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A novel parametric intuitionistic fuzzy entropy measure with applications to image edge detection

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Abstract

In digital image processing, edge detection is a fundamental operation used to identify intensity discontinuities in an image. In this paper, we propose a novel one-parameter generalization of an entropy measure for intuitionistic fuzzy sets (IFSs). IFSs provide additional flexibility by incorporating membership, non-membership, and hesitation information. A proof of validity is given for the proposed measure, together with numerical and graphical demonstrations. The entropy measure is applied to generate an edge map in which pixels exhibiting higher uncertainty are marked as image edges. Experimental results for standard benchmark images and real photographic images demonstrate superior edge localization and improved quantitative performance compared with existing fuzzy-entropy-based techniques.

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Keywords: Edge detection, Image processing, Rényi entropy, Intuitionistic fuzzy sets, Generalized fuzzy entropy

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1. Introduction

In image analysis, edge detection plays an important role because it highlights discontinuities in image features and structural contours. Many engineering applications require edge detection for advanced vision tasks, such as object recognition, medical diagnosis, scene interpretation, and image segmentation [1, 2]. Edge strength estimation and edge enhancement are key components of digital image edge detection. In the process of edge enhancement the local edges of an image are highlighted by using an edge operator. The various traditional gradient-based edge operators such as Sobel, Prewitt, Laplacian, and Canny have demonstrated considerable effectiveness across various applications; however, they are often susceptible to noise, parameter sensitivity, and degraded performance in low-contrast or textured scenarios [3, 4]. These limitations have motivated the exploration of uncertainty-driven methods that are inherently capable of modeling the ambiguity present in digital images.

Entropy is a fundamental concept used as a quantitative measure of information in probability distributions. This work was pioneered by Shannon [5], and several generalizations of entropy measures were later developed. Rényi [6], Havrda and Charvát [7], Mathai and Haubold [8–11] defined one-parameter entropy as a generalization of Shannon entropy. In parallel, the notion of a fuzzy set was introduced by Zadeh [12] to deal with non-statistical or vague data. This phenomenon became very important, as it is extensively used in different disciplines such as image processing, artificial intelligence, signal processing,

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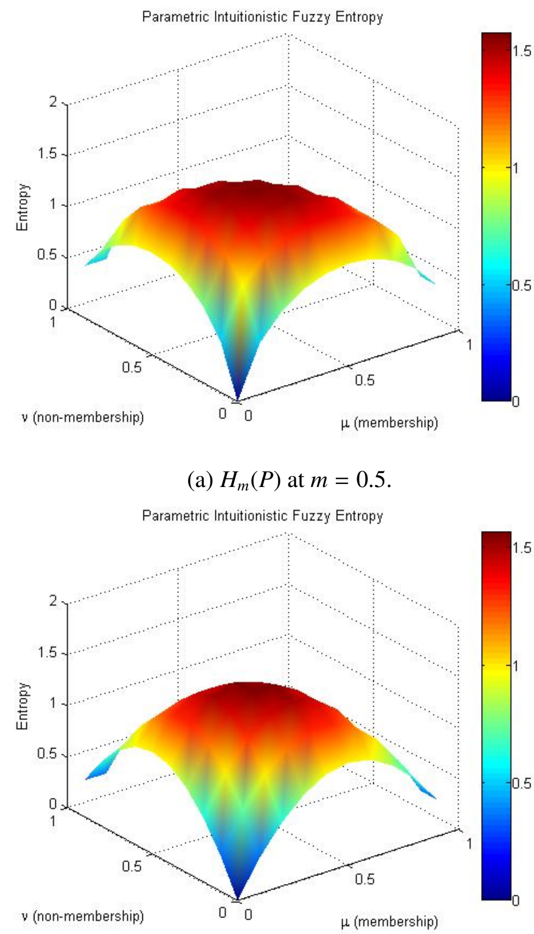
decision-making and many such real-life problems in engineering. De Luca and Termini [13] established a set of axioms for the entropy measure of fuzzy sets aligned with the axioms introduced by Shannon for checking the validity of entropy measures in probability distributions.

Classical fuzzy sets cannot handle hesitation or incomplete information because they assign only one membership value. Atanassov [14, 15], coined the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. An intuitionistic fuzzy set (IFS) is characterized by two elements, a membership function and a non-membership function such that their sum does not exceed one. It effectively represents uncertainty by incorporating a hesitation degree, reflecting the lack of precise knowledge about membership. Burillo and Bustince [16] were among the first to define an entropy function for intuitionistic fuzzy sets (IFSs) to quantify imprecision and hesitation. Szmidt and Kacprzyk [1] later formulated distance-based entropy measures to capture the divergence between an IFS and its crisp or complementary forms. Vlachos and Sergiadis [17] proposed a generalized IFS that incorporated both membership and non-membership degrees, emphasizing their interrelation and the role of hesitation in uncertainty modeling. Subsequently, various parametric entropy measures, including those developed by Hung and Yang [18], Arya Kumar [19], Chai *et al.* [20], and many others, extended the concept to Shannon-type, logarithmic, and power-based frameworks to enhance flexibility and adaptability in several real-life applications such as image processing, decision-making and pattern recognition. These developments collectively enriched the theoretical foundation of intuitionistic fuzzy information measures and broadened their practical relevance.

