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Weaker and deficiency of even vertex odd edge root square mean labeling graphs

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Abstract

This paper introduces and investigates a relaxed variant of even vertex odd edge root square mean labeling (EVOERSML), called weaker EVOERSML. For a graph G, a weaker EVOERSML is an injective labeling $f : V(G) \to \{0, 2, 4, \ldots, 2(q + k)\}$, where $k \in \mathbb{Z}^+$, such that all vertex labels are distinct non-negative even integers. The induced edge labels are obtained by applying either the floor or ceiling function to the square root of the average of the squares of the labels of the end vertices. This relaxed labeling framework extends the applicability of root square mean labelings to a broader class of graphs. The minimum labeling bound $K_{\min}(G)$ is defined as the least integer k for which G admits a weaker EVOERSML, and methods for determining this bound are discussed. The existence of weaker EVOERSML is established for several families of graphs, and their structural properties are analyzed. The deficiency associated with weaker EVOERSML is also examined, with particular emphasis on connected graphs of orders 3, 4, and 5, yielding complete classifications and illustrative examples.

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Keywords: Path corona, Cycles, Star graphs, Weaker EVOERSML, Deficiency.

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1. Introduction

In this study, we consider simple and undirected graphs. The terminology and notation adopted in this paper follow those in [1]. A graph that does not satisfy the EVOERSML condition requires an additional minimum number of positive integers to achieve it. This deficiency gives rise to the concept of a weaker EVOERSML graph. It is observed that any disconnected graph can be regarded as a weaker EVOERSML graph if each of its components is an EVOERSML graph. Therefore, the property of being a weaker EVOERSML naturally extends to disconnected graphs whose components individually satisfy the EVOERSML condition.

Various researchers have introduced several labeling schemes, including EVOERSML, Root Square Mean, OVEERSML, and Super Root Square Mean labelings [2–4]. The significance of this work lies in identifying the conditions under which graphs that do not admit an EVOERSML pattern can be transformed into one-either by reducing their size or by increasing the labels assigned to their vertices.

The notation of deficiency in graph labeling was initially introduced as a structural measure indicating the extent to which a graph deviates from admitting a specific labeling pattern. In particular, the concept was first formalized in the study of super edge-magic labelings [5], where the super edge-magic deficiency of a graph was defined as the smallest non-negative inte-

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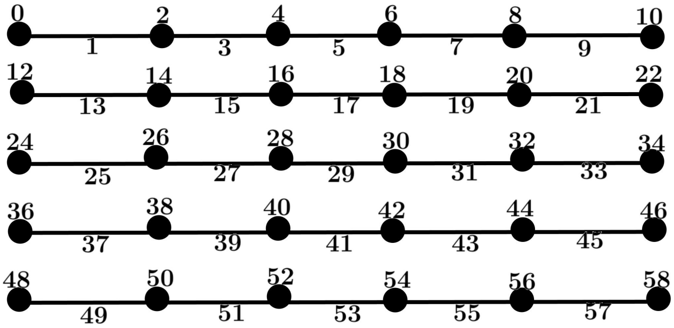

Figure 1. Weaker EVOERSML of $5P_6$.

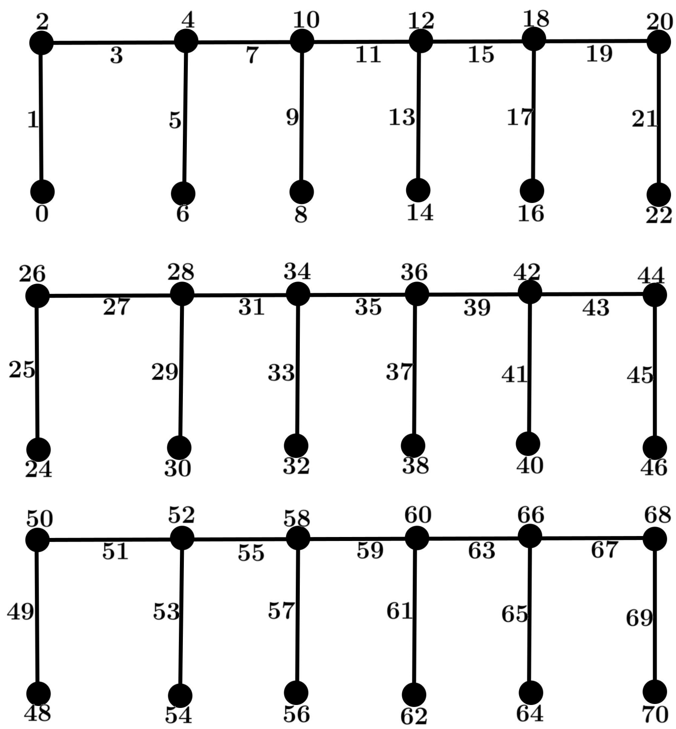

Figure 2. Weaker EVOERSML of $3(P_6 \odot K_1)$.

ger n such that the disjoint union $G \cup nK_1$ admits a super edge-magic labeling. This additive interpretation of deficiency subsequently became a standard framework across various labeling schemes, including edge-graceful, edge-antimagic, and cordial labelings. In accordance with this established framework, the present study extends the concept to the class of EVOERSML graphs, wherein the labeling deficiency quantifies the minimal augmentation or vertex-label shortfall required for achieving a complete labeling.

