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Mathematical model analysis for maize yield under co-infection of maize streak virus and maize stripe virus diseases with control measures

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Abstract

Maize is a staple food for billions of humans globally. It is a very important food crop for sustaining global food security, livelihoods, and economic development. Despite the valuable advantages of maize, its optimal production is affected by several challenges, such as climate change, socio-economic factors, sustainable practices, pests, and diseases. A mathematical epidemiological model for maize yield that takes into account the major factors influencing maize yield, such as co-infection of maize streak virus and maize stripe virus, is developed. The essential dynamical features of the model, such as the basic reproduction number and disease-free and endemic equilibria, are determined and analysed. The model is used to investigate how to improve maize yield under several challenging factors, using Nigeria as a case study. By fitting the model to real data on maize yield in Nigeria, possible long-term model predictions on maize yield in Nigeria were obtained, and parameters of the model were also estimated. Numerical simulations using the estimated parameters illustrated how various control interventions, such as the use of modern technologies, controlling of insect vectors, etc, can improve maize yield significantly. The results of this study are expected to aid farmers and policy-makers in adopting best farming practices that will eliminate maize diseases and consequently assist in achieving food security globally.

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1. Introduction

Maize is a staple food for billions of humans globally [1, 2]. It is a significant source of calories, protein, and essential nutrients for humans. Beyond human consumption, maize is a major source of animal feed and is increasingly being used in the production of biofuel [3]. Over 1.1 billion tons of maize are produced annually on approximately 197 million hectares globally [3]. Overall, maize is a critical food crop for global food security, livelihoods, and economic development [3]. Despite the valuable advantages of maize, several challenges influence optimal maize production, such as climate change, socio-economic factors, nutrient management, sustainable practices, pests and diseases. Maize streak virus (MSV) disease caused by a geminivirus is one of the major diseases that causes significant maize yield loss, particularly in sub-Saharan Africa [4, 5]. The virus, transmitted by leafhoppers, primarily Cicadulina mbila and other Cicadulina species, can lead to reduced maize yield, poor cob formation, and even complete crop

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failure, if the maize is infected during the early stages of growth (e.g., within the first three weeks after planting), especially for susceptible maize varieties [4, 5]. Maize stripe virus (MSpV) is another important maize disease that can cause significant yield losses in maize. The virus is transmitted by leafhoppers Peregrinus maidis, and causes severe stunting and reduced cob and grain production [4, 6]. It also causes premature death of infected maize plants and, in general, leads to significant maize yield loss [4, 6]. MSpV is found in Africa, the Americas, Australia, and Asia [6]. The MSpV can also co-infect with Maize Streak Virus (MSV), and this co-infection can lead to even greater yield losses than infection by either virus alone [4, 6].

In this study, a mathematical epidemiological model for the dynamics and control of co-infection of MSV and MSpV is developed. The model is used to analyse the impact of co-infection of MSV and MSpV together with other important control interventions that affect maize yield using Nigeria as a case study. Nigeria is chosen as a case study due to its significant challenges with maize yield (ie., about 2 Tonne/Ha), food insecurity and disease, representative of many Sub-Saharan African countries [7]. The country's diverse agricultural zones and varying socio-economic conditions provide a comprehensive framework for studying maize yield dynamics. Insights gained from this study can inform broader global strategies aimed at combating food insecurity in comparable settings.

Theoretical studies using mathematical models to analyse the dynamics and control of MSV or MSpV, including their co-infection, is incomplete. Some important research findings on this critical area of research are presented here. For instance, Alemneh et al. [8] formulated and analysed a mathematical model for the dynamics of MSV and investigated the optimal strategy to control MSV disease. Seidu [9] used a mathematical model to analyse the role of host-to-host transmission of MSV disease. Ackora-Prah et al. [10] investigated the interaction of leafhopper vectors on maize plants using a mathematical model of the form Holling's Type III functional response. Collins & Duffy [11] used a stochastic epidemic model to study the dynamics and control of MSV disease. They discovered that chemical and mechanical control measures, preferably together, are better than preventive controls in reducing disease prevalence. Ayembillah et al. [12] use a mathematical model to analyse the dynamics of MSV disease. Kumar et al. [13] use an optimised linearization-based predictor-corrector method to investigate MSV dynamics. The effect of co-infection by maize streak virus (MSV) and maize stripe virus (MSpV) on plant growth and grain yield was studied experimentally by Doyle & Autrey [4]. Their study revealed that co-infection by MSV and MSpV viruses causes greater yield loss than when the maize plants are infected by either virus [4].

Even though the above studies have contributed to the understanding of the dynamics and control of MSV and MSpV diseases, to the best of our knowledge, none of these studies used a mathematical model to analyse the co-infection of MSV and MSpV dynamics and control together with their effects on maize yield. In this study, a mathematical model is used to analyse the dynamics and control of co-infection of MSV and MSpV. The model is further extended and used to study the impact of control interventions on maize yield using Nigeria as a case study. The results of this study are expected to aid farmers and policy-makers in adopting best farming practices that will improve maize yield in the presence of challenges such as MSV, MSpV diseases, and consequently assist in achieving food security.

To enhance scientific depth, we illustrated how this study is connected with various areas of research (such as agriculture, epidemiology, mathematics, biomedical and materials science research), which is crucial for creating new understanding, fostering deeper learning by showing how these different fields relate to real-world topics. For instance, biomedical, materials science research, and bio-composite innovation are very helpful in adopting the best innovative control measures used in modelling disease control. More details on the usefulness of biochemical and medical innovations research can be found in [14]. The interdisciplinary relationship between agriculture, epidemiology and mathematics is systematically illustrated by showing how mathematical modelling can be used to analyse the dynamics and control of agricultural food crop diseases that aid in adopting best farming practices that can minimise maize diseases like MSV, MSpV, which is crucial in achieving and sustaining food security globally.

2. Model development

The mathematical model for maize yield in the presence of MSV and MSpV co-infection and control comprises two parts. The first part is an epidemiological model that describes the population dynamics of maize plants under MSV and MSpV co-infection and control. The second part involves developing a mathematical model that estimates maize yield. The estimation of maize yield by the second part is computed using the first part of the model. Below is a detailed description of the models.

2.1. Co-infection of MSV and MSpV disease model

The mathematical model development for maize plant population dynamics under MSV and MSpV disease co-infection is derived based on the following assumptions. The MSV is transmitted by a leafhopper of the species Cicadulina, while the MSpV is transmitted by leafhopper peregrinus maidis. These two viruses have different vectors. This means that a leafhopper that is a vector for MSV cannot be a vector for MSpV and vice versa. However, it is possible for a maize plant to be infected with both viruses simultaneously when it is fed upon by two different leafhoppers, each carrying one of the viruses. Based on these explanation, we consider a total maize plant population N and partition it into four sub-populations: namely healthy maize plants (susceptible) denoted by S, maize plants infected by only MSV denoted by I_a, maize plants infected by only MSpV denoted by I_b and maize plants co-infected by MSV and MSpV denoted by I_{ab}. Since any maize plant infected by either MSV or MSpV cannot recover but can be managed, we do not consider the recovered class. The mode of transmission of MSV is only through the infected leafhopper of the species Cicadulina, while the mode of transmission of MSpV is also through the leafhopper Peregrinus Maidis. Let M_a denote the total population of leafhoppers of the species Cicadulina, while M_b denotes the total population of leafhoppers of Peregrinus Maidis. The M_a is partitioned into susceptible and infected leafhoppers, denoted by X_a and Y_a, respectively. Similarly, the M_b is also partitioned into susceptible and infected leafhoppers, denoted by X_b and Y_b, respectively.

