751	0.9663	0.9568	0.9457	0.9344
SE	0.0001	0.0005	0.0012	0.0020	0.0031	0.0045	0.0061	0.0076	0.0095	0.0113
Width.CI	0.0006	0.0021	0.0047	0.0079	0.0122	0.0178	0.0240	0.0298	0.0373	0.0444
AV_BAYES	0.9462	0.9077	0.8740	0.8434	0.8146	0.7886	0.7632	0.7411	0.7190	0.6994
PSE	0.0045	0.0073	0.0099	0.0116	0.0132	0.0150	0.0168	0.0176	0.0184	0.0189
Width.HPD	0.0171	0.0285	0.0388	0.0450	0.0509	0.0588	0.0643	0.0681	0.0710	0.0737

Model-2

γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
AVAILABILITY	0.9994	0.9975	0.9944	0.9901	0.9846	0.9779	0.9702	0.9615	0.9518	0.9412
AV_MLE	0.9994	0.9974	0.9942	0.9898	0.9842	0.9774	0.9697	0.9612	0.9508	0.9400
SE	0.0001	0.0005	0.0011	0.0019	0.0031	0.0042	0.0052	0.0070	0.0092	0.0110
Width.CI	0.0005	0.0019	0.0044	0.0075	0.0120	0.0165	0.0206	0.0274	0.0360	0.0429
AV_BAYES	0.9993	0.9974	0.9941	0.9896	0.9838	0.9769	0.9691	0.9603	0.9498	0.9388
PSE	0.0001	0.0005	0.0011	0.0020	0.0031	0.0043	0.0053	0.0071	0.0094	0.0111
Width.HPD	0.0005	0.0018	0.0043	0.0076	0.0117	0.0167	0.0203	0.0274	0.0352	0.0417

Table 8. Numerical values of Availability for fixed $\gamma_{m2} = 0.5, q = 2.0$ and varying γ_1.

Model-1

γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
Availability	0.7859	0.7226	0.6805	0.6486	0.6228	0.6012	0.5826	0.5663	0.5518	0.5388
AV_MLE	0.7856	0.7223	0.6806	0.6489	0.6227	0.6011	0.5821	0.5661	0.5512	0.5386
SE	0.0079	0.0092	0.0101	0.0102	0.0105	0.0110	0.0113	0.0113	0.0113	0.0114
Width.CI	0.0310	0.0359	0.0394	0.0398	0.0410	0.0430	0.0444	0.0443	0.0444	0.0446
AV_BAYES	0.9643	0.9310	0.9007	0.8719	0.8444	0.8187	0.7945	0.7720	0.7504	0.7309
PSE	0.0036	0.0066	0.0092	0.0112	0.0131	0.0154	0.0167	0.0177	0.0189	0.0205
Width.HPD	0.0142	0.0244	0.0348	0.0427	0.0506	0.0579	0.0634	0.0674	0.0728	0.0812

Model-2

γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
Availability	0.8759	0.8387	0.8140	0.7954	0.7803	0.7677	0.7568	0.7473	0.7389	0.7313
AV_MLE	0.8758	0.8384	0.8140	0.7956	0.7805	0.7674	0.7572	0.7480	0.7390	0.7314
SE	0.0063	0.0074	0.0085	0.0088	0.0093	0.0095	0.0095	0.0100	0.0108	0.0106
Width.CI	0.0247	0.0289	0.0335	0.0347	0.0366	0.0372	0.0373	0.0393	0.0423	0.0414
AV_BAYES	0.8756	0.8380	0.8137	0.7953	0.7801	0.7670	0.7568	0.7476	0.7386	0.7310
PSE	0.0063	0.0074	0.0085	0.0088	0.0093	0.0095	0.0095	0.0100	0.0108	0.0106
Width.HPD	0.0242	0.0290	0.0333	0.0339	0.0371	0.0365	0.0367	0.0383	0.0419	0.0407

$$B_3(t) = Z_3(t) + q_{30}(t)©B_0(t)$$
$$B_4(t) = Z_4(t) + q_{40}(t)©B_0(t).$$

Similarly, recursive relations for model-2 are:

$$\begin{aligned} B_0(t) &= q_{01}(t)©B_1(t) \\ B_1(t) &= q_{12}(t)©B_2(t) \\ B_2(t) &= Z_2(t) + q_{20}(t)©B_0(t). \end{aligned} \tag{25}$$

For model-1: $Z_3(t) = e^{-(\gamma_{m1}t^q)}$ and $Z_4(t) = e^{-(\gamma_{m2}t^q)}$ and For model-2: $Z_2(t) = e^{-(\mu_i t^q)}$.