In the literature, gradient operator was pioneered by Sobel [21] developed in 1968. In addition to it, Prewitt [3], developed a method for edge detection of an image in 1970. Hildreth [22], in 1980 has given method for edge detection titled " Theory of Edge detection". Later, Canny [4], presented a method entitled " A computational approach to edge detection" in 1986. This method is considered as a standard edge detector operator. This paper investigates a one-parameter entropy measure for intuitionistic fuzzy sets as a generalization of the Zhang–Jiang entropy [23]. The proposed generalized entropy function is introduced within the intuitionistic fuzzy framework, and its mathematical validity and properties are established. The novelty of this work lies in formulating a flexible entropy measure for intuitionistic fuzzy information and demonstrating its applicability in image processing. In the domain of edge detection, intuitionistic fuzzy entropy has emerged as an effective tool due to its capability to highlight regions of high uncertainty, where structural transitions are most prominent [2, 9, 24–28]. Edge pixels inherently exhibit conflicting membership behavior and substantial hesitation, making entropy-enhanced IFS-based techniques desirable for producing noise-resilient and well-localized edge maps. Existing entropy measures may not be able to detect weak edges or reduce noise in an image. To address such issues, we introduce a novel one-parameter intuitionistic fuzzy entropy measure designed to magnify uncertainty around edge boundaries in image edge detection. This measure is incorporated into an edge detection framework, and the experimental outcomes on standard images show improved edge continuity, sharper localization, and enhanced robustness. The structure of this paper is as follows. In Section 2, the outline of the essential preliminaries related to entropy, fuzzy sets, intuitionistic fuzzy sets, and existing entropy measures in the intuitionistic fuzzy context is given. Section 3 introduces the proposed one-parameter generalized entropy measure for intuitionistic fuzzy sets, establishes its axiomatic framework, and presents its graphical representation, numerical examples, and key properties. Section 4 demonstrates the application of the proposed entropy measure to image edge detection, while Section 5 concludes the paper with a summary of findings and potential directions for future research.

The graphical, numerical, and experimental materials used in the subsequent sections are presented in Figures 1–13 and Tables 1–5.



(a) $H_m(P)$ at $m = 0.5$.

(b) $H_m(P)$ at $m = 1.5$.

Figure 1: Behavior of the proposed entropy measure for selected values of m.

Table 1: Proposed entropy for different values of m.

$m \rightarrow$	0.2	0.4	0.6	0.8	0.9	1.2	1.4	1.6
$E_m(A)$	0.7166	0.7237	0.7388	0.7634	0.7796	0.8477	0.9125	0.9968
$E_m(A^2)$	0.6337	0.6395	0.6527	0.6751	0.6907	0.7627	0.8424	0.9653
$E_m(A^3)$	0.5350	0.5399	0.5516	0.5724	0.5873	0.6603	0.7498	0.9082
$E_m(A^4)$	0.4520	0.4559	0.4663	0.4857	0.5001	0.5733	0.6677	0.8472

Table 2: Comparison of MSE and PSNR values obtained by different edge-detection methods.

Image	Proposed		Prewitt		Sobel		Canny	
	MSE	PSNR	MSE	PSNR	MSE	PSNR	MSE	PSNR
Cameraman	17962.0	5.583	17555.9	5.678	17568.9	5.679	18237.7	5.513
Lena	24414.4	4.262	24400.9	4.253	24362.0	4.254	24812.0	4.181
Cell	6566.5	9.954	6904.2	9.737	6885.3	9.747	7899.0	9.148
Man	14972.0	6.374	14907.8	6.469	14662.4	6.469	15624.8	6.188
Saturn	3963.3	12.145	3937.4	12.170	3950.5	12.161	7664.1	9.261
Cake	7124.8	9.601	7714.1	9.255	7762.0	9.229	10963.4	7.729
Coin	8010.3	9.091	7611.4	9.312	7599.8	9.319	9342.7	8.422
Butterfly	28356.3	3.602	29326.8	3.455	29318.2	3.456	29971.3	3.361

Table 3: Comparison of edge-detection methods using ground-truth-based metrics. Higher values indicate better performance.

Image	Method	Precision	Recall	F-measure	FOM
Cameraman	Proposed	**0.6842**	0.5931	**0.6354**	**0.7218**
	Prewitt	0.6512	0.5847	0.6162	0.6953
	Sobel	0.6489	**0.5963**	0.6214	0.7001
	Canny	0.6103	0.5412	0.5737	0.6544
Lena	Proposed	**0.6215**	**0.5738**	**0.5967**	**0.6891**
	Prewitt	0.5982	0.5521	0.5742	0.6712
	Sobel	0.5974	0.5508	0.5731	0.6703
	Canny	0.5641	0.5213	0.5419	0.6318
Cell	Proposed	**0.7423**	**0.6891**	**0.7147**	**0.7934**
	Prewitt	0.7151	0.6542	0.6834	0.7611
	Sobel	0.7138	0.6538	0.6825	0.7597
	Canny	0.6834	0.6012	0.6397	0.7102

Table 4: Recommended parameter m values for different image types and uncertainty scenarios.

Image/scenario type	Image characteristics	Recommended m	Rationale
High-contrast structured	Sharp intensity transitions, low noise (e.g., Saturn, coin)	0.2–0.6	High sensitivity to strong edges; suppresses background noise
Moderate-contrast general	Balanced uncertainty, mixed features (e.g., cameraman, Lena)	0.8–1.0	Reduces to Zhang–Jiang entropy at $m = 1$; stable performance
Low-contrast/textured	Subtle edges, high hesitation (e.g., rice grain, cake)	1.2–1.6	Less aggressive weighting preserves fine details
Medical/fine-structure	Weak boundaries, high noise susceptibility (e.g., cell)	0.6–0.8	Moderate sensitivity for weak-edge enhancement

Table 5: Sensitivity of F-measure to the parameter m for benchmark images. Bold values indicate the best performance per image.

	Parameter m							
Image	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
Cameraman	0.6101	0.6213	0.6289	**0.6354**	0.6308	0.6195	0.6042	0.5847
Lena	0.5712	0.5809	0.5891	**0.5967**	0.5934	0.5826	0.5693	0.5498
Cell	0.6934	0.7021	0.7094	**0.7147**	0.7089	0.6923	0.6741	0.6509
Saturn†	**0.8234**	0.8189	0.8051	0.7943	0.7802	0.7601	0.7388	0.7112
Rice grain‡	0.4812	0.5031	0.5289	0.5517	0.5734	**0.5921**	0.5876	0.5702

† High-contrast image: lower m yields higher F-measure, confirming Table 4.
‡ Textured image: higher m yields better performance, confirming Table 4.