The nature of interactions among these sub-populations of maize plants and leafhoppers is described as follows. The susceptible maize S increase logistically to its carrying capacity K at a growth rate r and decrease through MSV or MSpV infection through contact with infected leafhoppers I_a or I_b at a rate β_a^* and β_b^* respectively. The maize plants already infected with MSV (i.e, I_a) can become co-infected when interacted with leafhopper infected with MSpV (i.e, Y_b) at a rate β_{ab}^*. Similarly, maize plants already infected with MSpV (i.e, I_b) will become co-infected if contact with leafhoppers infected with MSV (i.e, Y_a) at a rate β_{ba}^*. The infected maize plants I_a, I_b and I_{ab} can die due to the disease at a rate μ_a^*, μ_b^*, and μ_{ab}^* respectively. The susceptible leafhopper X_a increase through recruitment at a rate Λ_a^* and decrease as they get infected with infected maize plants I_a. Both the X_a and Y_a die naturally at a rate δ_a^*. Similarly, susceptible leafhopper X_b increase through recruitment at a rate Λ_b^* and decrease as they get infected with infected maize plants I_b. Both the X_b and Y_b die naturally at a rate δ_b^*. To consider control measures, we assume that use of resistant varieties and good management practices like early planting, weed control, crop rotation and removal of crop residues decreases infections rates $\beta_a^*, \beta_b^*, \beta_{ab}^*, \beta_{ba}^*$ at rate c_a, c_b, c_{ab}, c_{ba} respectively. Use of appropriate insecticides reduces the vectors population M_a and M_b at a rate γ_a^* and γ_b^* respectively. Use of modern technology (fertilizer and irrigation) enhances the growth of healthy (susceptible) maize S at a rate ρ. Based on these formulations, the following MSV and MSpV co-infection model with control measures is obtained

$$\begin{aligned}
\frac{dS}{dt} &= \rho r S\left(1-\frac{S}{K}\right) - (1-c_a)\beta_a^* Y_a S - (1-c_b)\beta_b^* Y_b S,\\
\frac{dI_a}{dt} &= (1-c_a)\beta_a^* Y_a S - (1-c_{ab})\beta_{ab}^* Y_b I_a - \mu_a^* I_a,\\
\frac{dI_b}{dt} &= (1-c_b)\beta_b^* Y_b S - (1-c_{ba})\beta_{ba}^* Y_a I_b - \mu_b^* I_b,\\
\frac{dI_{ab}}{dt} &= (1-c_{ab})\beta_{ab}^* Y_b I_a + (1-c_{ba})\beta_{ba}^* Y_a I_b - \mu_{ab}^* I_{ab},\\
\frac{dX_a}{dt} &= \Lambda_a^* - (1-c_a)\alpha_a^* I_a X_a - (\gamma_a^* + \delta_a^*) X_a, \qquad (1)\\
\frac{dY_a}{dt} &= (1-c_a)\alpha_a^* I_a X_a - (\gamma_a^* + \delta_a^*) Y_a,\\
\frac{dX_b}{dt} &= \Lambda_b^* - (1-c_b)\alpha_b^* I_b X_b - (\gamma_b^* + \delta_b^*) X_b,\\
\frac{dY_b}{dt} &= (1-c_b)\alpha_b^* I_b X_b - (\gamma_b^* + \delta_b^*) Y_b.
\end{aligned}$$

The description of variables and parameters are presented in Tables 1 and 2.

Table 1: Description for variables for model (7)

Variables	Description	Unit
N	Total population of maize plants	Plants Ha^{-1}
S	Susceptible maize plants	Plants Ha^{-1}
I_a	Maize plants infected with MSV	Plants Ha^{-1}
I_b	Maize plants infected with MSpV	Plants Ha^{-1}
I_{ab}	Maize plants co-infected with MSV and MSpV	Plants Ha^{-1}
M_a	Total population of vectors that host MSV	Vectors Ha^{-1}
X_a	Susceptible vectors that host MSV	Vectors Ha^{-1}
Y_a	Infected vectors that host MSV	Vectors Ha^{-1}
M_b	Total population of vectors that host MSpV	Vectors Ha^{-1}
X_b	Susceptible vectors that host MSpV	Vectors Ha^{-1}
Y_b	Infected vectors that host MSpV	Vectors Ha^{-1}

2.2. Maize yield model

The total quantity of maize yield Q in Tonne Per Hectare (Tonne/Ha) depends on the total population of maize plants N per Hectare. Mathematically, the total quantity of maize yield Q (Tonne/Ha) is proportional to the total population of maize plants N per Hectare. Since $N = S + I_a + I_b + I_{ab}$, and each of the sub-population have different yield rates depending on the severity of the disease on the plant. The total quantity of maize yield Q (Tonne/Ha) will depend on the proportion of S, I_a, I_b and I_{ab} per Hectare. Based on these explanations, the mathematical model describing the quantity of susceptible maize yield Q_s (Tonne/Ha) can be estimated as

$$Q_s = \sigma_s S, \qquad (2)$$

where σ_s is constant of proportionality that converts S to susceptible maize yield Q_s (Tonne/Ha). Similarly, the mathematical model for estimating maize yields from I_a, I_b and I_{ab} can be described as

$$\begin{aligned}
Q_a &= \sigma_a I_a, \qquad (3)\\
Q_b &= \sigma_b I_b, \qquad (4)\\
Q_{ab} &= \sigma_{ab} I_{ab}, \qquad (5)
\end{aligned}$$

where the parameters σ_a, σ_b and σ_{ab} are proportionality constants that converts maize plants (I_a, I_b and I_{ab}) to maize yield (Q_a, Q_b and Q_{ab}) (Tonne/Ha) respectively. From equations (2)–(5), the total quantity of maize yield Q (Tonne/Ha) can be estimated as

$$Q = Q_s + Q_a + Q_b + Q_{ab}. \qquad (6)$$

From equation (6), it is very clear that the analysis of model (7) is very crucial in determining maize yield Q. Therefore, in the subsequent analysis, we shall present dynamical system analysis of model (7).

Table 2: Description of parameters for model (7)

Parameters	Description	Unit
K	Carrying capacity of maize plants	Plants Ha^{-1}
r	Growth rate of S	Year^{-1}
β_a^*	Contact rate of S with Y_a	Ha Vector^{-1} Year^{-1}
β_b^*	Contact rate of S with Y_b	Ha Vector^{-1} Year^{-1}
β_{ab}^*	Contact rate of I_a with Y_b	Ha Vector^{-1} Year^{-1}
β_{ab}^*	Contact rate of I_b with Y_a	Ha Vector^{-1} Year^{-1}
μ_a^*	Mortality rate of I_a	Year^{-1}
μ_b^*	Mortality rate of I_b	Year^{-1}
μ_{ab}^*	Mortality rate of I_{ab}	Year^{-1}
ρ	Enhance growth rate of S due to modern technology	Dimensionless
Λ_a^*	Recruitment rate of X_a	Vectors Ha^{-1} Year^{-1}
Λ_b^*	Recruitment rate of X_b	Vectors Ha^{-1} Year^{-1}
α_a^*	Contact rate of X_a with I_a	Ha Plant^{-1} Year^{-1}
α_b^*	Contact rate of X_b with I_b	Ha Plant^{-1} Year^{-1}
δ_a^*	Natural death rate of M_a	Year^{-1}
δ_b^*	Natural death rate of M_b	Year^{-1}
γ_a^*	Removal of M_a using control measures (CM)	Year^{-1}
γ_b^*	Removal of M_b using CM	Year^{-1}
c_a	Reduction of β_a due to use of resistant varieties (RV)	Dimensionless
c_b	Reduction of β_b due to RV	Dimensionless
c_{ab}	Reduction of β_{ab} due to RV	Dimensionless
c_{ba}	Reduction of β_{ba} due to RV	Dimensionless