2. Preliminaries and definitions

In this section, we recall the basic terminology and introduce the definitions required for the subsequent results.

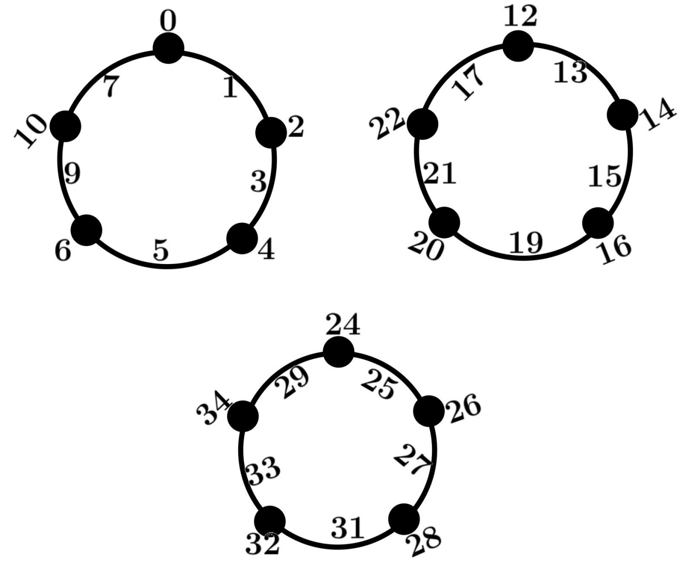

Figure 3. Weaker EVOERSML of $3C_5$.

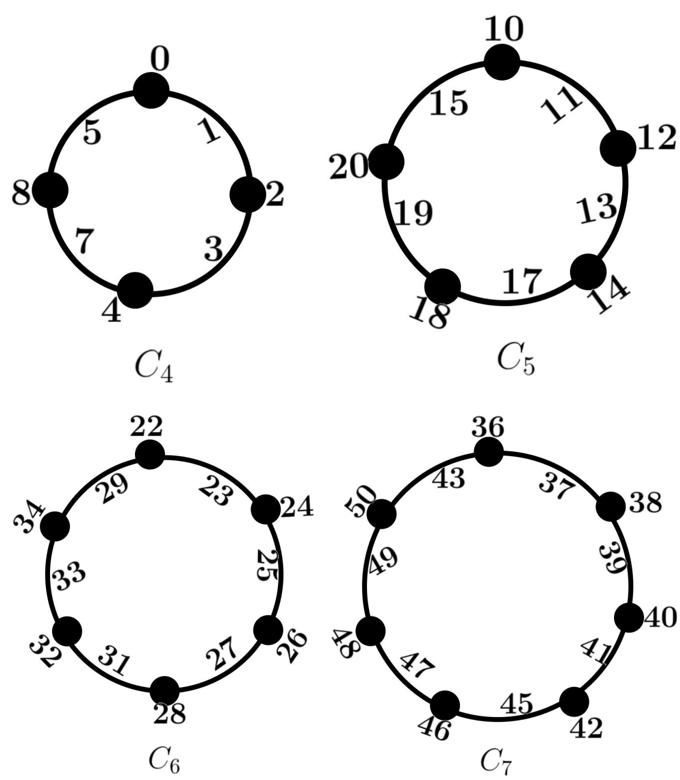

Figure 4. Weaker EVOERSML of $C_4 \cup C_5 \cup C_6 \cup C_7$.

2.1. Definition

According to Ref. [6], a function $f : V(G) \rightarrow \{0, 1, 2, \ldots, 2q\}$ is said to be an EVOERSML (Even Vertex Odd Edge Root Square Mean Labeling) if the edge labels, denoted by $f^*(e)$, are either the ceiling or floor value of the square root of the average of the squares of the labels of the end vertices of each edge, where all vertex labels are even numbers and all

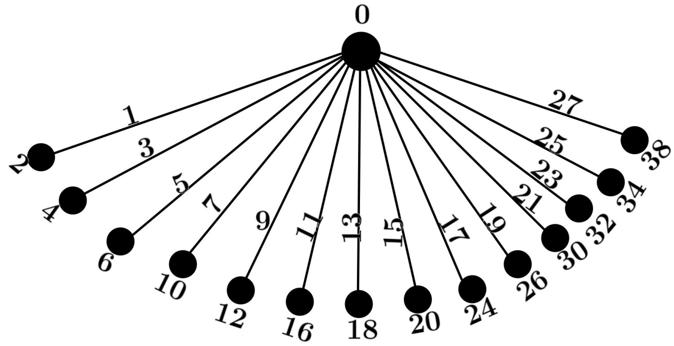

Figure 5. Weaker EVOERSML of $K_{1,14}$.