By taking Laplace Transformation of relations (24), (25) and solving for $B_0^*(s)$, omitting the argument 's' for brevity. We get,

$$B_0^*(s) = \frac{N_2(s)}{D_1(s)}, \tag{26}$$

where

$$N_2(s) = q_{01}^* q_{14}^* Z_4^*(s) + q_{23}^* q_{02}^* Z_3^*(s)\ (for model-1),$$

$$N_2(s) = q_{01}^* q_{12}^* Z_2^*(s)\ (for model-2),$$

Table 9. Numerical values of Busy period for fixed $\gamma_{m2} = 0.5, q = 1.0$ and varying γ_1.

Model-1										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
BUSY	0.0193	0.0373	0.0540	0.0697	0.0844	0.0983	0.1113	0.1236	0.1352	0.1462
Busy_MLE	0.0195	0.0374	0.0541	0.0696	0.0846	0.0982	0.1118	0.1236	0.1362	0.1468
SE	0.0020	0.0038	0.0053	0.0066	0.0079	0.0090	0.0102	0.0109	0.0115	0.0126
Width.CI	0.0080	0.0148	0.0208	0.0258	0.0310	0.0351	0.0401	0.0428	0.0450	0.0496
Busy_BAYES	0.0195	0.0376	0.0544	0.0699	0.0849	0.0986	0.1121	0.1239	0.1365	0.1472
PSE	0.0020	0.0038	0.0053	0.0066	0.0079	0.0090	0.0102	0.0109	0.0115	0.0126
Width.HPD	0.0078	0.0147	0.0207	0.0259	0.0306	0.0348	0.0392	0.0419	0.0433	0.0482
Model-2										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
BUSY	0.9646	0.9317	0.9009	0.8721	0.8451	0.8197	0.7958	0.7732	0.7519	0.7317
Busy_MLE	0.9644	0.9310	0.9004	0.8715	0.8443	0.8187	0.7954	0.7736	0.7513	0.7309
SE	0.0032	0.0057	0.0083	0.0102	0.0123	0.0135	0.0141	0.0159	0.0176	0.0185
Width.CI	0.0124	0.0223	0.0326	0.0398	0.0483	0.0531	0.0552	0.0624	0.0691	0.0727
Busy_BAYES	0.9642	0.9307	0.900	0.8710	0.8437	0.8182	0.7947	0.7729	0.7506	0.7302
PSE	0.0032	0.0057	0.0083	0.0102	0.0123	0.0136	0.0141	0.0159	0.0176	0.0185
Width.HPD	0.0126	0.0218	0.0324	0.0398	0.0484	0.0533	0.0535	0.0609	0.0674	0.0713

Table 10. Numerical values of Busy period for fixed $\gamma_{m2} = 0.5, q = 0.5$ and varying γ_1.

Model-1										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
BUSY	0.0004	0.0005	0.0001	0.0004	0.0258	0.0026	0.0100	0.0004	0.0005	0.0001
Busy_MLE	0.0017	0.0017	0.0004	0.0014	0.0457	0.0045	0.0178	0.0017	0.0017	0.0004
SE	0.0037	0.0038	0.0008	0.0031	0.0636	0.0062	0.0246	0.0037	0.0038	0.0008
Width.CI	0.0065	0.0065	0.0014	0.0054	0.0797	0.0075	0.0293	0.0065	0.0065	0.0014
Busy_BAYES	0.0100	0.0102	0.0021	0.0084	0.0951	0.0088	0.0343	0.0100	0.0102	0.0021
PSE	0.0143	0.0145	0.0030	0.0119	0.1087	0.0098	0.0389	0.0143	0.0145	0.0030
Width.HPD	0.0192	0.0197	0.0042	0.0164	0.1227	0.0113	0.0442	0.0192	0.0197	0.0042
Model-2										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
BUSY	0.9993	0.9973	0.9939	0.9893	0.9833	0.9762	0.9679	0.9584	0.9480	0.9365
Busy_MLE	0.9993	0.9972	0.9938	0.9890	0.9829	0.9756	0.9673	0.9581	0.9469	0.9352
SE	0.0001	0.0005	0.0012	0.0020	0.0032	0.0044	0.0055	0.0073	0.0096	0.0115
Width.CI	0.0005	0.0019	0.0046	0.0079	0.0126	0.0174	0.0216	0.0287	0.0377	0.0451
Busy_BAYES	0.9993	0.9971	0.9937	0.9888	0.9825	0.9751	0.9667	0.9572	0.9459	0.9340
PSE	0.0001	0.0005	0.0012	0.0020	0.0033	0.0045	0.0056	0.0075	0.0098	0.0117
Width.HPD	0.0005	0.0019	0.0045	0.0079	0.0123	0.0175	0.0213	0.0287	0.0366	0.0438