3. Analyses and results

3.1. Model transformation

Model (1) involves the insect vectors population and the maize plant population, which have different measuring units. Thus, to analyse the model, it is advisable that we non-dimensionalize the model, converting all variables and parameters into proportions. This helps to reveal hidden relationships and dependencies between variables, and identifying key dimensionless parameters and consequently leading to a deeper understanding of system dynamics [15, 16]. By rescaling model (1) such that $\tau = rt$, $s = \frac{S}{K}$, $i_a = \frac{I_a}{K}$, $i_b = \frac{I_b}{K}$, $i_{ab} = \frac{I_{ab}}{K}$, $x_a = \frac{X_a}{M_a^0}$, $y_a = \frac{Y_a}{M_a^0}$, $x_b = \frac{X_b}{M_b^0}$, $y_b = \frac{Y_b}{M_b^0}$, $\alpha_a = \frac{\alpha_a^* K}{r}$, $\alpha_b = \frac{\alpha_b^* K}{r}$, $\beta_a = \frac{\beta_a^* M_a^0}{r}$, $\beta_b = \frac{\beta_b^* M_b^0}{r}$, $\beta_{ab} = \frac{\beta_{ab}^* M_b^0}{r}$, $\beta_{ba} = \frac{\beta_{ba}^* M_a^0}{r}$, $\Lambda_a = \frac{\Lambda_a^*}{rM_a^0}$, $\Lambda_b = \frac{\Lambda_b^*}{rM_b^0}$, $\mu_a = \frac{\mu_a^*}{r}$, $\mu_b = \frac{\mu_b^*}{r}$, $\mu_{ab} = \frac{\mu_{ab}^*}{r}$, $\delta_a = \frac{\delta_a^*}{r}$, $\delta_b = \frac{\delta_b^*}{r}$, $\gamma_a = \frac{\gamma_a^*}{r}$, $\gamma_b = \frac{\gamma_b^*}{r}$, $M_a^0 = X_a(0)+Y_a(0)$ and $M_b^0 = X_b(0)+Y_b(0)$ the dimensionless version of model (1) is obtained as:

$$\begin{aligned}
\frac{ds}{d\tau} &= \rho s(1-s) - (1-c_a)\beta_a y_a s - (1-c_b)\beta_b y_b s,\\
\frac{di_a}{d\tau} &= (1-c_a)\beta_a y_a s - (1-c_{ab})\beta_{ab} y_b i_a - \mu_a i_a,\\
\frac{di_b}{d\tau} &= (1-c_b)\beta_b y_b s - (1-c_{ba})\beta_{ba} y_a i_b - \mu_b i_b,\\
\frac{di_{ab}}{d\tau} &= (1-c_{ab})\beta_{ab} y_b i_a + (1-c_{ba})\beta_{ba} y_a i_b - \mu_{ab} i_{ab},\\
\frac{dx_a}{d\tau} &= \Lambda_a - (1-c_a)\alpha_a i_a x_a - (\gamma_a+\delta_a)x_a, \qquad (7)\\
\frac{dy_a}{d\tau} &= (1-c_a)\alpha_a i_a x_a - (\gamma_a+\delta_a)y_a,\\
\frac{dx_b}{d\tau} &= \Lambda_b - (1-c_b)\alpha_b i_b x_b - (\gamma_b+\delta_b)x_b,\\
\frac{dy_b}{d\tau} &= (1-c_b)\alpha_b i_b x_b - (\gamma_b+\delta_b)y_b.
\end{aligned}$$

3.2. MSV sub-model

The MSV only sub-model is a special case of the MSV and MSpV co-infection model (7) where MSV is the only disease affecting the maize plants. This special case is considered in this section, and the MSV only sub-model is obtained by setting $i_b = i_{ab} = x_b = y_b = 0$ in the model (7), to get

$$\begin{aligned}
\frac{ds}{d\tau} &= \rho s(1-s) - (1-c_a)\beta_a y_a s,\\
\frac{di_a}{d\tau} &= (1-c_a)\beta_a y_a s - \mu_a i_a, \qquad (8)\\
\frac{dx_a}{d\tau} &= \Lambda_a - (1-c_a)\alpha_a i_a x_a - (\gamma_a+\delta_a)x_a,\\
\frac{dy_a}{dt} &= (1-c_a)\alpha_a i_a x_a - (\gamma_a+\delta_a)y_a.
\end{aligned}$$

The analysis of MSV only sub-model (8) will reveal the maize plants dynamics in the presence of MSV only. For the model analysis, the following assumptions are made where applicable: $\phi_1 = \rho$, $\phi_2 = (1-c_a)\beta_a$, $\phi_3 = (1-c_a)\alpha_a$, $\phi_4 = (\gamma_a+\delta_a)$.

3.2.1. Equilibrium points of MSV sub-model (8)

The MSV sub-model (8) has two important equilibrium points namely: the disease-free equilibrium (DFE) and endemic equilibrium (EE). The DFE can be understood as the equilibrium point of the model when there is no MSV disease in the system, while the EE is the equilibrium point of the model when MSV disease is in the system. The analytical representation of the DFE of sub-model (8) is given by:

$$(s^0,\ i_a^0,\ x_a^0,\ y_a^0) = \left(1,\ 0,\ \frac{\Lambda_a}{\phi_4},\ 0\right). \qquad (9)$$

Similarly, the analytical representation of the EE of sub-model (8) is given by:

$$(s^*,\ i_a^*,\ x_a^*,\ y_a^*) = \left(s^*,\ A_2 s^*(1-s^*),\ x_a^0 - y_a^*,\ A_1(1-s^*)\right), \qquad (10)$$

where $s^* = \frac{-b_1+\sqrt{(b_1)^2-4a_1c_1}}{2a_1}$, $a_1 = \phi_3 A_1 A_2$, $b_1 = \phi_3 A_2(x_a^0 - A_1)$, $c_1 = -\phi_4 A_1$, $A_1 = \frac{\phi_1}{\phi_2}$, $A_2 = \frac{\phi_1}{\mu_a}$.

3.2.2. The basic reproduction number of MSV sub-model (8)

In epidemiological modelling, the basic reproduction number denoted by $\mathcal{R}_0$ of an infection can be understood as the expected number of cases directly generated by one case in a population where all individuals are susceptible to infection [17]. The value of the basic reproduction number indicates whether an outbreak will be eradicated or persist. The basic reproduction number for the sub-model (8) is computed using the next generation matrix method [17] and is given by

$$\mathcal{R}_0^a = \sqrt{\frac{(1-c_a)\beta_a}{\mu_a}\frac{(1-c_a)\alpha_a x_a^0}{\gamma_a+\delta_a}}. \tag{11}$$

This can be simplified to:

$$(\mathcal{R}_0^a)^2 = \frac{x_a^0\phi_3\phi_2}{\mu_a\phi_4}. \tag{12}$$

3.2.3. Stability analysis of the MSV sub-model (8)

The stability analysis of a model about its equilibrium point describes the dynamics of the model about the equilibrium point. The stability analysis of the MSV sub-model (8) is summarized in the following theorems below.

Theorem 1. The sub-model (8) is locally asymptotically stable about the DFE (9) provided $\mathcal{R}_0^a < 1$.