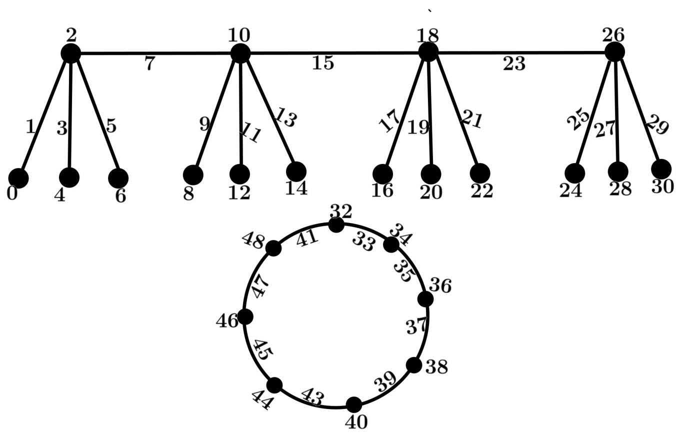

Figure 6. Weaker EVOERSML of $(P_4 \odot K_{1,3}) \cup C_8$.

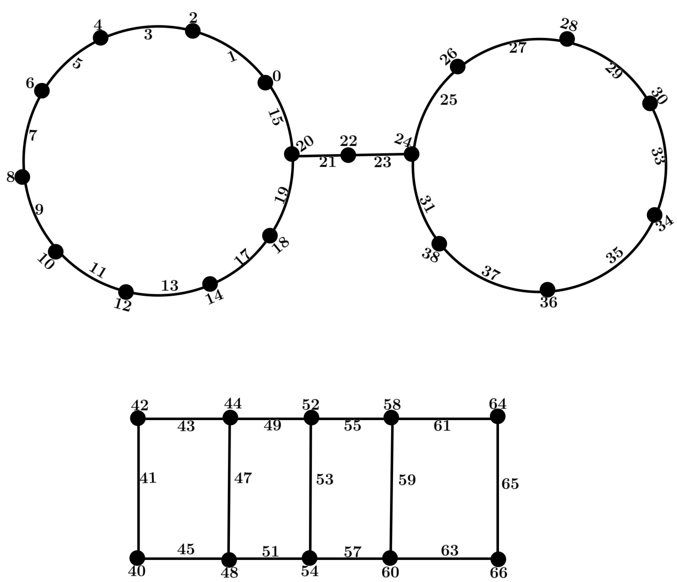

Figure 7. Weaker EVOERSML of $DB(C(10,7)) \cup L_5$.

edge labels are odd numbers, with all labels being distinct.