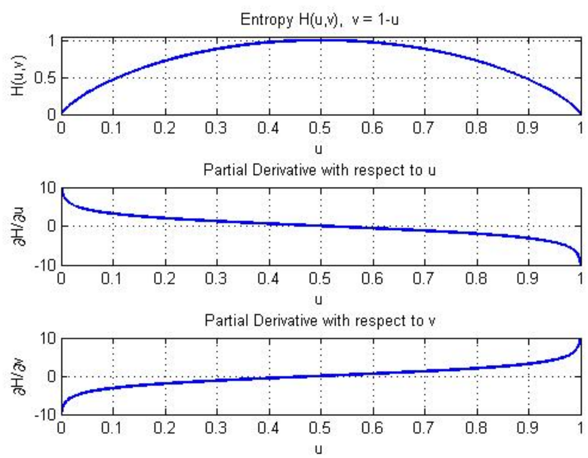


Figure 2: Behavior of the entropy function and partial derivative in (0, 1).

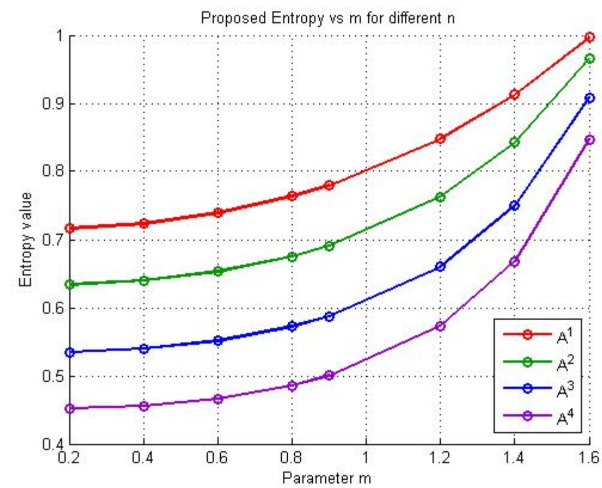


Figure 4: Proposed entropy versus the parameter m for different values of n.

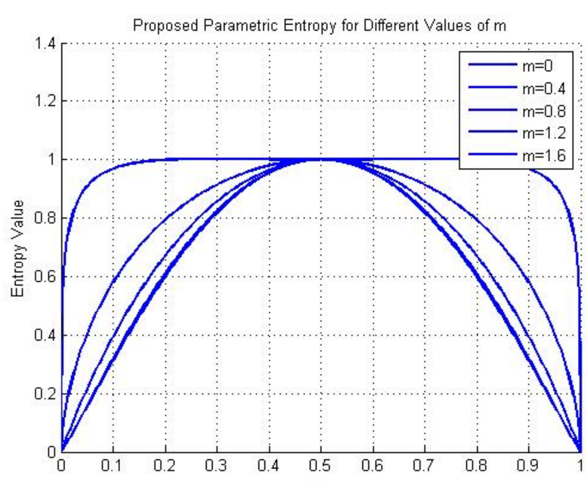


Figure 3: Entropy curve for various values of the parameter m.

2. Preliminaries

In this section, the fundamental concepts of fuzzy set theory, fuzzy entropy, and intuitionistic fuzzy set theory with their corresponding entropy measures are reviewed, providing a foundation for the subsequent development of the proposed work.

To measure the uncertainty in information content, Shannon [5] introduced a mathematical formula as $H(T) = -\sum_{k=1}^{n} t_k \log(t_k)$, where t_k is the probability corresponding to an event e_k, for $k = 1, 2, ..., n$ on $T = \{t_1, t_2, ..., t_n\}$. Various generalizations of Shannon entropy were studied. Some of them are listed below: Rényi entropy [6], of order m.

$$H^m(T) = \frac{1}{1-m} \log \sum_{k=1}^{n} t_k^m, m \neq 1, m > 0. \quad (1)$$

Havrda-Charvat [7] entropy of order m,

$$H^m(T) = \frac{1}{2^{1-m}-1} \sum_{k=1}^{n} t_k^m - 1, m \neq 1, m > 0. \quad (2)$$

Mathai–Haubold entropy [8], of order m

$$H^m(T) = \frac{1}{m-1} \sum_{k=1}^{n} t_k^{2-m} - 1, m \neq 1, -\infty < m < 2. \quad (3)$$

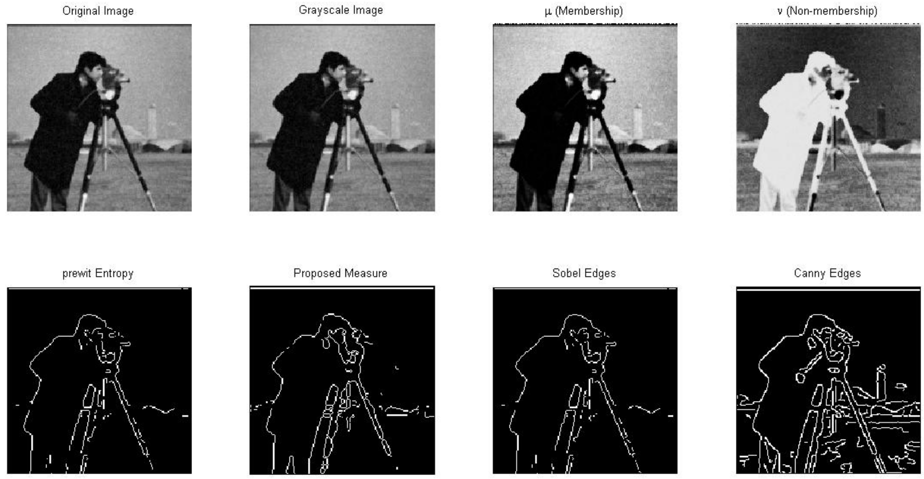


Figure 5: Edge detection of the cameraman image using the proposed entropy measure.