Proof. The proof of Theorem 1 can be established by showing all the eigenvalues of the Jacobian of sub-model (8) about the disease-free equilibrium point (9) have a negative real part. The Jacobian of sub-model (8) about the DFE (9) is by:

$$J_0 = \begin{pmatrix} -\phi_1 & -\phi_1 & 0 & -\phi_2 \\ 0 & -\mu_a & 0 & \phi_2 \\ 0 & -x_a^0\phi_3 & -\phi_4 & 0 \\ 0 & x_a^0\phi_3 & 0 & -\phi_4 \end{pmatrix}. \tag{13}$$

The eigenvalues of the Jacobian are:

$$\begin{aligned} \lambda_1 &= -\phi_1, \\ \lambda_2 &= -\phi_4, \\ \lambda_3 &= \frac{-(\mu_a+\phi_4)-\sqrt{(\mu_a+\phi_4)^2+4(x_a^0\phi_2\phi_3-\mu_a\phi_4)}}{2}, \\ \lambda_4 &= \frac{-(\mu_a+\phi_4)+\sqrt{(\mu_a+\phi_4)^2+4(x_a^0\phi_2\phi_3-\mu_a\phi_4)}}{2}. \end{aligned} \tag{14}$$

Clearly, $\lambda_1, \lambda_2, \lambda_3$ are negative. For λ_4 we have that by taking equation (12) into consideration and simplify, we obtain $\lambda_4 = \frac{-(\mu_a+\phi_4)+\sqrt{(\mu_a+\phi_4)^2+4\mu_a\phi_4((\mathcal{R}_0^a)^2-1)}}{2}$. Consequently, $\lambda_4 < 0$ if $\mathcal{R}_0^a < 1$. Based on this, we conclude that the sub-model (8) is locally asymptotically stable about the DFE (9) provided $\mathcal{R}_0^a < 1$. □

The implication of Theorem 1 is that for the situation when only MSV is present in the system, the disease can be eradicated if the initial infected population (maize plants and leafhoppers) is small (within the neighbourhood of the disease-free equilibrium) and provided the control measures are effective enough to keep the basic reproduction number below unity. However, if the initial infected population is large, to determine the dynamics of the model about the DFE requires the investigation of the global stability of the model. The global stability analysis of sub-model (8) is presented in Theorem 2.

Theorem 2. The sub-model (8) is globally asymptotically stable about the DFE (9) provided $\mathcal{R}_0^a < 1$.

The proof of Theorem 2 will be established using a stability result in Castillo-Chavez et al. [18], which is stated in Lemma 1.

Lemma 1. Consider a model system written in the form:

$$\begin{aligned} \frac{dZ_1}{dt} &= F(Z_1, Z_2), \\ \frac{dZ_2}{dt} &= G(Z_1, Z_2), \quad G(Z_1, 0) = 0, \end{aligned} \tag{15}$$

where $Z_1 \in R^m$ and $Z_2 \in R^n$. $Z_0 = (Z_1^*, 0)$ denotes the disease-free equilibrium of the system. Assume that:

(H1) For $\frac{dZ_1}{dt} = F(Z_1, 0)$, Z_1^* is globally asymptotically stable;

(H2) $G(Z_1, Z_2) = AZ_2 - \hat{G}(Z_1, Z_2)$, $\hat{G}(Z_1, Z_2) \geq 0$ for $(Z_1, Z_2) \in \Omega$, where the Jacobian $A = \frac{\partial G}{\partial Z_2}(Z_1, 0)$ is an M–matrix (the off diagonal elements of A are non-negative) and Ω is the region where the model makes biological sense.

Then the disease-free equilibrium Z_0 is globally asymptotically stable provided that $\mathcal{R}_0 < 1$ [18].

Proof. To apply Lemma 1, we must show that the disease-free part of model (8) is globally stable (condition (H1)), whereas the disease part of model (8) satisfies conditions H2 of the Lemma. From sub-model equation (8), let $Z_1 = (S, X_a)$, $Z_2 = (I_a, Y_a)$. So, disease free part of model (8) is given by

$$\frac{dZ_1}{d\tau} = F(Z_1, 0) = \begin{pmatrix} s\,(1-s) \\ \Lambda_a - \phi_4 x_a \end{pmatrix}, \tag{16}$$

whereas the disease part of model (8) is

$$\frac{dZ_2}{dt} = G(Z_1, Z_2) = \begin{pmatrix} (1-c_a)\beta_a y_a s - \mu_a i_a \\ (1-c_a)\alpha_a i_a x_a - (\gamma_a+\delta_a)y_a \end{pmatrix}. \tag{17}$$

Clearly, disease free part (16) comprises of a set of linear ordinary differential equations and its exact solution can be determined as $s(\tau) = \frac{1}{1+\eta e^{-\tau}}$, $x_a(\tau) = \frac{\Lambda_a}{\phi_4} + \psi e^{-\phi_4\tau}$, where $\psi = x_a(0) - \frac{\Lambda_a}{\phi_4}$ and $\eta = \frac{1-s(0)}{s(0)}$. As $\tau \longrightarrow \infty$, $s(\tau) \longrightarrow s^0 = 1$, $x_a(\tau) \longrightarrow x_a^0 = \frac{\Lambda_a}{\phi_4}$. Thus, the disease free part of model (8) is globally asymptotically stable.

For the disease part (17), we compute the Jacobian of (17) about the disease-free equilibrium as

$$A = \begin{pmatrix} -\mu_a & (1-c_a)\beta_a \\ (1-c_a)\alpha_a x_a^0 & -(\gamma_a+\delta_a) \end{pmatrix}, \tag{18}$$

where A is an M–matrix with all its off-diagonal elements non-negative. From equations (17) and (18), the term $\hat{G}(Z_1, Z_2)$ is determined as

$$\hat{G}(Z_1, Z_2) = \begin{pmatrix} (1-c_a)\beta_a y_a(1-s) \\ (1-c_a)\alpha_a i_a(x_a^0 - x_a) \end{pmatrix}. \tag{19}$$

Clearly, $\hat{G}(Z_1, Z_2) \geq 0$, since $1 \geq s$ and $x_a^0 \geq x_a$. This shows that condition (H_2) holds and hence completes the proof. □

The epidemiological implications of these results is that MSV disease could possibly be eradicated irrespective of the size of the infected maize plants and leafhoppers at the initial stage of the outbreak, provided the control measures are effective to keep the basic reproduction number $\mathcal{R}_0^a$ below unity.

3.3. MSpV sub-model

The MSpV sub-model is a special case of the MSV and MSpV co-infection model (7) where MSpV is the only disease affecting the maize plants. This special case is considered in this section. The MSpV only sub-model is obtained by setting $I_a = I_{ab} = X_a = Y_b = 0$ in the model (7), to get:

$$\begin{aligned} \frac{ds}{d\tau} &= \rho s(1-s) - (1-c_b)\beta_b y_b s, \\ \frac{di_b}{d\tau} &= (1-c_b)\beta_b y_b s - \mu_b i_b, \\ \frac{dx_b}{d\tau} &= \Lambda_b - (1-c_b)\alpha_b i_b x_b - (\gamma_b+\delta_b)x_b, \\ \frac{dy_b}{d\tau} &= (1-c_b)\alpha_b i_b x - (\gamma_b+\delta_b)y_b. \end{aligned} \tag{20}$$

The analysis of MSpV sub-model (20) will reveal the maize plants' dynamics in the presence of MSpV only. For the model analysis, the following assumptions are made where applicable: $\theta_2 = (1-c_b)\beta_b,\ \ \theta_3 = (1-c_b)\alpha_b,\ \ \theta_4 = (\gamma_b+\delta_b)$.

3.3.1. Equilibrium points of the MSpV sub-model (20)

The MSpV sub-model (20) has two important equilibrium points, namely: the DFE and EE. The analytical representation of the DFE is given by:

$$(1^0,\ i_b^0,\ x_b^0,\ y_b^0) = \left(1,\ 0,\ \frac{\Lambda_b}{\theta_4},\ 0\right). \tag{21}$$

Similarly, the analytical representation of the EE of the MSpV sub-model (20) is given by:

$$(s^*,\ i_b^*,\ x_b^*,\ y_b^*) = \left(s^*,\ B_2 s^*(1-s^*),\ x_b^0 - y_b^*,\ B_1(1-s^*)\right), \tag{22}$$

where $s^* = \frac{-b_2+\sqrt{(b_2)^2-4a_2c_2}}{2a_2}$, $a_2 = \theta_3 B_1 B_2$, $b_2 = \theta_3 B_2(x_b^0 - B_1)$, $c_2 = -\theta_4 B_1$, $B_1 = \frac{\phi_1}{\theta_2}$, $B_2 = \frac{\phi_1}{\mu_b}$.