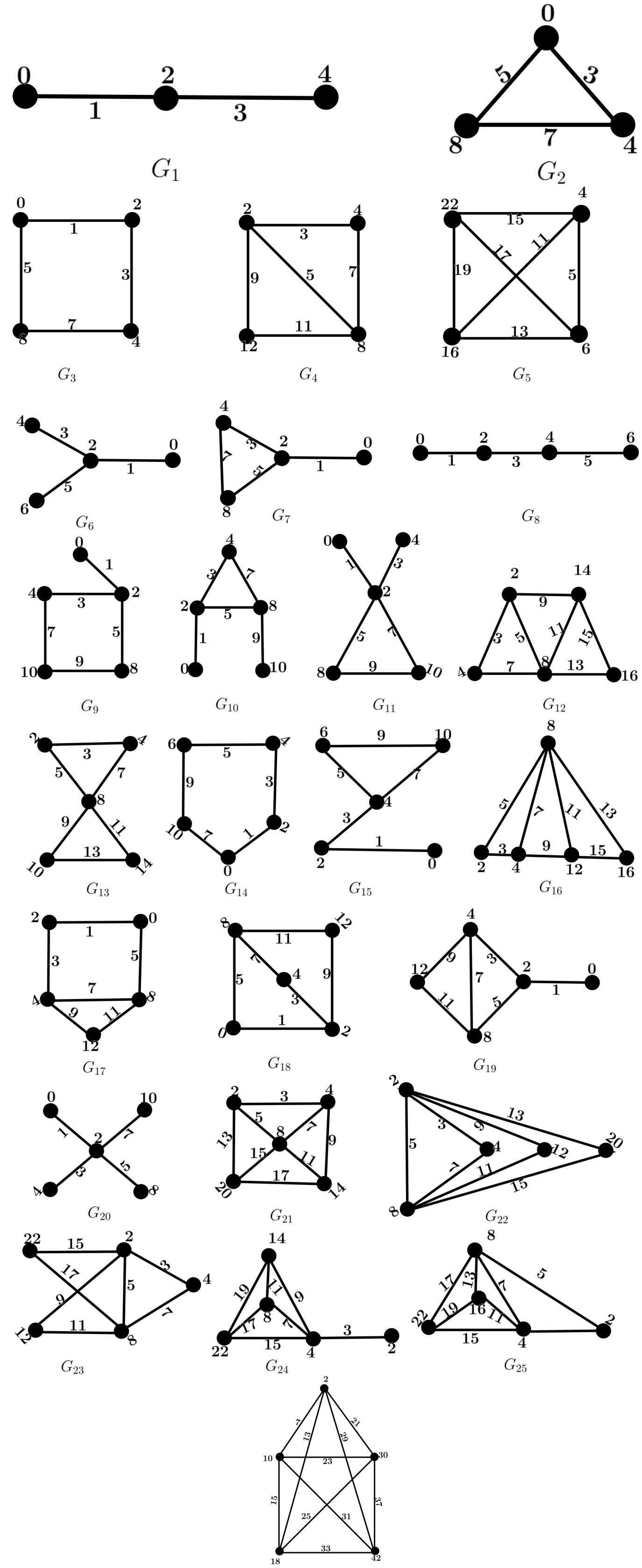

Figure 8. The collection of all connected graphs of order 3, 4 and 5.

Table 1. Results for various graphs of EVOERSML and weaker EVOERSML.

Graph	EVOERSML	Weaker EVOERSML	Number of Deficiency
G_1	Yes	–	0
G_2	–	Yes	1
G_3	Yes	–	0
G_4	–	Yes	1
G_5	–	Yes	5
G_6	Yes	–	0
G_7	Yes	–	0
G_8	Yes	–	0
G_9	Yes	–	0
G_{10}	Yes	–	0
G_{11}	Yes	–	0
G_{12}	–	Yes	1
G_{13}	–	Yes	1
G_{14}	Yes	–	0
G_{15}	Yes	–	0
G_{16}	–	Yes	1
G_{17}	Yes	–	0
G_{18}	Yes	–	0
G_{19}	Yes	–	0
G_{20}	–	Yes	1
G_{21}	–	Yes	2
G_{22}	–	Yes	3
G_{23}	–	Yes	4
G_{24}	–	Yes	4
G_{25}	–	Yes	3
G_{26}	–	Yes	11

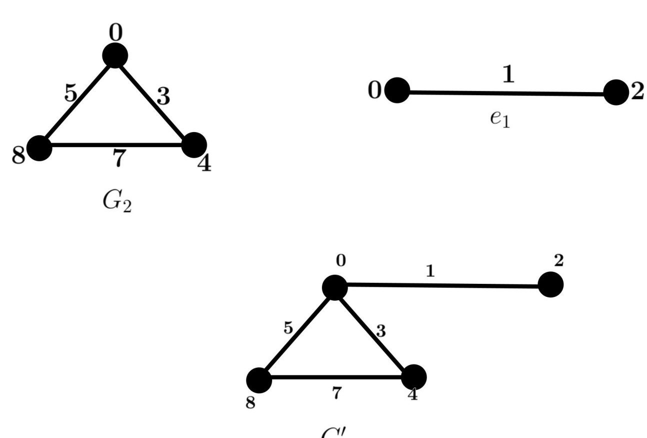

Figure 9. Transformation of weaker EVOERSML of G_2.

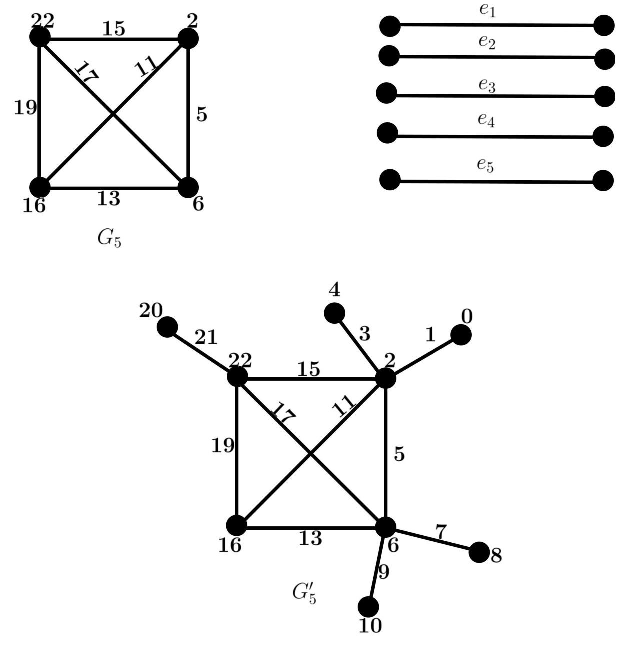

Figure 10. Transformation of weaker EVOERSML of G_5.