and $D_1(s)$ is identical to what was provided in the availability analysis of model-1 & 2 respectively. The probability that the repairman will be busy at a specific period for given values of the parameters can be obtained by using the Inverse Laplace Transformation to (26). In long run, the probability that the repairman will be busy in repair of failed unit is given by

$$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} sB_0^*(s) = \lim_{s\to 0} \frac{sN_2(s)}{D_1(s)} = \frac{N_2}{D_1'}, \qquad (27)$$

where

$$N_2 = p_{01}p_{14}\Phi_4 + p_{23}p_{02}\Phi_3,$$

$$D_1'(s) = \Phi_0 + p_{01}\Phi_1 + p_{01}\Phi_4 + p_{02}\Phi_2 + p_{02}\Phi_3 \ (model-1),$$

$$N_2 = p_{01}p_{12}\Phi_2$$

and

$$D_1'(s) = \Phi_0 p_{12}p_{20} + p_{01}p_{20}\Phi_1 + p_{01}p_{12}\Phi_2 \ (model-2).$$

Table 11. Numerical values of Busy period for fixed $\gamma_{m2} = 0.5, q = 2.0$ and varying γ_1.

Model-1										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
BUSY	0.1077	0.1422	0.1648	0.1819	0.1956	0.2071	0.2169	0.2256	0.2332	0.2401
Busy_MLE	0.1078	0.1422	0.1648	0.1816	0.1956	0.2068	0.2171	0.2253	0.2338	0.2404
SE	0.0050	0.0061	0.0068	0.0071	0.0076	0.0079	0.0083	0.0084	0.0082	0.0088
Width.CI	0.0197	0.0240	0.0265	0.0280	0.0299	0.0309	0.0325	0.0329	0.0323	0.0345
Busy_BAYES	0.0193	0.0375	0.0542	0.0697	0.0848	0.0985	0.1117	0.124	0.1363	0.1469
PSE	0.0023	0.0043	0.0058	0.0074	0.0087	0.0101	0.0111	0.0119	0.0128	0.0141
Width.HPD	0.0089	0.0165	0.0220	0.0290	0.0333	0.0386	0.0425	0.0456	0.0497	0.0547
Model-2										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
BUSY	0.7871	0.7233	0.6809	0.6489	0.6231	0.6015	0.5828	0.5665	0.552	0.5389
Busy_MLE	0.7869	0.7226	0.6807	0.6487	0.6228	0.6011	0.5829	0.5671	0.5519	0.5387
SE	0.0077	0.0089	0.0100	0.0103	0.0109	0.0109	0.0105	0.0112	0.0116	0.0117
Width.CI	0.0303	0.0348	0.0393	0.0403	0.0429	0.0427	0.0412	0.0438	0.0456	0.046
Busy_BAYES	0.7867	0.7224	0.6805	0.6485	0.6226	0.6009	0.5827	0.5670	0.5518	0.5386
PSE	0.0077	0.0089	0.0100	0.0103	0.0109	0.0109	0.0105	0.0112	0.0116	0.0117
Width.HPD	0.0306	0.0339	0.039	0.0401	0.0429	0.0427	0.0397	0.0429	0.0445	0.0451

Table 12. Numerical values of Profit function for fixed $\gamma_{m2} = 0.5, q = 1.0$ and varying γ_1.

Model-1										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
PROFIT	4803.5920	4628.5940	4465.1400	4312.1230	4168.5750	4033.6440	3906.5760	3786.7030	3673.4320	3566.2300
Profit_MLE	4802.0960	4626.1780	4463.9730	4312.0920	4165.7000	4030.8090	3900.2030	3782.9800	3665.7240	3561.9020
SE	17.6439	31.6789	44.5670	53.6524	62.9658	72.7693	81.4237	86.3918	91.7042	96.6108
Width.CI	69.1641	124.1812	174.7028	210.3173	246.8258	285.2558	319.1808	338.6559	359.4806	378.7145
Profit_BAY	4801.0970	4624.4330	4461.6910	4309.3650	4162.7300	4027.5990	3896.8800	3779.5790	3662.3340	3558.4430
PSE	17.7353	31.7926	44.7373	53.7387	63.1291	72.8986	81.4778	86.4498	91.6717	96.6120
Width.HPD	67.9970	121.5751	167.7454	200.3991	244.8778	283.5078	317.2676	336.0813	345.2292	384.8473
Model-2										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
PROFIT	4292.6050	4192.5470	4099.0990	4011.6280	3929.5780	3852.4590	3779.8410	3711.3400	3646.6170	3585.3660
Profit_MLE	4291.9480	4190.5360	4097.6470	4010.5300	3927.7410	3849.1980	3779.3570	3713.2930	3644.8380	3583.3670
SE	11.4406	20.4963	30.3344	37.0631	44.7199	49.6167	52.4716	59.1197	66.8148	69.2209
Width.CI	44.8471	80.3456	118.9109	145.2875	175.3019	194.4974	205.6887	231.7492	261.9139	271.3458
Profit_BAY	4291.1270	4189.0480	4095.6410	4008.0740	3924.9670	3846.1460	3776.0420	3709.8290	3641.2070	3579.6740
PSE	11.5089	20.5975	30.4556	37.1929	44.8458	49.7751	52.5410	59.2328	66.8984	69.2519
Width.HPD	44.4623	80.0371	116.2271	142.2653	175.4543	192.1937	199.7820	227.8008	257.0581	262.2312