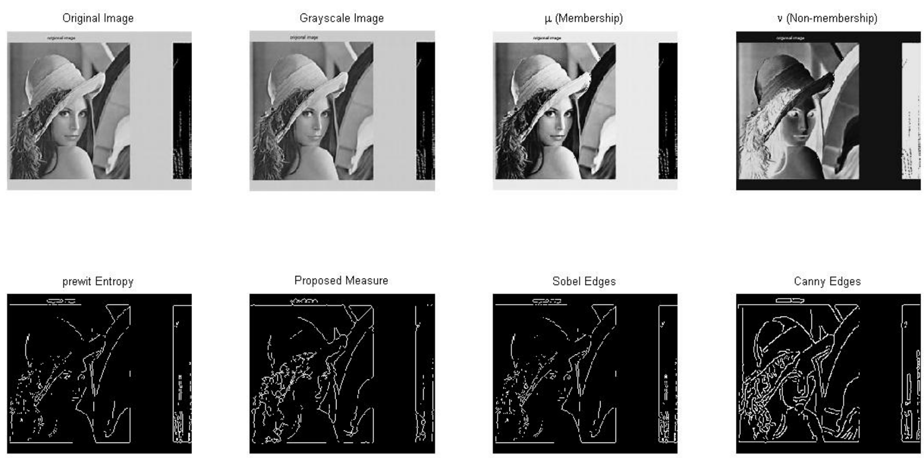


Figure 6: Edge detection of the Lena image using the proposed entropy measure.

$$H^m(T) = \frac{1}{m-1} log \sum_{k=1}^{n} t_k^{2-m}, m \neq 1, -\infty < m < 2. \tag{4}$$

Definition 2.1. A fuzzy set P, defined on finite universe of discourse $T = \{t_1, t_2, ..., t_n\}$ is given by $P = \{(t, u_P(t))/t \in T\}$ where $u_P(t) : T \to [0, 1]$ is the membership function of P and $u_P(t)$ defines the membership of belongingness of $t \in T$ in a fuzzy set P.

De Luca and Termini [13], introduced the measure of Fuzzy entropy corresponding to Shannon's entropy as

$$H(P) = -\sum_{k=1}^{n} u_P(t_k) log(u_P(t_k)) + (1 - u_P(t_k)) log(1 - u_P(t_k)). \tag{5}$$

Later on, Bhandari and Pal [11], defined generalized parametric measure of fuzzy entropy

$$H(P) = -\sum_{k=1}^{n} (u_P(t_k) log(u_P(t_k))^m + (1 - u_P(t_k)) log(1 - u_P(t_k))^m. \tag{6}$$

J. Kapur [10], measure of fuzzy entropy

$$H(P) = -\sum_{k=1}^{n} (u_P(t_k)^m + (1 - u_P(t_k))^m - 1). \tag{7}$$

In practical problems, complement of membership value is not same as the non-membership value. To resolve this, Atanassov instigated the generalization of fuzzy sets which is known as intuitionistic fuzzy sets(IFSs). It is defined as below:

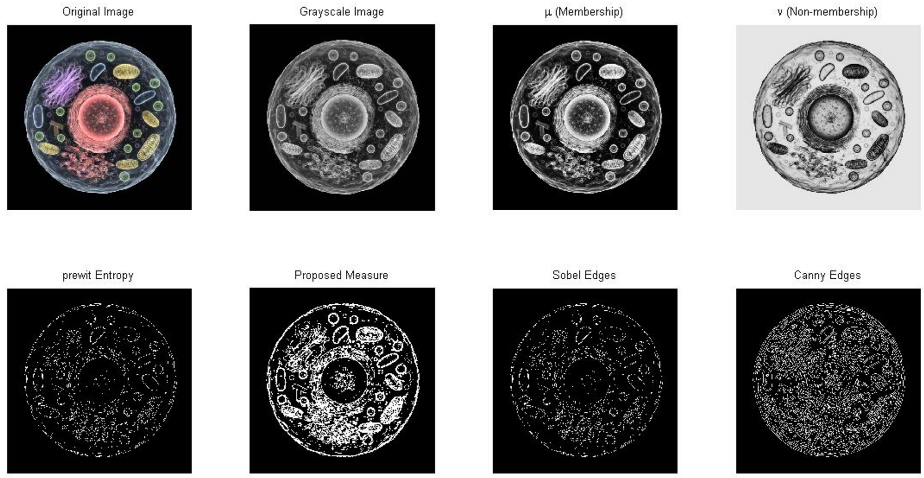


Figure 7: Edge detection of the two-dimensional cell image using the proposed entropy measure.

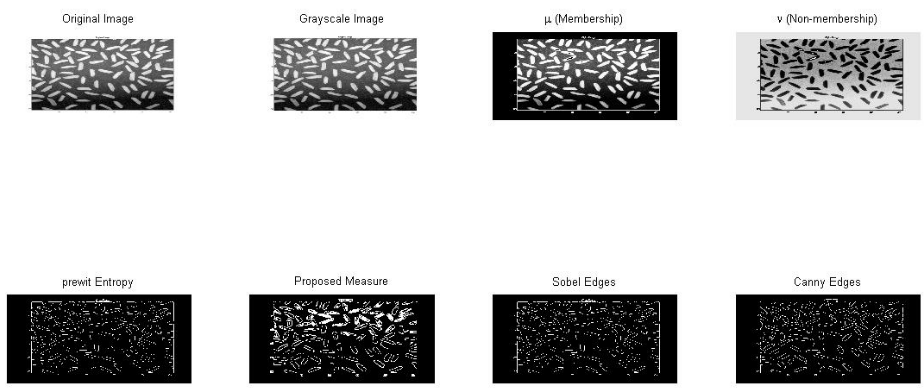


Figure 8: Edge detection of the rice-grain image using the proposed entropy measure.