3.3.2. The basic reproduction number of the MSpV sub-model (20)

The basic reproduction number for the MSpV sub-model (20) is computed using the next generation matrix method [17] and is given by:

$$\mathcal{R}_0^b = \sqrt{\frac{(1-c_b)\beta_b}{\mu_b}\frac{(1-c_b)\alpha_b x_b^0}{\gamma_b+\delta_b}}. \tag{23}$$

The $\mathcal{R}_0^b$ can be simplified as:

$$(\mathcal{R}_0^b)^2 = \frac{x_b^0\theta_3\theta_2}{\mu_b\theta_4}. \tag{24}$$

3.3.3. Stability analysis of the MSpV sub-model (20)

The stability analysis of the MSpV only sub-model is summarized in the following theorems.

Theorem 3. The MSpV sub-model (20) is locally asymptotically stable about the DFE point (21) provided $\mathcal{R}_0^b < 1$.

The proof of Theorem 3 can be established using a similar approach used in the proof of Theorem 1.

The epidemiological implication of Theorem 3 is that for the situation when only MSpV is present in the system, the disease can be eradicated if the initial MSpV infected population (maize plants and leafhoppers) is small (within the neighbourhood of the disease free equilibrium) and provided the control measures are effective enough to keep basic reproduction number below unity. However, if the initial infected population is large, to determine the dynamics of the model about the DFE required the investigation of the global stability of the model. The global stability analysis of the MSpV sub-model (20) is presented in Theorem 4.

Theorem 4. The MSpV sub-model (20) is globally asymptotically stable about the DFE point (9) provided $\mathcal{R}_0^b < 1$.

The proof of Theorem 4 will be established using a similar approach used in the proof of Theorem 2. The epidemiological implication of these results is that MSpV disease could possibly be eradicated irrespective of the size of the MSpV infected maize plants and leafhoppers at the initial stage of the outbreak, provided the control measures are effective to keep the basic reproduction number $\mathcal{R}_0^b$ below unity.

3.4. Analysis of the co-infection MSV and MSpV model (7)

The qualitative analysis of the co-infection MSV and MSpV model (7) is presented in this section.

3.4.1. Equilibrium points of the co-infection MSV and MSpV model (7)

The MSV and MSpV co-infection model (7) has important equilibrium points, namely: DFE and EE. The analytical representation of the DFE of the co-infection MSV and MSpV model (7) is given by

$$\left(s^0,\ i_a^0,\ i_b^0,\ i_{ab}^0,\ x_a^0,\ y_a^0,\ x_b^0,\ y_b^0\right) = \left(1,\ 0,\ 0,\ 0,\ \frac{\Lambda_a}{\phi_4},\ 0,\ \frac{\Lambda_b}{\theta_4},\ 0\right). \tag{25}$$

The EE of the co-infection MSV and MSpV model (7) is complex to represent analytically, so it is not presented here.

3.4.2. Basic reproduction number of the co-infection MSV and MSpV model (7)

The basic reproduction number of the co-infection MSV and MSpV model (7) is computed using the next generation matrix method [17] and is given by

$$\mathcal{R}_0 = \max\{\mathcal{R}_0^a, \mathcal{R}_0^b\}, \tag{26}$$

where

$$\mathcal{R}_0^a = \sqrt{\frac{x_a^0\phi_3\phi_2}{\mu_a\phi_4}}, \quad \mathcal{R}_0^b = \sqrt{\frac{x_b^0\theta_3\theta_2}{\mu_b\theta_4}}. \tag{27}$$

Table 3: Maize production in Nigeria from 2015–2025 [19].

Year	Area (Ha)	Production (Tonne)	Yield (T/Ha)
2015	6,771	10,562	1.56
2016	7,312	11,548	1.58
2017	6,389	10,632	1.66
2018	6,541	10,934	1.67
2019	6,543	12,599	1.93
2020	6,049	12,400	2.05
2021	6,205	12,745	2.05
2022	5,800	12,949	2.23
2023	5,700	11,053	1.94
2024	5,500	11,200	2.04
2025	5,800	12,000	2.07

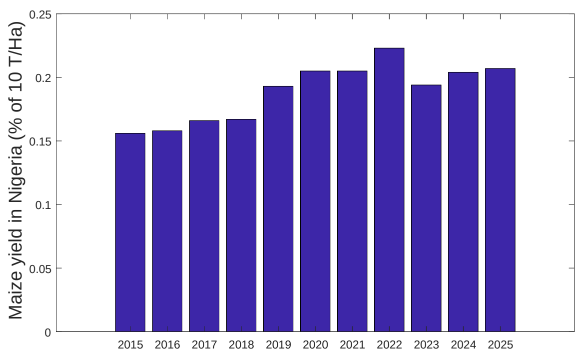


Figure 1: Bar chart illustrating the proportion of maize yield in Nigeria (% of 10 Tonne/Ha) from 2015 to 2025 [19].

This shows that the basic reproduction number of the co-infection MSV and MSpV model (7) is greater than the basic reproduction number of the MSV sub-model (8) and MSpV sub-model (20). Since an increase in the basic reproduction number implies an increase in the severity of the infection. This supports the findings that co-infection by MSV and MSpV leads to greater severity of infections and greater reductions in maize yield than single infection by either virus [4].

3.4.3. Stability analysis of the co-infection MSV and MSpV model (7)

Theorem 5. The co-infection MSV and MSpV model (7) is not globally asymptotically stable about the disease free equilibrium point (25).

The proof of Theorem 4 will be established using Lemma 1.

Proof. The proof of Theorem 5, it suffices to show that at least one of the conditions of the global stability result in Castillo-Chavez et al. [18], which is stated in Lemma 1, does not hold. Particularly, we show that condition H2 of the Lemma 1 is not satisfied by the co-infection MSV and MSpV model (7). From co-infection MSV and MSpV model (7), let $Z_1 = (s(\tau), x_a(\tau), x_b(\tau))$, $Z_2 = (i_a(\tau), i_b(\tau), i_{ab}(\tau), y_a(\tau), y_b(\tau))$. So, disease disease-free part of the co-infection MSV and MSpV model (7) is given by:

$$\frac{dZ_1}{dt} = F(Z_1, 0) = \begin{pmatrix} \phi_1 s(1-s) \\ \Lambda_a - \phi_4 x_a \\ \Lambda_b - \theta_4 x_b \end{pmatrix}, \tag{28}$$

whereas the disease part of co-infection MSV and MSpV model (7) is

$$\frac{dZ_2}{dt} = G(Z_1, Z_2) = \begin{pmatrix} \phi_2 y_a s - \xi_1 y_b i_a - \mu_a i_a \\ \theta_2 y_b s - \xi_2 y_a i_b - \mu_b i_b \\ \xi_1 y_b i_a + \xi_2 y_a i_b - \mu_{ab} i_{ab} \\ \phi_3 i_a x_a - \phi_4 y_a \\ \theta_3 i_b x_b - \theta_4 y_b \end{pmatrix}. \tag{29}$$