During the interval (0,t), the server's anticipated busy time is

$$\mu_b(t) = \int_0^t B_0(u)du,$$

such that

$$\mu_b^*(s) = \frac{B_0^*(s)}{s}. \tag{28}$$

4.4. Profit analysis

Let K_0 = The system's revenue per unit uptime and K_1 = Cost per unit when the repairman is busy in repair of a failed unit or unit under preventive maintenance.
The system's net expected profit function throughout the time interval (0, t) is thus determined by:

$$P(t) = \text{Expected total revenue in } (0,t)$$

$$- \text{Expected total cost of repair in } (0,t)$$

$$= K_0\mu_{up}(t) - K_1\mu_b(t). \tag{29}$$

In steady state, the system's net expected profit per unit of time is determined by:

$$P = K_0A_0 - K_1B_0, \tag{30}$$

where A_0 and B_0 have been given in (21) and (27) for model-1 & 2 respectively.

5. Estimation of parameters, MTSF and profit function

5.1. Classical estimation

The failure, maximum operative time, arrival of server, preventive maintenance and repair rate independently fol-

Table 13. Numerical values of Profit function for fixed $\gamma_{m2} = 0.5, q = 0.5$ and varying γ_1.

Model-1										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
PROFIT	4995.8590	4984.5120	4966.0370	4940.5830	4908.3510	4869.5920	4824.6000	4773.7110	4717.2930	4655.7410
Profit_MLE	4995.7450	4984.1140	4965.4490	4939.8640	4906.4600	4867.0160	4819.7530	4769.1200	4709.4360	4648.6090
SE	0.7841	2.9030	6.4138	10.8052	16.7417	24.2941	32.8828	40.8799	51.0910	60.9254
Width.CI	3.0736	11.3796	25.1419	42.3566	65.6276	95.2327	128.9004	160.2494	200.2767	238.8277
Profit_BAY	4715.4940	4511.0220	4331.7740	4169.2930	4015.9860	3877.7720	3742.2010	3625.0560	3507.0040	3402.7540
PSE	23.9148	38.8606	52.3820	61.5487	70.3090	79.5188	89.4414	94.0045	98.2150	101.2614
Width.HPD	91.2720	150.6730	207.2376	238.9113	272.7044	310.9210	341.2638	364.5626	382.4294	395.1454
Model-2										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
PROFIT	4397.2700	4389.1010	4375.5590	4356.7530	4332.8330	4303.9830	4270.4250	4232.4080	4190.2080	4144.1210
Profit_MLE	4397.2010	4388.7450	4374.9550	4355.7270	4331.0590	4301.4360	4268.3000	4230.9560	4185.9770	4138.6810
SE	0.5393	2.0628	4.8534	8.3603	13.3957	18.4414	22.9246	30.5123	40.1956	47.8699
Width.CI	2.1142	8.0863	19.0254	32.7725	52.5113	72.2903	89.8644	119.6082	157.5669	187.6500
Profit_BAY	4397.1330	4388.4750	4374.3610	4354.6870	4329.4760	4299.2180	4265.3790	4227.3110	4181.4970	4133.4110
PSE	0.5522	2.1110	4.9633	8.5453	13.6823	18.8351	23.3646	31.0973	40.9044	48.6263
Width.HPD	2.1071	7.8355	18.7759	32.9716	51.3384	72.7517	88.3020	119.8274	153.4780	181.4735

Table 14. Numerical values of Profit function for fixed $\gamma_{m2} = 0.5, q = 2.0$ and varying γ_1.