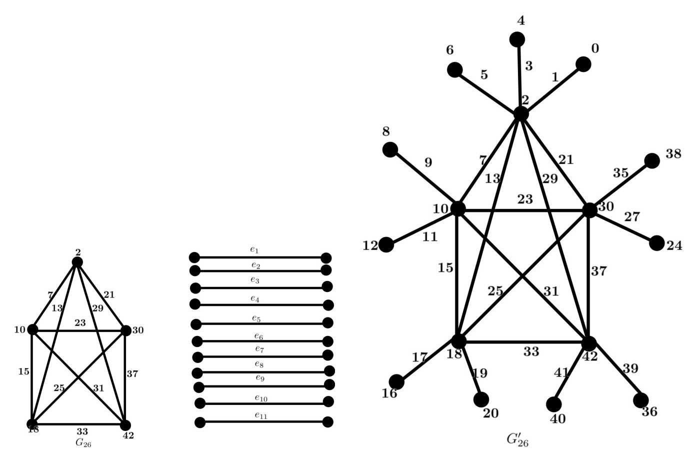

Figure 11. Transformation of weaker EVOERSML of G_{26}.

2.2. Definition

A map $f : V(G) \to \{0, 2, 4, \ldots, 2(q + k)\}$, $k \in \mathbb{Z}^+$, is said to be a weaker EVOERSML if the induced edge labeling satisfies the EVOERSML conditions for a minimal value of k.

2.3. Definition

According to Ref. [7], consider a graph G with vertex set $V(G) = \{v_i \mid 1 \leq i \leq n\}$. Fix a vertex, say v_1, of G. For each integer $k \geq 1$, consider $G_1, G_2, \ldots, G_{m^{k-1}}$, which are the m^{k-1} disjoint isomorphic copies of G, with $V(G_i) = \{v^i_j \mid 1 \leq i \leq m^{k-1},\ 1 \leq j \leq n\}$, and let v^i_j be the isomorphic copy of v_j in G_i. A new graph is generated by attaching the vertex $v^i_1(G_i)$ with x^{k-1}_i of $T(m, k)$, for all i. It is denoted by $G(v_1, m, k)$.

2.4. Definition

The notion of deficiency in graph labeling is employed to quantify the extent to which a graph fails to satisfy the conditions of a specific labeling scheme. Various researchers have proposed distinct definitions of deficiency, depending on the labeling framework under consideration. Broadly, two principal interpretations of this concept are prevalent in the literature.

1. Structural deficiency:
   For a given labeling L, the deficiency of a graph G, denoted by $\mu_L(G)$, is defined as the smallest non-negative integer n for which the disjoint union $G \cup nK_1$ admits the labeling L. Formally, $\mu_L(G) = \min\{n \geq 0 \mid G \cup nK_1 \text{ admits } L\}$.
2. Label-count deficiency:
   Another interpretation focuses on the shortfall in label usage rather than structural augmentation. For a partial or incomplete labeling $f : V(G) \to S$, where S is the admissible label set, the label-count deficiency $\delta(f)$ is defined as $\delta(f) = |S| - |f(V(G))|$.

3. Main results

3.1. Theorem:

The union of paths is a weaker EVOERSML.

Proof. Let mP_n be the disjoint union of m paths, and let this graph be G. The ordinary labeling of the elements of G is given below: $V(G) \cup E(G) = \{v_i^j \cup e_k^j : 1 \leq i \leq n,\ 1 \leq j \leq m,\ 1 \leq k \leq n-1\}$. A map $f : V(G) \to \{0, 2, 4, \ldots, 2(q+k)\}$ is defined by

For $1 \leq i \leq n$,

$$f(v_i^1) = \{0,\ 2,\ \ldots,\ 2(n-1)\}$$
$$f(v_i^2) = \{2n,\ 2n+2,\ \ldots,\ 4n-2\}$$
$$f(v_i^3) = \{4n,\ 4n+2,\ \ldots,\ 6n-2\}$$
$$\vdots$$
$$f(v_i^m) = \{2n(m-1),\ 2n(m-1)+2,\ \ldots,\ 2mn-2\}$$

For $1 \leq k \leq n-1$,

$$f(e_k^1) = \{1,\ 3,\ \ldots,\ 2n-3\}$$
$$f(e_k^2) = \{2n+1,\ 2n+3,\ \ldots,\ 4n-3\}$$
$$f(e_k^3) = \{4n+1,\ 4n+3,\ \ldots,\ 6n-3\}$$
$$\vdots$$
$$f(e_k^m) = \{2n(m-1)+1,\ 2n(m-1)+3,\ \ldots,\ 2mn-3\}.$$

The above labeling pattern shows that G is a weaker EVOERSML graph. Therefore, the minimum bound of k is $m-1$.

3.1.1. Illustration:

Weaker EVOERSML of $5P_6$ is presented in Figure 1.

3.2. Theorem:

The graph obtained by taking the union of path corona graphs is a weaker EVOERSML graph.