The Jacobian of the disease part (29) about the disease free equilibrium is

$$A = \begin{pmatrix} -\mu_a & 0 & 0 & \phi_2 & 0 \\ 0 & -\mu_b & 0 & 0 & \theta_2 \\ 0 & 0 & -\mu_{ab} & 0 & 0 \\ x_a^0\phi_3 & 0 & 0 & -\phi_4 & 0 \\ 0 & x_b^0\theta_3 & 0 & 0 & -\theta_4 \end{pmatrix}, \tag{30}$$

where A is an M–matrix with all its off diagonal elements non-negatives. From equations (29) and (30), the term $\hat{G}(Z_1, Z_2)$ is determined as

$$\hat{G}(Z_1, Z_2) = \begin{pmatrix} \xi_1 y_b i_a + \phi_2 y_a(1-s) \\ \xi_2 y_a i_b + \theta_2 y_b(1-s) \\ -\xi_1 y_b i_a - \xi_2 y_a i_b \\ \phi_3 i_a(x_a^0 - x_a) \\ \theta_3 i_b(x_b^0 - x_b) \end{pmatrix}. \tag{31}$$

Clearly, $\hat{G}(Z_1, Z_2)$ is not non-negative. This shows that condition (H_2) does not hold and hence completes the proof. Therefore, the co-infection MSV and MSpV model (7) is not globally asymptotically stable about the disease-free equilibrium point (25). □

The implication of these results is that it will be difficult to eradicate both MSV and MSpV co-infection simultaneously from the system by lowering the basic reproduction number below unity. This could explain why co-infection of MSV and MSpV is more severe and leads to greater yield loss. In the next section, we shall examine the impact of control measures on maize yield for a situation where the maize plants are exposed to MSV and MSpV co-infection.

4. Numerical simulations

In this section, numerical simulations of the co-infection MSV and MSpV model (7) are presented using Nigeria as a case study. Nigeria is considered as a case study due to its significant challenges with maize yield, which is currently about 2 Tonne/Ha. The simulation will involve both disease co-infection MSV and MSpV model (7) together with the model that estimates the maize yield, which is captured in equation (6). These two equations illustrate the connection between the disease co-infection and maize yield in this study. The data on

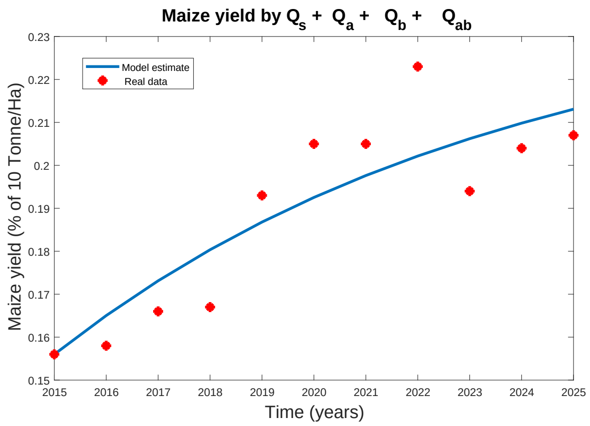


Figure 2: Model fit of the proportion of maize yield in Nigeria (% of 10 Tonne/Ha) from 2015 to 2025, where bold lines represent the model fit and stars mark the actual data.

maize yield (Tonne/Ha) in Nigeria from 2015–2025 extracted from the International Production Assessment Division (IPAD), Foreign Agricultural Service, US Department of Agriculture, presented in Table 3 [19]. Insights gained from this study can inform broader global strategies aimed at combating food insecurity.

From Table 3, it can be seen that the total area planted with maize in Nigeria from 2015 to 2025, according to [19], is decreasing. A possible explanation for this decrease could be due to urbanisation. Thus, the need to increase maize yield by maximising the limited area available for maize plants becomes essential.

4.1. Parameter estimation and model fitting

A study by Doyle & Autrey [4] revealed that at the early stage (3–5 leaf stage), maize plants singly infected with MSV, MSpV, or co-infected with MSV and MSpV have (91%, 100% and 100% reduction, respectively). However, at the later stage (7–10 leaf stage), maize plants infected either with MSV, MSpV, or co-infected with MSV and MSpV have (65%, 80% and 100% reduction, respectively) [4]. For the purpose of this study, 65%, 80% and 95% reduction, respectively, will be considered for maize plants singly infected with MSV, MSpV, or co-infected with MSV and MSpV, respectively. Based on this explanation, the maize yield Q_a, Q_b, Q_{ab} can be estimated as $35\%, 20\%, 5\%$ respectively of the expected maize yield. Note that the expected maize yield is the yield by the susceptible S (healthy) maize plants.

The maximum maize yield per hectare can vary greatly depending on many factors, but with optimal conditions and practices, yields of 10 tons per hectare or more are achievable [23]. Based on these, the carrying capacity is taken as $K = 10$ tons per hectare in this study. Since model 7 is dimensionless with variables as proportion, the data on maize yield from Nigeria need to be converted to proportions by dividing them by 10

Table 4: Parameters values used in the simulation

Parameters	Value	Reference
β_a	0.4500	[8, 20]
β_b	0.4500	[8, 20]
β_{ab}	0.4500	[8, 20]
β_{ab}	0.4500	[8, 20]
μ_a	0.0192	Estimated from model fitting
μ_b	0.0192	Estimated from model fitting
μ_{ab}	0.0836	Estimated from model fitting
ρ	0.1325	Estimated from model fitting
Λ_a	0.0200	[8, 21]
Λ_b	0.0200	[8, 21]
α_a	0.0400	[8, 20]
α_b	0.0400	[8, 20]
δ_a	0.0303	[21, 22]
δ_b	0.0303	[21, 22]
γ_a	0.0012	Estimated from model fitting
γ_b	0.0012	Estimated from model fitting
c_a	0.0029	Estimated from model fitting
c_b	0.0029	Estimated from model fitting
c_{ab}	0.0148	Estimated from model fitting
c_{ba}	0.0150	Estimated from model fitting

Tonne/Ha. Based on this, the proportion of maize yield in Nigeria (% of 10 Tonne/Ha) from 2015 to 2025 extracted from [19] is given in Figure 1. The figure demonstrated a very poor maize yield (Tonne/Ha) that is slowly increasing. This shows there is a need for urgent intervention that can improve yield. This study is an attempt to use a mathematical model to illustrate theoretically how to improve maize yield.

All the parameters used in this study (see Table 4) are extracted from the literature or estimated from actual data of maize yield in Nigeria (Tonne/Ha) from 2015 to 2025 [19]. Those parameters extracted from the literature have references showing where they were extracted. For those estimated from real data, the details of how they were estimated from model fitting are presented below.

Model fitting and parameter estimation are carried out in this section using actual data on maize yield in Nigeria (Tonne/Ha) from 2015 to 2025 [19]. The model fitting is carried out using the least squares fitting algorithm. The least squares fitting algorithm is considered because it provides a robust method for finding the best fit to a set of data points, minimizing the sum of squared differences between observed and predicted values. Some of the parameters in the co-infection MSV and MSpV model (7) are obtained from the literature, while the remaining parameters are estimated using the fitting algorithm and are presented in Table 4. Results of the model fit given in Figure 2 show that model (7) is a good fit for the maize yield in Nigeria (Tonne/Ha) from 2015 to 2025. Hence, the model is used further for predictions of maize yield trends in Nigeria.