Model-1										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
PROFIT	3865.0760	3527.9270	3303.5250	3133.6450	2996.6300	2881.7860	2782.9780	2696.3400	2619.2640	2549.9070
Profit_MLE	3863.0900	3525.9680	3304.0150	3135.4460	2995.9190	2881.3450	2780.1010	2695.4940	2615.8650	2548.8370
SE	42.0479	48.8761	53.5958	54.2056	55.8364	58.3561	60.4061	60.2435	60.3167	60.8998
Width.CI	164.8278	191.5942	210.0956	212.4860	218.8789	228.7561	236.7919	236.1544	236.4414	238.7272
Profit_BAY	4810.0040	4632.4320	4470.7680	4317.8000	4170.9490	4034.2120	3905.4540	3785.5060	3670.2370	3566.5870
PSE	19.2220	35.3875	49.3242	59.8460	69.8954	82.0074	88.9775	94.2617	100.8618	109.8460
Width.HPD	76.1381	131.1859	184.9738	226.8240	268.9153	310.3990	333.6998	364.2152	391.2318	436.4484
Model-2										
γ_1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
PROFIT	3907.2080	3759.5700	3661.5600	3587.4300	3527.6690	3477.5920	3434.5160	3396.7510	3363.1570	3332.9300
Profit_MLE	3906.9600	3758.2260	3661.7290	3588.9280	3528.6340	3476.4930	3436.1450	3399.8950	3363.6620	3333.7010
SE	27.1166	31.8847	37.0627	38.5176	40.6279	41.4829	41.8630	44.0598	47.6809	46.3982
Width.CI	106.2972	124.9882	145.2858	150.9890	159.2613	162.6128	164.1030	172.7142	186.9092	181.8809
Profit_BAY	3905.7210	3756.7430	3660.1290	3587.2120	3526.8780	3474.6900	3434.2680	3398.0090	3361.7370	3331.7910
PSE	27.1348	31.8928	37.0628	38.5276	40.6193	41.5106	41.8292	44.0580	47.6691	46.3584
Width.HPD	104.5434	123.6950	145.5473	151.0204	161.0510	160.3984	163.3172	168.9135	184.0776	176.3505

lows Weibull distribution having common shape parameter 'q' and different scale parameters $\gamma_1, \gamma_2, \mu_i, \gamma_{m1}, \gamma_{m2}$ respectively: where

$$f_i(t) = \gamma_i q t^{q-1}, \quad h_i(t) = \mu_i q t^{q-1} \text{ and } m_i(t) = \gamma_{mi} q t^{q-1}; t \geq 0,$$
$$\gamma_i, \mu_i, \gamma_{mi}, q > 0 \ (i = 1, 2).$$

One of the most significant classical processes that we are interested in in our study is the ML estimation procedure.

5.1.1. Maximum likelihood estimation

Let

$$\begin{aligned} X_1 &= (x_{11}, x_{12}, \ldots, x_{1n}) \\ X_2 &= (x_{21}, x_{22}, \ldots, x_{2n}) \\ X_3 &= (x_{31}, x_{32}, \ldots, x_{3n}) \\ X_4 &= (x_{41}, x_{42}, \ldots, x_{4n}) \\ X_5 &= (x_{51}, x_{52}, \ldots, x_{5n}). \end{aligned} \tag{31}$$

Be five independent random sample of size $n_i (i = 1, 2, 3, 4, 5)$ drawn from Weibull distribution with failure rate, maximum operative time, arrival rate of server, preventive maintenance and repair rates $\gamma_1, \gamma_2, \mu_i, \gamma_{m1}, \gamma_{m2}$ respectively. The Likelihood function of the combined sample is:

$$\begin{aligned} &L(X_1, X_2, X_3, X_4, X_5 | \gamma_1, \gamma_2, \mu_i, \gamma_{m1}, \gamma_{m2}) \\ &= \gamma_1^{n_1} \gamma_2^{n_2} \mu_i^{n_3} \gamma_{m1}^{n_4} \gamma_{m2}^{n_5} \times q^{n_1+n_2+n_3+n_4+n_5} \\ &\quad \times \left[\prod_{i=1}^{n1} x_i^{q-1} \times \prod_{i=1}^{n2} x_i^{q-1} \times \prod_{i=1}^{n3} x_i^{q-1} \times \prod_{i=1}^{n4} x_i^{q-1} \times \prod_{i=1}^{n5} x_i^{q-1} \right] \end{aligned} \tag{32}$$

$$\times \exp -[\gamma_1 \sum_{i=1}^{n1} x_i^q + \gamma_2 \sum_{i=1}^{n2} x_i^q + \mu_i \sum_{i=1}^{n3} x_i^q$$
$$+ \gamma_{m1} \sum_{i=1}^{n4} x_i^q + \gamma_{m2} \sum_{i=1}^{n5} x_i^q],$$

put $Y_i = \prod_{i=1}^{n_i} X_i^{q-1}$ and $W_i = \sum_{i=1}^{n_1} X_i^q : i = 1, 2, 3, 4, 5$

$$\begin{aligned} &L(X_1, X_2, X_3, X_4, X_5|\gamma_1, \gamma_2, \mu_i, \gamma_{m1}, \gamma_{m2}) \\ &= \gamma_1^{n_1}\gamma_2^{n_2}\mu_i^{n_3}\gamma_{m1}^{n_4}\gamma_{m2}^{n_5} \times q^{n1+n2+n3+n4+n5} \times Z_1Z_2Z_3Z_4Z_5 \\ &\quad \times \exp -[\gamma_1 W_1 + \gamma_2 W_2 + \mu_i W_3 + \gamma_{m1} W_4 + \gamma_{m2} W_5]. \end{aligned} \tag{33}$$