Definition 2.2. An intuitionistic fuzzy set defined as $P = \{(t, u_P(t), v_P(t))/t \in T\}$, $X = \{t_1, t_2, ..., t_n\}$ where $u_P(t) : T \to [0,1], v_P(t) : T \to [0,1]$ and $0 \le u_P(t) + v_P(t) \le 1, \forall t \in T$. Here the numbers $u_P(t)$ and $v_P(t)$ are membership and non-membership of $t \in T$ in fuzzy set P.

Definition 2.3. For each intuitionistic fuzzy set P in T, the intuitionistic fuzzy index / hesitation degree is defined as $\pi_P(t) = 1 - u_P(t) - v_P(t)$.

If the intuitionistic fuzzy index is zero then, intuitionistic fuzzy set becomes a traditional fuzzy set, hence fuzzy sets are the special case of intuitionistic fuzzy sets.

Definition 2.4. Set Operations in intuitionistic fuzzy sets: Let P and Q be any two intuitionistic fuzzy sets defined on universe of discourse T, $P = \{(t, u_P(t), v_P(t))/t \in T\}$ and $Q = \{(t, u_Q(t), v_Q(t))/t \in T\}$ then some operations on sets are defined as follows: (i) $P \subseteq Q$ if and only if $u_P(t) \le u_Q(t)$ and $v_P(t) \ge v_Q(t)$, $\forall t \in T$ (ii) $P = Q$ if and only if $P \subseteq Q$ and $Q \subseteq P$ (iii) $P^c = \{(t, v_P(t), u_P(t))/t \in T\}$ (iv) $P \cup Q = \{(t, u_P(t) \vee u_Q(t), v_P(t) \wedge v_Q(t))/t \in T\}$ (v) $P \cap Q = \{(t, u_P(t) \wedge u_Q(t), v_P(t) \vee v_Q(t))/t \in T\}$ where $\vee$ and $\wedge$ denote the maximum and minimum operators.

Later, Szmidt and Kacprzyk [1], defined the axioms for entropy in intuitionistic fuzzy sets. These axioms are stated as below.

Definition 2.5. Axioms of Entropy in intuitionistic fuzzy sets: A real valued function $H : IFS(T) \to [0,1]$ is called entropy on intuitionistic fuzzy set (IFS) on set T, if it satisfies the following four axioms **A1** : $H(P) = 0$ if and only if P is a crisp set, that is $u_P(t) = 0, v_P(t) = 1$ or $u_P(t) = 1, v_P(t) = 0$ **A2** : $H(P) = 1$ if and only if $u_P(t) = v_P(t), \forall t \in T$ **A3** :$H(P) \le H(Q)$ if and

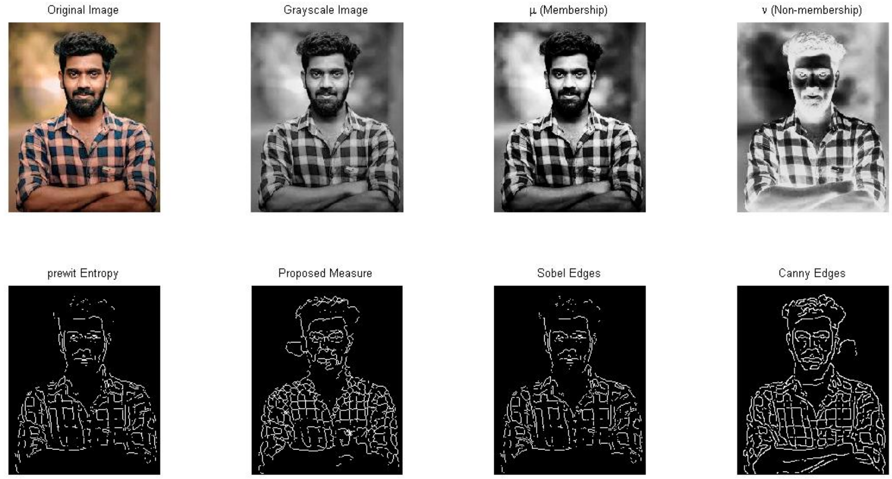


Figure 9: Edge detection of the person photograph using the proposed entropy measure.

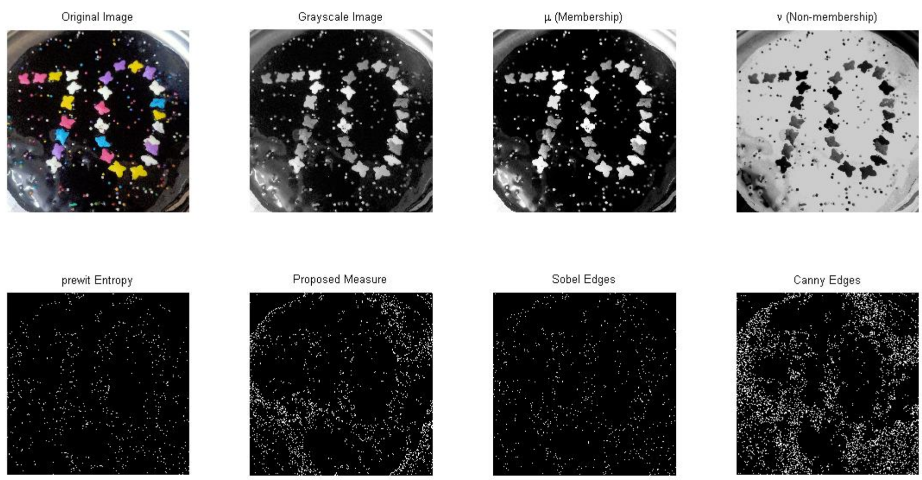


Figure 10: Edge detection of the cake image using the proposed entropy measure.

only if $P \subseteq Q$ that is if $u_P(t) \leq u_Q(t)$ and $v_P(t) \geq v_Q(t)$ for $u_Q(t) \leq v_Q(t)$ or if $u_P(t) \geq u_Q(t)$ and $v_P(t) \leq v_Q(t)$ for any $t \in T$.
A4 : $H(P) = H(P^c)$

Li *et al.* [20], introduced a method for transforming Atanassov's IFSs into fuzzy sets by equal distribution of hesitation degree with membership and non-membership values. It is defined as,

Definition 2.6. Let $P = \{(t, u_P(t), v_P(t))/t \in T\}$ be an intuitionistic fuzzy set defined on finite set T, the fuzzy membership function $u_{P^*}(t)$ to P^* is defined as

$$u_{P^*}(x) = u_P(t) + \frac{\pi_P(t)}{2} = \frac{u_P(t) + 1 - v_P(t)}{2}, \tag{8}$$

where P^* be the fuzzy set corresponding to intuitionistic fuzzy set P.