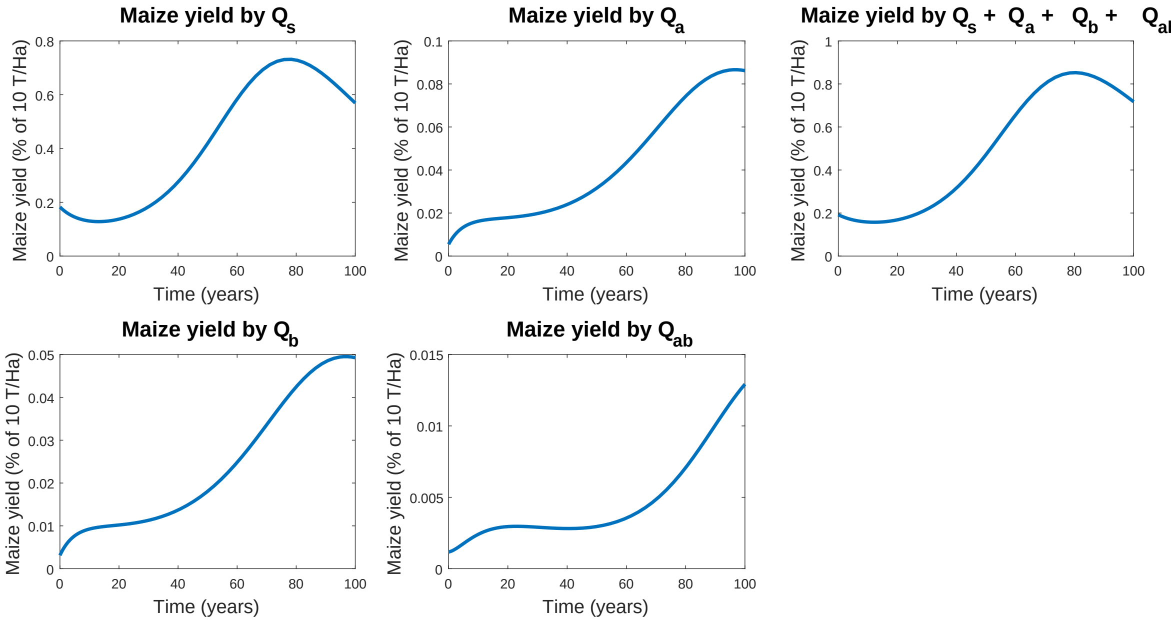


Figure 3: Plot showing a possible long-term dynamics of the proportion of maize yield in Nigeria (% of 10 Tonne/Ha).

4.2. Model predictions

Mathematical models can predict the potential severity and spread of disease outbreaks, allowing for timely interventions and resource allocation. By incorporating real-time data on factors influencing the disease, models can serve as early warning systems, alerting farmers and policy-makers to potential disease risks. A possible long-term prediction of maize yield in Nigeria using MSV and MSpV co-infection model 7 and parameter values from Table 4 is presented in Figure 3. Figure 3 reveals a very slow increase in maize yield in Nigeria. Despite running the simulation for a very long time of 100 years, the model predictions show that it will be difficult for maize yield in Nigeria to reach 10 Tonne/Ha, which is the target in this study, unless a very effective intervention is implemented. From the Figure, we noticed that the total maize yield increased continuously from about 2 Tonne/Ha to a peak of about 8 Tonne/Ha before it start decreasing. The period when the total maize yield was increasing was as a result of the susceptible (healthy) maize, which account for about 90% of the total yield. However, when infected maize increases significantly, the total maize yield begins to decrease as a result of the decrease in healthy maize. This reveals the impact of infected maize on reducing yield. Thus, implementation of effective control intervention that will enhance maize yield in Nigeria to about 10 Tonne/Ha is urgently needed to achieve food security. In the next section, the impact of various control intervention measures is explored and showing how they can be implemented to enhance maize yield. The results can serve as a guide for both farmers and policymakers for improving the management of maize farming towards enhancing yield and achieving food security.

4.3. Effects of control measures on model dynamics

Modern technologies are increasingly being adopted for maize farming to boost productivity and resilience. Studies show a positive correlation between the adoption of modern technologies and increased maize yields [24]. Here, the effects of adoption of modern technologies (ϕ) on the maize yield dynamics using the estimated parameter values is presented in Figure 4. The figure shows that an increase in the adoption of modern technologies leads to an increase in total maize yield. Specifically, the figure revealed that effective adoption of modern technology can increase maize yield in Nigeria beyond 10 Tonne/Ha within a very short period. This result agrees with other similar studies in the literature [24]. Despite the valuable advantages of the adoption of modern technologies, factors like access to information, affordability of inputs, and farmer awareness can affect the adoption rate [24, 25]. Thus, it is strongly recommended that the government should support the farmers' adoption of modern technologies by providing the following: strengthening extension services, promoting affordable inputs, encouraging mechanization and supporting research and development [24, 25].

Use of varieties resistant to MSV is one of the methods of controlling MSV, and it offers a sustainable, cost-effective, and environmentally friendly approach to maize farming [2]. Apart from the economic impact on maize yield, other valuable advantage of MSV resistant maize varieties include: reduced disease incidence, reduced reliance on pesticides, sustainable and cost-effective [5]. Theoretically, it is shown in Figure 5(a) that the use of resistant varieties for MSV only (c_a) increases maize yield. Specifically, effective use of resistant varieties for MSV only can increase maize yield in Nigeria to about 4 Tonne/Ha. Similarly, Figure 5(b) shows that the use of resistant varieties

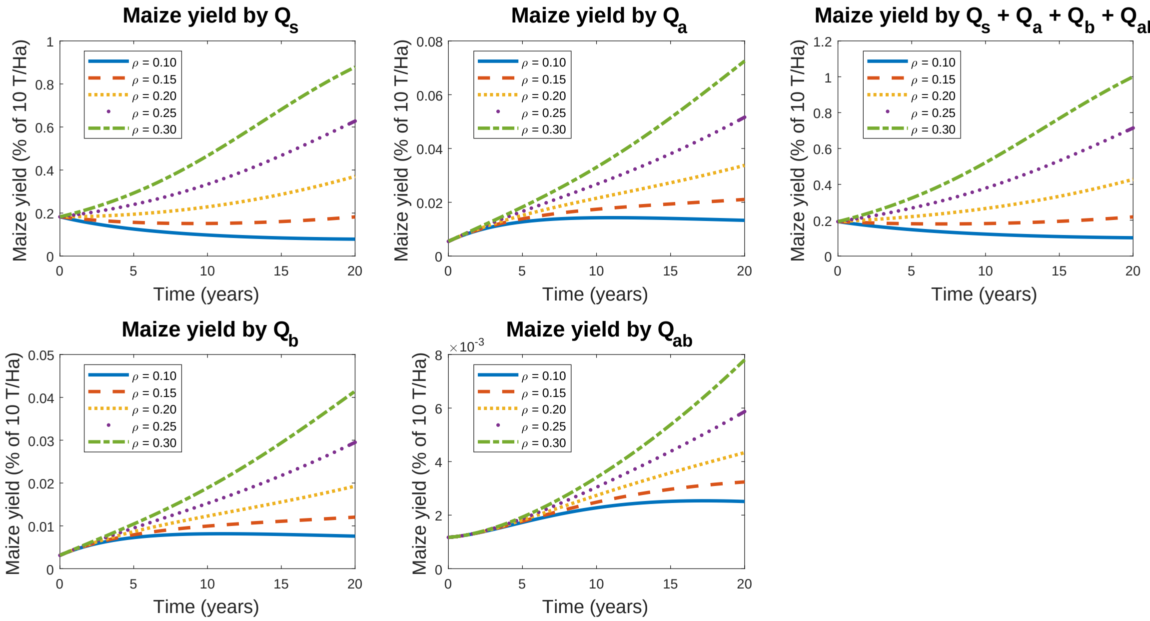


Figure 4: Plot showing the effects of use of modern technologies (ϕ) on the maize yield dynamics.

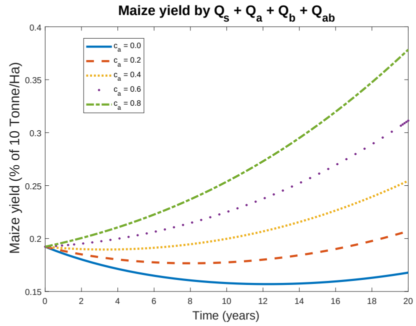
for MSpV only (c_b) increases maize yield. Specifically, effective use of resistant varieties for MSpV only can increase maize yield in Nigeria to about 4 Tonne/Ha. However, Figure 5(c) shows that the use of resistant varieties for co-infection MSV and MSpV only (c_{ab}, c_{ba}) increases maize yield. Specifically, effective use of resistant varieties for co-infection of MSV and MSpV can only increase maize yield in Nigeria to about 3 Tonne/Ha. These results show that implementing the use of resistant varieties only without other control intervention cannot lead to significant maize yield. Hence, we recommend combining the use of resistant varieties with other control interventions to maximize yield.