The M.L estimates (say $\hat{\gamma}_1, \hat{\gamma}_2, \hat{\mu}_i, \hat{\gamma}_{m1}, \hat{\gamma}_{m2}$) of the parameters $\gamma_1, \gamma_2, \mu_i, \gamma_{m1}, \gamma_{m2}$ are based on commonly used maximization likelihood approach are:

$$\begin{aligned} \hat{\gamma}_1 &= \frac{n_1}{W_1}, \\ \hat{\gamma}_2 &= \frac{n_2}{W_2}, \\ \hat{\mu}_i &= \frac{n_3}{W_3}, \\ \hat{\gamma}_{m1} &= \frac{n_4}{W_4}, \\ \hat{\gamma}_{m2} &= \frac{n_5}{W_5}. \end{aligned} \tag{34}$$

Therefore, the MLE's of the MTSF and profit function say, $\hat{M}$ and $\hat{P}$ can be determined by applying the invariance property of MLE. The asymptotic sampling distribution of

$$\begin{pmatrix} \hat{\gamma}_1 - \gamma_1 \\ \hat{\gamma}_2 - \gamma_2 \\ \hat{\mu}_i - \mu_i \\ \hat{\gamma}_{m1} - \gamma_{m1} \\ \hat{\gamma}_{m2} - \gamma_{m2} \end{pmatrix} \sim N_5(0, I^{-1}), \tag{35}$$

where I represents the Fisher's Information matrix with diagonal elements

$$I_{11} = \frac{n_1}{\gamma_1^2}; I_{22} = \frac{n_2}{\gamma_2^2}; I_{33} = \frac{n_3}{\mu_i^2}; I_{44} = \frac{n_4}{\gamma_{m1}^2}; I_{55} = \frac{n_5}{\gamma_{m2}^2}, \tag{36}$$

and non-diagonal elements are all zero. Also the asymptotic distribution of $(\hat{M} - M) \sim N_2(0, A'I^{-1}A)$ and $(\hat{P} - P) \sim N_5(0, B'I^{-1}B)$, where

$$A' = \left(\frac{\partial M}{\partial \gamma_1}, \frac{\partial M}{\partial \gamma_2}\right), \tag{37}$$

and

$$B' = \left(\frac{\partial P}{\partial \gamma_1}, \frac{\partial P}{\partial \gamma_2}, \frac{\partial P}{\partial \mu_i}, \frac{\partial P}{\partial \gamma_{m1}}, \frac{\partial P}{\partial \gamma_{m2}}\right). \tag{38}$$

5.2. *Bayesian estimation*

Since the Weibull distribution, when the shape parameter is known, has a gamma distribution as its natural family of conjugate priors of scale parameter. In our scenario, the prior distributions of scale parameters $\gamma_1, \gamma_2, \mu_i, \gamma_{m1}, \gamma_{m2}$ when the shape parameter is q is known as considered to be gamma with parameters (a_i, b_i) where $(i = 1, 2, 3, 4, 5)$ and are given as follows:

$$\begin{aligned} \gamma_1 &\sim Gamma(a_1, b_1) \\ \gamma_2 &\sim Gamma(a_2, b_2) \\ \mu_i &\sim Gamma(a_3, b_3) \\ \gamma_{m1} &\sim Gamma(a_4, b_4) \\ \gamma_{m2} &\sim Gamma(a_5, b_5). \end{aligned} \tag{39}$$

The parameters (a_i, b_i) of prior distributions are known as hyper parameters. By using the likelihood function in (33) and prior distribution of $\gamma_1, \gamma_2, \mu_i, \gamma_{m1}, \gamma_{m2}$ from set of (34) the posterior distribution of these parameters are obtained as follows:

$$\begin{aligned} \gamma_1|X_1 &\sim Gamma(a_1 + n_1, b_1 + W_1) \\ \gamma_2|X_2 &\sim Gamma(a_2 + n_2, b_2 + W_2) \\ \mu_i|X_3 &\sim Gamma(a_3 + n_3, b_3 + W_3) \\ \gamma_{m1}|X_4 &\sim Gamma(a_4 + n_4, b_4 + W_4) \\ \gamma_{m2}|X_5 &\sim Gamma(a_5 + n_5, b_5 + W_5). \end{aligned} \tag{40}$$