This area has attracted attention of many researchers for further studies. Some generalized measure of intuitionistic fuzzy entropy based on a generalization of the fuzzy entropy are listed below:

(a) Zhang and Jiang [23]

$$H_m(P) = \frac{-1}{n}\sum_{k=1}^{n}\left[\left(\frac{u_P(t_k) + 1 - v_P(t_k)}{2}\right)\log\left(\frac{u_P(t_k) + 1 - v_P(t_k)}{2}\right) + \left(\frac{v_P(t_k) + 1 - u_P(t_k)}{2}\right)\log\left(\frac{v_P(t_k) + 1 - u_P(t_k)}{2}\right)\right]. \tag{9}$$

(b) Ye [27]

$$H_m(P) = \frac{1}{n(2^{0.5} - 1)}\sum_{k=1}^{n}\left[\sin\pi\left(\frac{u_P(t_k) + 1 - v_P(t_k)}{4}\right) + \right.$$

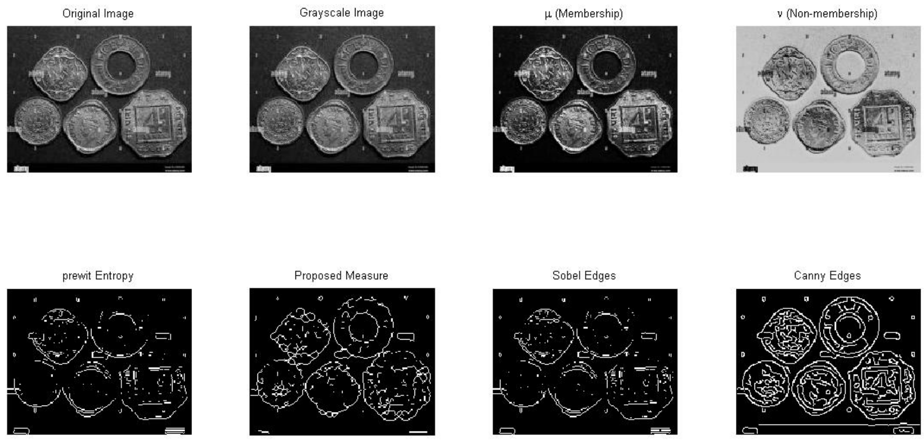


Figure 11: Edge detection of the coin image using the proposed entropy measure.

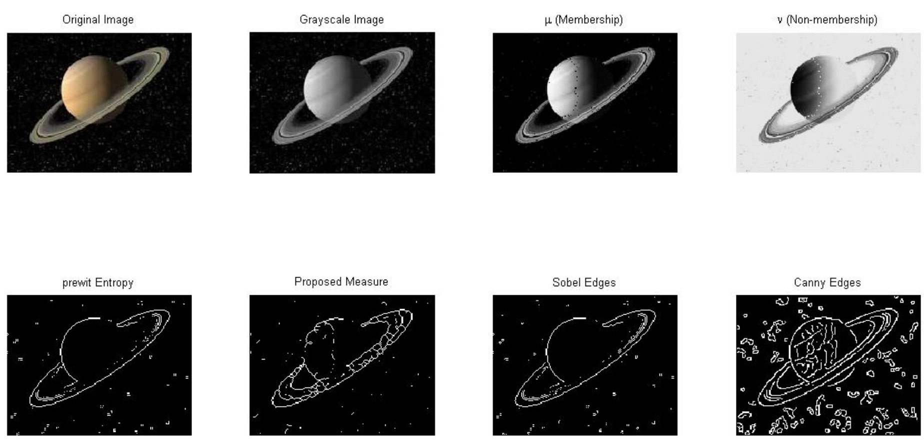


Figure 12: Edge detection of the Saturn image using the proposed entropy measure.

$$\sin\pi\left(\frac{v_P(t_k)+1-u_P(t_k)}{4}\right)-1\Bigg]. \tag{10}$$

$$H_m(P)=\frac{1}{n(2^{0.5}-1)}\sum_{i=1}^{n}\Bigg[\cos\pi\left(\frac{u_P(t_k)+1-v_P(t_k)}{4}\right)+\cos\pi\left(\frac{v_P(t_k)+1-u_P(t_k)}{4}\right)-1\Bigg]. \tag{11}$$

(c) Wei *et al.* [28]

$$H_m(P)=\frac{1}{n(2^{0.5}-1)}\sum_{k=1}^{n}\Bigg[(2^{0.5})\cos\pi\left(\frac{u_P(t_k)-v_P(t_k)}{4}\right)-1\Bigg]. \tag{12}$$

(d) Hung and Yang[18]

$$H_m(P)=\frac{1}{n(1-m)}\sum_{k=1}^{n}\Bigg[\log_2(u_P^m(t_k)+v_P^m(t_k)+\pi_P^m(t_k))\Bigg],\quad 0<m<1. \tag{13}$$

(e) Arya and Kumar[19]

$$H_m(P)=\frac{1}{n(m-m^{-1})}\sum_{k=1}^{n}\Bigg[(u_P^{m^{-1}}(t_k)+v_P^{m^{-1}}(t_k)+\pi_P^{m^{-1}}(t_k))-(u_P^m(t_k)+v_P^m(t_k)+\pi_P^m(t_k))\Bigg],m>0,m\neq 1. \tag{14}$$