Proof. Let $G = P_n \odot K_1$. Define $V(G) = \{x_i^j, y_i^j \mid 1 \leq i \leq n,\ 1 \leq j \leq m\}$ and $E(G) = \{x_i^j y_i^j,\ x_k^j x_{k+1}^j \mid 1 \leq i \leq n,\ 1 \leq j \leq m,\ 1 \leq k \leq n-1\}$. Let $f : V(G) \to \{0, 2, 4, \ldots, 2(q+k)\}$ be a labeling function, where k is a positive integer. The vertices in each copy are labeled in the following order: $\{y_1^j, x_1^j, x_2^j, y_2^j, y_3^j, x_3^j, x_4^j, y_4^j, \ldots\}$. If the number of vertices in the path P_n is odd, then the vertices of the form x_i^j are assigned labels of the form $2q$. If n is even, the vertices of the form y_i^j are assigned labels of the form $2q$.

Accordingly, in the first copy, the vertex labels are $\{0, 2, 4, 6, \ldots, 2q\}$ and the corresponding edge labels are $\{1, 3, 5, 7, \ldots, (2q-1)\}$.

In the second copy, the first pendant vertex is assigned the label $2q+2$. The remaining vertices in that copy follow the same alternating pattern as above, with vertex labels $\{(2q+4), (2q+6), \ldots, (4q+2)\}$, and corresponding edge labels $\{(2q+3), (2q+5), \ldots, (4q+1)\}$.

This labeling process continues analogously for the remaining copies. In the last copy (i.e., the m^{th} copy), the first pendant vertex receives the label $2(m-1)(q+1)$. The remaining vertices in that copy are labeled as $\{2(m-1)(q+1)+2,\ 2(m-1)(q+1)+4,\ \ldots,\ 2(m(q+1)-1)\}$, and the corresponding edge labels are $\{2(m-1)(q+1)+1,\ 2(m-1)(q+1)+3,\ \ldots,\ 2m(q+1)-3\}$.

Hence, the minimum bound of k is $m-1$. Therefore, G is a weaker EVOERSML graph.

3.2.1. Illustration:

Figure 2 illustrates the weaker EVOERSML of $3(P_6 \odot K_1)$.

3.3. Theorem:

The union of cycles is a weaker EVOERSML.

Proof. Let $V(G) = \{u_i^j \mid 1 \leq i \leq n,\ 1 \leq j \leq m\}$ and $E(G) = \{u_i^j u_{i+1}^j,\ u_1^j u_{i+1}^j \mid 1 \leq i \leq n-1,\ 1 \leq j \leq m\}$. Define a labeling function $f : V(G) \to \{0, 2, 4, \ldots, 2(q+k)\}$, where $k > 0$.

Case 1: Cycles of equal size.

Let

$$a_i = \sqrt{\frac{f(u_1^i)^2 + f(u_n^i)^2}{2}},$$

where each a_i is an odd integer.

For the first copy of the cycle, the vertex labels are assigned as follows: $\{0, 2, 4, 6, \ldots, (a_1-1),\ (a_1+3),\ (a_1+5), \ldots, 2q\}$.

For the second copy, the vertex labels are:
$\{(2q+2),\ (2q+4), \ldots, (a_2-1),\ (a_2+3),\ (a_2+5), \ldots, (4q+2)\}$.

For the m^{th} copy, the vertex labels are given by:
$\{2(m-1)(q+1),\ 2(m-1)(q+1)+2,\ 2(m-1)(q+1)+4,\ \ldots,\ a_m-1,\ a_m+3,\ a_m+5,\ \ldots,\ 2[m(q+1)-1]\}$. Clearly, the edge labels are distinct. Hence, the minimum bound of k is $m-1$.

Case 2: Cycles of unequal size.

Let

$$a_i = \sqrt{\frac{f(u_1^i)^2 + f(u_n^i)^2}{2}},$$

where each a_i is an odd integer.

For the first copy of the cycle, the vertex labels are: $\{0, 2, 4, 6, \ldots, a_1-1,\ a_1+3,\ a_1+5, \ldots, 2q_1\}$.

For the second copy, the vertex labels are: $\{(2q_1+2),\ (2q_1+4), \ldots,\ (a_2-1),\ (a_2+3),\ (a_2+5), \ldots,\ (2[q_1+q_2]+2)\}$.

For the m^{th} copy, the vertex labels are assigned as: $2\sum_{i=1}^{m-1} q_i + 2(m-1),\ 2\sum_{i=1}^{m-1} q_i + 2(m-1)+2, \ldots,\ 2\sum_{i=1}^{m} q_i + 2(m-1)$. Clearly, the edge labels are distinct. Therefore, the minimum bound of k is $m-1$.

3.3.1. Illustration:

$3C_5$ is a weaker EVOERSML graph as given in Figure 3 .