Under squared error loss function (which minimizes the expected posterior loss), Bayes estimates of $\gamma_1, \gamma_2, \mu_i, \gamma_{m1}, \gamma_{m2}$ are respectively the means of posterior distribution given in (40) and are as follows

$$\begin{aligned} \hat{\gamma}_1 &= \frac{b_1 + W_1}{a_1 + n_1}; \\ \hat{\gamma}_2 &= \frac{b_2 + W_2}{a_2 + n_2}; \\ \hat{\mu}_i &= \frac{b_3 + W_3}{a_3 + n_3}; \\ \hat{\gamma}_{m1} &= \frac{b_4 + W_4}{a_4 + n_4}; \\ \hat{\gamma}_{m2} &= \frac{b_5 + W_5}{a_5 + n_5}. \end{aligned} \tag{41}$$

6. Simulation study

The MLE and Bayes estimates of scale parameters $\gamma_1, \gamma_2, \mu_i, \gamma_{m1}, \gamma_{m2}$ where shape parameter q is known and common are found in the section-5 by maximizing log likelihood function. Additionally, we have estimates of the profit function, busy period, availability, and MTSF. A Monte Carlo Markov Chain simulation study is carried out in order to evaluate the statistical performances of these estimates. Using a random sample of size $n_i = 180$ where $(i = 1, 2, 3, 4, 5)$ for model 1 and for model 2 $(i = 1, 2, 3)$ selected from the Weibull distribution for a variety of parameter values and ML estimates for the MTSF, availability, busy period, and profit function are derived.

In Bayesian estimation, we used Metropolis-Hastings (M-H) algorithm within a Markov Chain Monte Carlo (MCMC). To make sure that the Markov chains reach the stationary posterior distribution, we generated 10,000 realizations from the posterior densities with N=1000 replications. Also, we have taken $K_0 = 5000$ and $K_1 = 600$. The hyper-parameters are set to these values $\gamma_1 = b_1/a_1; \gamma_2 = b_2/a_2; \mu_i = b_3/a_3; \gamma_{m1} = b_4/a_4; \gamma_{m2} = b_5/a_5$ to produce Bayes estimates of the parameters using gamma prior. The simulation study's outcome is summed up in Tables (3-14). Every computation is carried out utilizing the Python programming language. Validation of the estimators by the use of the "statistical tests" was based on two major measures of precision and accuracy, averaged over N=1,000 replications:

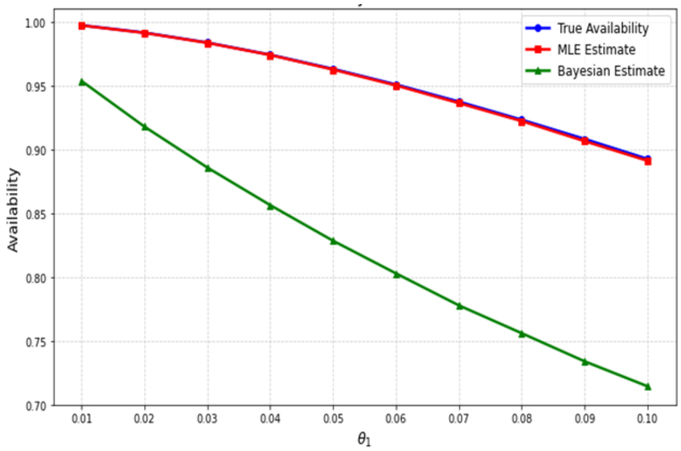

Figure 3a. Availability at $q = 0.6$.

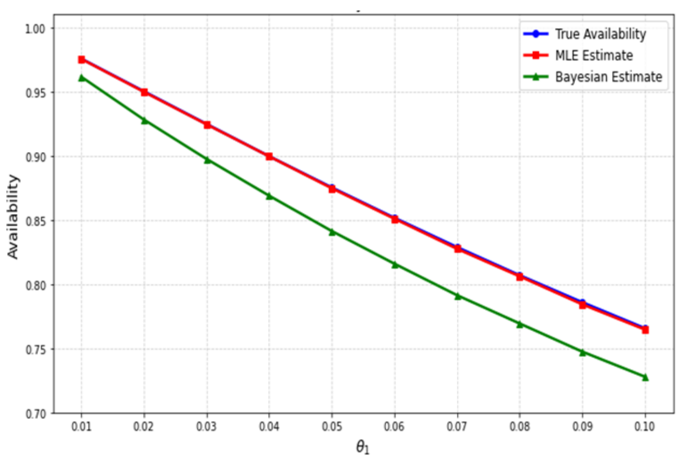

Figure 3b. Availability at $q = 0.9$.