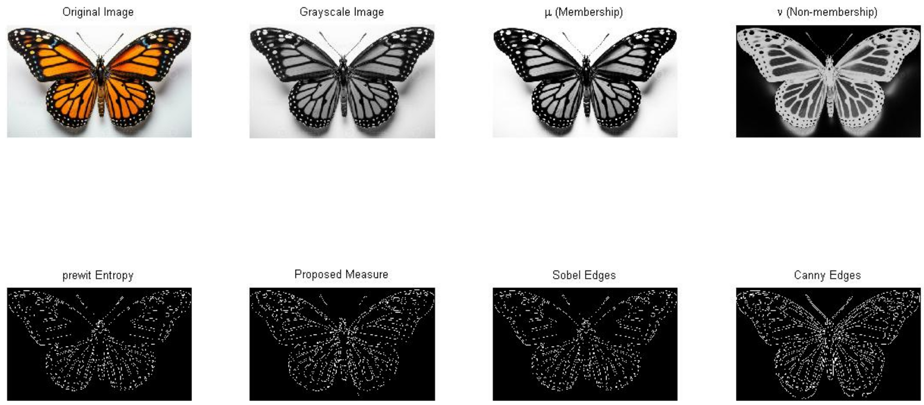


Figure 13: Edge detection of the butterfly image using the proposed entropy measure.

(f) Burillo and Bustince [16]

$$H(P) = \frac{1}{n}\sum_{k=1}^{n} |1 - u_P(t_k) - v_P(t_k)|. \tag{15}$$

With these basic concepts, we have introduced an entropy measure in IFSs of order m which is given in the next section.

3. Proposed intuitionistic fuzzy entropy of order *m*

A proposed parametric entropy in IFSs as a generalization of Zhang and Jiang entropy as follows.

Let P be an IFS on finite set T. We propose an entropy for intuitionistic fuzzy sets (IFSs) of order m as

$$H_m(P) = -\frac{1}{n2^{m-1}}\sum_{k=1}^{n}\left[\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{2-m} \times \log_2\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right) + \left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{2-m} \log_2\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)\right], 0 < m < 2. \tag{16}$$

Theorem 3.1. The generalized IFSs entropy of order m,

$$H_m(P) = -\frac{1}{n2^{m-1}}\sum_{k=1}^{n}\left[\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{2-m} \log_2\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right) + \left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{2-m} \times \log_2\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)\right], 0 < m < 2,$$

is a valid measure of intuitionistic fuzzy entropy.

Proof 3.1. To prove $H_m(P)$ a valid fuzzy entropy measure. Axiom 1: Let P be a crisp set that is $u_P(t_k) = 0, v_P(t_k) = 1$ or $u_P(t_k) = 1, v_P(t_k) = 0$, $\forall t_k \in T$ Let $t_k = \frac{u_P(t_k)+1-v_P(t_k)}{2}$, as t_k tends to zero $(u_P(t_k) = 0, v_P(t_k) = 1)$ the value of $(t_k)^m \log_2(t)$ tends to zero (using L'Hospital rule). This implies $H_m(P) = 0$ Conversely, if $H_m(P) = 0$ then, $\Longrightarrow$

$$-\frac{1}{n2^{m-1}}\sum_{k=1}^{n}\left[\left(\frac{u_p(t_k)+1-v_P(t_k)}{2}\right)^{2-m} \log_2\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right) + \left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{2-m} \log_2\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)\right] = 0.$$

$\Longrightarrow$

$$\sum_{k=1}^{n}\left[\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{2-m} \log_2\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right) + \left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{2-m} \log_2\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)\right] = 0.$$

$\Longrightarrow$

$$\left[\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{2-m} \log_2\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right) = 0,\right.$$

and

$$\left.\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{2-m} \log_2\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)\right] = 0,$$

$u_P(t_k) = 1, v_P(t_k) = 0$ or $u_P(t_k) = 0, v_P(t_k) = 1$. Therefore, P is a crisp set. Axiom 2: We prove that $H_m(P) = 1$ if and only if $u_P(t_k) = v_P(t_k)$ Substituting $u_P(t_k) = v_P(t_k)$ in the proposed measure

$$H_m(P) = -\frac{1}{n2^{m-1}}\sum_{k=1}^{n}\left[\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{2-m}\right.$$

$$\times \log_2\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)+\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{2-m}$$
$$\times \log_2\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)\Bigg] = 1.$$

Thus, if $H_m(P) = 1$ then

$$-\frac{1}{n2^{m-1}}\sum_{k=1}^{n}\Bigg[\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{2-m}$$
$$\times \log_2\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)+\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{2-m}$$
$$\times \log_2\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)\Bigg] = 1.$$

$$\frac{1}{2^{m-1}}\Bigg[\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{2-m}\log_2\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)$$
$$+\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{2-m}\log_2\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)\Bigg] = 1.$$

This holds when $u_P(t_k) = v_P(t_k)$. Differentiating partially $H_m(P)$ with respect to $u_P(t_k)$, we obtain

$$\frac{\partial H_m(P)}{\partial u_P(t_k)} = -\frac{1}{n2^{m-1}}\sum_{k=1}^{n}\frac{2-m}{2}\Bigg[\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{1-m}$$
$$\times \log_2\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)-\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{1-m}$$
$$\times \log_2\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)\Bigg]+\frac{1}{2}\Bigg[\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{1-m}$$
$$-\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{1-m}\Bigg].$$

Again differentiating with respect to $u_P(t_k)$,

$$\frac{\partial^2 H_m(P)}{\partial u_P^2(t_k)} = -\frac{1}{n2^{m-1}}\sum_{k=1}^{n}\frac{(2-m)(1-m)}{4}\Bigg[\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{-m}$$
$$\times \log_2\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)-\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{-m}$$
$$\log_2\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)\Bigg]$$
$$+\frac{(3-2m)}{4}\Bigg[\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{-m}+\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{-m}\Bigg].$$

The value of $\frac{\partial^2 H_m(P)}{\partial u_P^2(t_k)} \leq 0$, for all $t_k \in T$. Hence $H_m(P)$ is a concave function and has a maximum value at $u_P(t_k) = v_P(t_k)$. (Figure 3) Axiom 3: To prove that the proposed entropy function is increasing with respect to $u_P(t)$ and decreasing with respect to $v_P(t_k)$ respectively, where $u_P(t_k) \in [0,1]$ and $v_P(t_k) \in [0,1]$ (Figure 2).