3.3.2. Illustration:

Figure 4 shows that the union of four vertex-disjoint cycles of orders 4, 5, 6, and 7, respectively, and admits a weaker EVOERSML.

3.4. Theorem:

A star graph is a weaker EVOERSML graph precisely when it does not satisfy the EVOERSML condition.

Proof. Suppose that G is not an EVOERSML graph. Let $f : V(G) \to \{0, 2, 4, \ldots, 2(q+k)\}$, where k denotes the minimum positive integer. Without loss of generality, assume $f(u) = 0$, where u is the apex vertex, and let $f(u_n) = 2(q + k)$. The remaining vertices can then be labeled using even integers.

If each edge label is distinct and odd, and the vertex labels are restricted to $\{0, 2, 4, \ldots, 2q\}$, then G satisfies the conditions of an EVOERSML graph. Otherwise, additional vertex labels are required, and such a graph is termed a weaker EVOERSML graph. Our objective is to determine the approximate value of k. For instance, let $f(uu_n) = 2q - 1$. Then,

$$2q - 1 = \sqrt{\frac{(2(q + k))^2}{2}}.$$

This implies, $k = \frac{2q-1}{\sqrt{2}} - q$. By using the derived value of k, the theorem can be illustrated as follows.

The graphs $K_{1,1}$, $K_{1,2}$, and $K_{1,3}$ are EVOERSML graphs since $k = 0$. In $K_{1,4}$, the vertex labels are $\{0, 2, 4, 6, 8\}$. However, edge repetition occurs between $\{0, 6\}$ and $\{0, 8\}$, requiring the addition of the vertex label 10 to satisfy the EVOERSML condition. Hence, $K_{1,4}$ is a weaker EVOERSML graph with at least one additional vertex label.

Similarly, for $K_{1,5}$ with vertex labels $\{0, 2, 4, 6, 8, 10\}$, edge repetition occurs between $\{0, 6\}$ and $\{0, 8\}$, necessitating the vertex label 12. Thus, $K_{1,5}$ is a weaker EVOERSML graph with $k = 1$.

For $K_{1,6}$, with vertex labels $\{0, 2, 4, 6, 8, 10, 12\}$, edge repetition again occurs between $\{0, 6\}$ and $\{0, 8\}$, requiring exactly one vertex label 14. Therefore, $K_{1,6}$ is a weaker EVOERSML graph with $k = 1$. Hence, the value of k can be determined similarly for any star graph.

3.4.1. Illustration:

The star graph $K_{1,14}$ is a weaker EVOERSML, and its minimum bound k is 5, as shown in Figure 5.

4. General results

4.1. Theorem:

If G_1 and G_2 are EVOERSML graphs, their union forms a weaker EVOERSML graph.

Proof. Assume that $f : V(G_1) \cup E(G_1) \to \{0, 1, 2, \ldots, 2q_1\}$, where q_1 denotes the size of the graph G_1, and $g : V(G_2) \cup E(G_2) \to \{0, 1, 2, \ldots, 2q_2\}$, where q_2 denotes the size of the graph G_2, are EVOERSML functions. We aim to prove that $G_1 \cup G_2$ is a weaker EVOERSML graph. Define a combined mapping $f \circ g : V(G_1 \cup G_2) \cup E(G_1 \cup G_2) \to \{0, 1, 2, \ldots, 2(q_1 + q_2 + k)\}$, where k is a positive integer.

The vertex labeling of G_1 and G_2 is assigned as follows: $f(v_1) = 0$, $f(v_2) = 2$, $\ldots$, $f(v_n) = 2q_1$, and $g(v_1) = 2q_1 + 2$, $g(v_2) = 2q_1 + 4$, $\ldots$, $g(v_n) = 2(q_1 + q_2 + 1)$.

From this construction, it follows that $f \circ g$ defines a weaker EVOERSML for $G_1 \cup G_2$. The minimum bound of k is 1. Hence, $G_1 \cup G_2$ is a weaker EVOERSML graph.

4.1.1. Illustration:

The graph $(P_4 \odot K_{1,3}) \cup C_8$ is a weaker EVOERSML graph as shown in Figure 6.

4.1.2. Illustration:

The $DB(C(10, 7)) \cup L_5$ is a weaker EVOERSML graph as shown in Figure 7.

4.2. Theorem:

The cycle graph is a weaker EVOERSML only for $n = 3$.