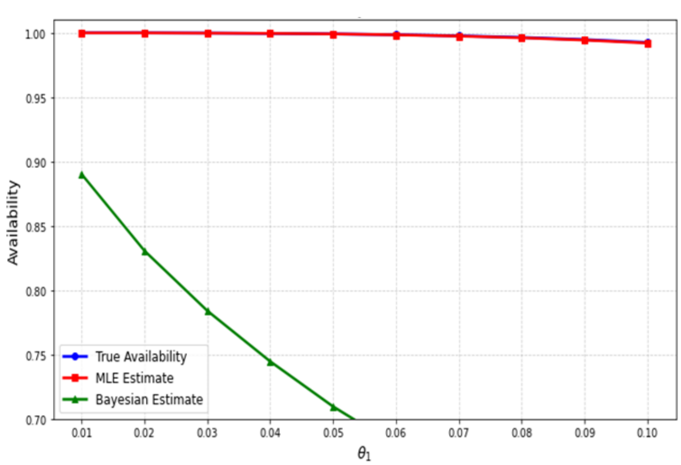

Figure 3c. Availability at $q = 0.3$.

- Precision: Comparison of Standard Error (SE) MLE to the Posterior Standard Error (PSE) Bayesian estimates.
- Interval Estimation: Comparison of asymptotic Confidence Interval (CI) widths with Bayesian Highest posteriory density (HPD) intervals.

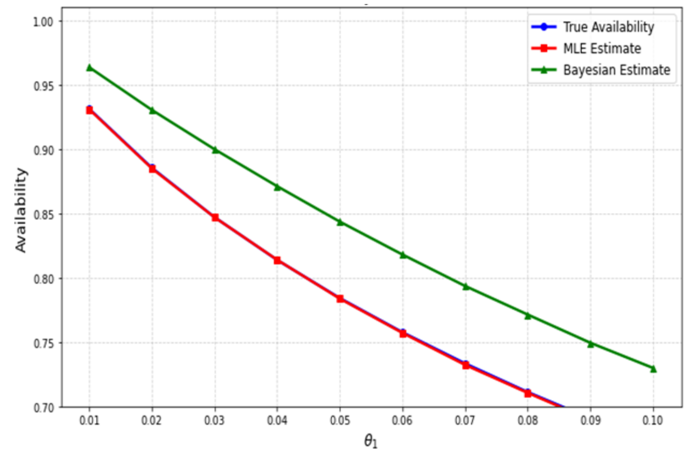

Figure 3d. Availability at $q = 1.2$.

In order to gain a more detailed understanding of the system, we plot the true availability, its MLE and Bayes estimate curves shown in Figures 3a- 3d, with respect to various shape parameter ($q = 0.6, 0.9, 0.3, 1.2$) and failure rate γ_1. And all other parameters are kept fixed ($\gamma_2 = 0.001, \mu_i = 0.6, \gamma_{m1} = 20, \gamma_{m2} = 0.5$).

7. Conclusion

The main aim of the study was to assess and compare the reliability aspects of a unit repairable system in two different models (Model 1 and Model 2) that made use of preventive maintenance (Model 1) and completely ignoring the preventive maintenance (Model 2). Through the Semi-Markovian Approach and the Regenerative Point Technique, the paper was able to measure the main metrics of Mean Time to System Failure (MTSF) and availability and profit using a dual-estimation model of Maximum Likelihood Estimation (MLE) and Bayesian estimation.

In Model 1, where we incorporated preventive maintenance, we found from Tables 3-14 that with a constant repair rate of 0.5 and shape parameter q $(0.5, 1, 2)$, both MTSF and availability, as well as profit, decrease as the failure rate increases. Additionally, the busy period increases with increase in the failure rate. It is also observed that the highest levels of availability and profit occur when $q = 0.5$, indicating the Weibull distribution. The standard error and posterior standard error for all reliability metrics increase as the failure rate rises. In Model 2, where preventive maintenance is not utilized, we observed that with a constant repair rate of 0.5 and shape parameter q $(0.5, 1, 2)$, MTSF, availability, busy period, and profit all decline with an increase in failure rate. Also, it is noticed that both MTSF and availability are greater when preventive maintenance is not employed. The availability curves in Figures 3a-3d indicate that the actual availability is roughly equivalent to the MLE availability. Since Model 2 eliminates PM-related downtime, it results in higher availability. Unless there is a substantial increase in system uptime, preventive maintenance may lead to fewer failures but can also cause increased planned downtime, potentially reducing availability.

Future studies would be able to extrapolate this model to multi-unit or standby systems. Moreover, the practical use of the industry planning would be enhanced further by the use of optimization tools to determine the specific best time to do the maintenance balancing the costs against the variable repair rates.

Data availability

In this study, no primary datasets were generated or collected.

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