Controlling leafhopper vectors is one of the methods for mitigating the impact of MSV, MSpV on maize farming [5]. Controlling Leafhopper Vectors can be done through the following methods: chemical control (use of approved fungicide), host plant resistance in maize varieties, cultural practices, and integrated disease management. Benefits of controlling leafhopper vectors include: reduced yield losses, reduced input costs, sustainable farming practices and ultimately improved food security. Theoretically, the impact of controlling vectors (γ_a, γ_b) on the maize yield is presented in Figure 6. The Figure shows that controlling vectors has a significant impact on maize yield. Specifically, the figure revealed that controlling vectors effectively can lead to a maize yield of about 10 Tonne/Ha in Nigeria. Thus, effective control of vectors is strongly recommended to enhance yield and achieve food security.

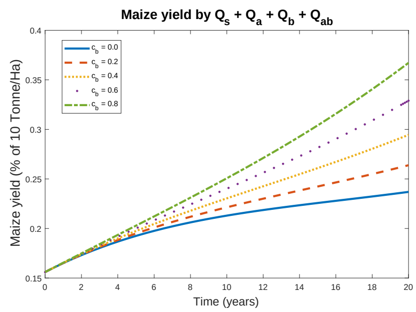
5. Conclusion

Maize is a critical crop for global food security, livelihoods, and economic development [3]. Despite the valuable advantages of maize, several challenges influence the optimal maize production. In this study, the challenges that influence the optimal maize production are considered to develop a mathematical model that can be used to demonstrate how to improve maize yield using Nigeria as a case study. Nigeria is chosen as a case study due to its significant challenges with maize yield (about 2 Tonne/Ha), food insecurity, and disease, representative of many Sub-Saharan African countries. Insights gained from this study can inform broader global strategies aimed at combating food insecurity.

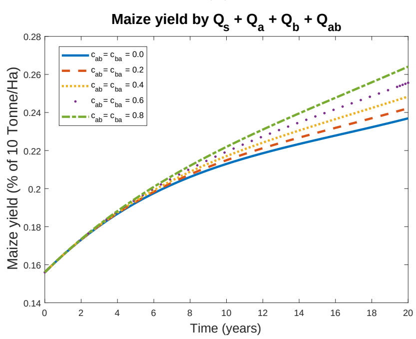
A mathematical epidemiological model for maize yield in the form of a differential equation is developed. The model considered the co-infection of maize streak virus (MSV) and maize stripe virus (MSpV), which are two major diseases affecting maize yield. The model also considered other control interventions such as the adoption of modern technologies, use of approved chemical control, use of resistant varieties, etc., which aimed at improving maize yield. The analysis of the MSV sub-model shows that it is possible to eradicate the maize streak virus if it is the only disease affecting the farm, provided the associated basic reproduction number ($\mathcal{R}_0^a$) is kept below unity. This was done by showing that the model is globally asymptotically stable about the disease-free equilibrium when $\mathcal{R}_0^a < 1$. Similarly, the analysis of the sub-model with only MSpV shows that it is possible to eradicate the maize stripe virus if it is the only disease affecting the system, provided the associated basic reproduction number ($\mathcal{R}_0^b$) is kept below unity. Unfortunately, the situation where both diseases, MSV and MSpV, co-infected the system was shown to be impossible to eradicate when the associated basic reproduction number is below unity. This supports other findings in the literature that reported that greater disease severity is due to co-infection of MSV and MSpV, which is more difficult to control [4].



(a)



(b)



(c)

Figure 5: Plot showing the effects of use of resistant varieties for: (a) MSV (c_a) (b) MSpV (c_b) and (c) Co-infection of MSV and MSpV (c_{ab}, c_{ba}) on the maize yield dynamics.

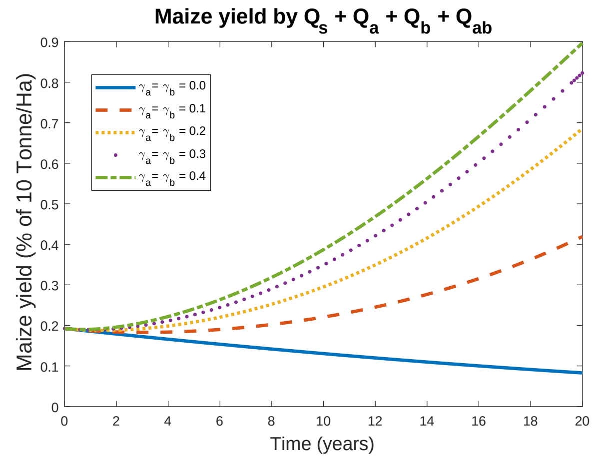


Figure 6: Plot showing the effects of controlling vectors (γ_a, γ_b) on the maize yield dynamics.

Numerical simulations are used further to analyse the model using real data on maize yield from Nigeria as a case study. The data on maize yield (Tonne/Ha) in Nigeria from 2015 to 2025 was extracted from the International Production Assessment Division (IPAD), Foreign Agricultural Service, US Department of Agriculture (2025) [19]. According to the data, the maize yield in Nigeria is about 2 Tonne/Ha, which is very small compared with the 10 Tonne/Ha yield that is achievable with improved modern farming practices [23]. To use Nigeria as a case study, the data on maize yield in Nigeria were fitted to our epidemiological model (7). Results of the model fit show that model (7) is a good fit for the maize yield in Nigeria (Tonne/Ha) from 2015 to 2025. Hence, the model is used further for predictions of maize yield trends in Nigeria. The results of the model prediction reveal a very slow increase in maize yield in Nigeria. Despite running the simulation for an extended time of 100 years, the model predictions show that it will be difficult for maize yield in Nigeria to reach 10 Tonne/Ha, which was our target in this study, unless a very effective intervention is implemented. Next, the impact of various control intervention measures is explored to show how they can be effectively implemented to enhance maize yield.

The effects of the adoption of modern technologies (ϕ) on the maize yield show that an increase in the adoption of modern technologies leads to a significant increase in maize yield. Specifically, the results revealed that effective adoption of modern technology can increase maize yield in Nigeria beyond 10 Tonne/Ha within a very short period. This result agrees with other similar studies in the literature [24]. Despite the valuable advantages of the adoption of modern technologies, factors like access to information, affordability of inputs, and farmer awareness can affect the adoption rate [24, 25]. Thus, it is strongly recommended that the government should support the farmers' adoption of modern technologies by providing the following: strengthening extension services, promoting affordable inputs, encouraging mechanization, and supporting research and development [24, 25]. Effective use of resistant varieties for MSV or MSpV only can increase maize yield in Nigeria to about 4 Tonne/Ha. Whereas, effective use of resistant varieties for co-infection of MSV and MSpV only can increase maize yield in Nigeria to about 3 Tonne/Ha. These show that implementing the use of resistant varieties only without other control interventions cannot lead to significant maize yield. Hence, combining the use of resistant varieties with other control interventions is strongly recommended to maximize maize yield. Controlling leafhopper vectors effectively can lead to a maize yield of about 10 Tonne/Ha in Nigeria. Thus, effective control of vectors is strongly recommended to enhance yield and achieve food security. Overall, the recommendations based on the results of this study are expected to aid farmers and policymakers in adopting best farming practices that will improve maize yield and consequently assist in achieving food security globally.

Data availability

All data used in this work can be found within the manuscript.

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