Proof. Consider the cycle graph C_3 with vertex set $V(C_3) = \{v_1, v_2, v_3\}$. Let $f : V(C_3) \to \{0, 2, 4, 6\}$ be a labeling function assigning distinct even integers to the vertices. Assign labels as follows: $f(v_1) = 0$, $f(v_2) = 2$, $f(v_3) = 4$. Then the induced edge labels are $f(v_1v_2) = 1$, $f(v_2v_3) = 3$, $f(v_3v_1) = 3$. Here, all edge labels are not distinct under the EVOERSML conditions, since repetitions may occur depending on the operation used. Similarly, if we assign $f(v_1) = 0$, $f(v_2) = 2$, $f(v_3) = 6$, then the edge labels become $f(v_1v_2) = 1$, $f(v_2v_3) = 5$, $f(v_3v_1) = 5$, which again leads to repetition or violation of the required labeling conditions. To avoid this issue, extend the labeling set to include an additional even integer. Therefore, $f : V(C_3) \to \{0, 2, 4, 6, 8\}$. Now assign $f(v_1) = 0$, $f(v_2) = 4$, $f(v_3) = 8$. Then the induced edge labels are all distinct and satisfy the weaker EVOERSML conditions. Hence, C_3 admits a weaker EVOERSML. Therefore, the minimum bound of k is 1.

4.3. Theorem:

Let G be a connected EVOERSML graph with a labeling function $f : V(G) \to \{0, 2, 4, \ldots, 2(q+k)\}$ such that every vertex label is distinct and each edge label is obtained by the ceiling or floor of the root square mean of its end vertex labels. If there exists at least one edge $e \in E(G)$ whose removal disconnects G and reduces the vertex labeling deficiency below the admissible bound of $2q$, then the resulting graph $G - e$ is a weaker EVOERSML graph.

Proof. When an edge e is deleted from G, the graph becomes disconnected, resulting in a reduction in the number of vertex and edge pairs available for root square mean labeling. The injective property of f is preserved, but the labeling fails to satisfy the complete EVOERSML condition due to an insufficient number of valid label pairs (less than $2q$). Hence, $G - e$ satisfies the weaker EVOERSML criteria.

4.4. Lemma:

Let G be a graph that admits an EVOERSML and let $f : V \to \{0, 2, 4, \ldots, 2(q + k)\}$ for some $k \in \mathbb{Z}^+$. Then for every edge $e \in E(G)$, the graph $G - e$ admits a weaker EVOERSML (with the same k). In particular, the restriction of f to $V(G-e)$ is a valid weaker EVOERSML of $G - e$. Consequently, $K_{\min}(G - e) \leq K_{\min}(G)$.

Proof. Let f be an EVOERSML of G. By definition, f is injective on $V(G)$ with values in $\{0, 2, 4, \ldots, 2(q+k)\}$, and for each $uv \in E(G)$, the induced label is defined in the EVOERSML.

Fix $e_0 \in E(G)$ and form $G' = G - e_0$. The vertex set of G' is still $V(G)$, and the same map f, viewed on $V(G')$, remains injective with values in $\{0, 2, 4, \ldots, 2(q + k)\}$.

For every edge $uv \in E(G')$, the induced label computed from $f(u), f(v)$ is unchanged from the original graph and hence remains odd and distinct among the remaining edges. Therefore, f restricted to G' satisfies the EVOERSML requirements (possibly with fewer edges), which proves the claim.

4.5. Theorem:

All connected graphs of orders 3, 4, and 5 admit either EVOERSML or weaker EVOERSML. The corresponding vertex and edge labelings are illustrated below.

Proof. The collection of all connected graphs of order 3, 4 and 5 is presented in graphs G_1 *to* G_{26}. (see Figure 8). Based on these graphs, the EVOERSML and weaker EVOERSML properties of the graphs are analyzed and the results are summarized in Table 1.

4.6. Theorem:

A graph labeled with deficiency attains the EVOERSML property by attaching a number of pendant edges equal to the number of its missing edge labels.

Proof. Let G be a deficiency of an EVOERSML graph that does not satisfy the EVOERSML condition due to the absence of certain edge labels. Let the number of missing edge labels be k. To eliminate this deficiency, attach k pendant edges to suitable vertices of G, and denote the resulting graph by G'. For each new pendant edge, assign one of the missing labels from the set of unused edge labels of G. After this operation, the set of edge labels in G' becomes complete and consists of distinct consecutive values that satisfy the root square mean labeling conditions. Furthermore, the addition of k pendant edges preserves the parity of the vertex set, by keeping the vertices even and increases the number of edges to an odd count, which is required for an EVOERSML graph. Hence, the modified graph satisfies all the conditions of an EVOERSML.

4.6.1. Illustration:

Figures 9, 10 and 11 show examples of the transformation of weaker EVOERSML to EVOERSML. This can be verified from Table 1.

5. Conclusion

This study introduced and investigated the concept of a weaker EVOERSML graph, wherein the vertex labels are restricted to non-negative even integers, and the edge labels are derived from root square mean conditions. Various classes of graphs, including paths, cycles, corona graphs, and star graphs, were examined to establish the conditions under which they admit weaker EVOERSML. Furthermore, we analyzed the corresponding deficiency graphs and discussed the transformation process from weaker EVOERSML graphs to EVOERSML graphs.

Data availability

No data were used in the research described in this article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this manuscript.

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