Now, Differentiating the proposed measure partially with respect to $u_P(t_k)$ and $v_P(t_k)$ respectively, we get

$$\frac{\partial H_m(P)}{\partial u_P(t_k)} = -\frac{1}{n2^{m-1}}\sum_{k=1}^{n}\frac{2-m}{2}\Bigg[\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{1-m}$$
$$\times \log_2\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)-\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{1-m}$$
$$\times \log_2\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)\Bigg]+$$
$$\frac{1}{2}\Bigg[\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{1-m}-\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{1-m}\Bigg].$$

$$\frac{\partial H_m(P)}{\partial v_P(t_k)} = -\frac{1}{n2^{m-1}}\sum_{k=1}^{n}\frac{2-m}{2}\Bigg[-\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{1-m}$$
$$\times \log_2\left(\frac{u_P(t)+1-v_P(t)}{2}\right)+\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{1-m}$$
$$\times \log_2\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)\Bigg]+\frac{1}{2}\Bigg[-\left(\frac{u_P(t_k)+1-v_P(t_k)}{2}\right)^{1-m}+$$
$$\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{1-m}\Bigg].$$

In order to find the critical point of $H_m(P)$, we set $\frac{\partial H_m(P)}{\partial u_P(t_k)} = 0$ and $\frac{\partial H_m(P)}{\partial v_P(t_k)} = 0$. This gives $u_P(t_k) = v_P(t_k)$.

From these equations, we have

$$\frac{\partial H_m(P)}{\partial u_P(t_k)} \geq 0,\ \text{when } u_P(t_k) \leq v_P(t_k), \tag{17}$$

$$\frac{\partial H_m(P)}{\partial u_P(t_k)} \leq 0,\ \text{when } u_P(t_k) \geq v_P(t_k), \tag{18}$$

for any $u_P(t_k), v_P(t_k) \in [0,1]$. Thus, $H_m(P)$ increases with respect to $u_P(t_k)$ when $u_P(t_k) \leq v_P(t_k)$ and decreases when $u_P(t_k) \geq v_P(t_k)$.

Similarly, we obtain

$$\frac{\partial H_m(P)}{\partial v_P(t_k)} \leq 0,\ \text{when } u_P(t_k) \leq v_P(t_k) \tag{19}$$

$$\frac{\partial H_m(P)}{\partial v_P(t_k)} \geq 0,\ \text{when } u_P(t_k) \geq v_P(t_k) \tag{20}$$

Consider two sets P and Q in IFSs X with $P \subseteq Q$. Suppose that the set T is partitioned into two disjoint sets T_1 and T_2 such that $T_1 \cup T_2 = T$. Suppose that the set T_1 is defined as $T_1 = \{t_i | u_P(t_i) \leq u_Q(t_i) \leq v_Q(t_i) \leq v_P(t_i)\}$ while $T_2 = \{t_i | u_P(t_i) \geq u_Q(t_i) \geq v_Q(t_i) \geq v_P(t_i)\}$ holds, then from monotonicity of the function $H_m(P) \leq H_m(Q)$. Axiom 4: To prove $H_m(P) = H_m(P^c)$ Since the complement of $P = \{(t_k, u_P(t_k), v_P(t_k))/t_k \in T\}$ is $P^c = \{(t_k, v_P(t_k), u_P(t_k))/t_k \in T\}$ that is $[u_P(t_k)]^c = v_P(t_k)$ and $[v_P(t_k)]^c = u_P(t_k)$.

$$H_m(P^c) = -\frac{1}{n2^{m-1}}\sum_{k=1}^{n}\Bigg[\left(\frac{u_P^c(t_k)+1-v_P^c(t_k)}{2}\right)^{2-m}$$
$$\times \log_2\left(\frac{u_P^c(t_k)+1-v_P^c(t_k)}{2}\right)+\left(\frac{v_P^c(t_k)+1-u_P^c(t_k)}{2}\right)^{2-m}$$
$$\times \log_2\left(\frac{v_P^c(t_k)+1-u_P^c(t_k)}{2}\right)\Bigg] = H_m(P).$$

Therefore, we get $H_m(P) = H_m(P^c)$. Hence, the proposed measure in intuitionistic fuzzy sets is a valid intuitionistic fuzzy entropy measure.

Particular Case: It is interesting to note that for $m = 1$, this proposed measure reduces to the Zhang and Jiang entropy [23].

$$H_m(P) = -\frac{1}{n2^{m-1}} \sum_{k=1}^{n} \left[\left(\frac{u_P(t_k) + 1 - (1 - u_P(t_k))}{2} \right)^{2-m} \times \log_2 \left(\frac{u_P(t_k) + 1 - (1 - u_P(t_k))}{2} \right) + \left(\frac{(1 - u_P(t_k)) + 1 - u_P(t_k)}{2} \right)^{2-m} \times \log_2 \left(\frac{(1 - u_P(t_k)) + 1 - u_P(t_k)}{2} \right) \right]$$

The proposed measure is expressed in terms of membership and nonmembership functions, the influence of hesitation degree is inherently embedded in the transformed quantities appearing in the proposed measure. The uncertainty arises from hesitation degree directly contributes to the entropy value, even though it does not appears explicitly in the equation.

If an intuitionistic fuzzy set is an ordinary fuzzy set, that is, for all $t_k \in T$, $v_P(t_k) = 1 - u_P(t_k)$ then the proposed measure of intuitionistic fuzzy entropy reduces to De Luca and Termini fuzzy entropy [13]

$$H_m(P) = -\frac{1}{n} \sum_{k=1}^{n} \left[u_P(t_k) log(u_P(t_k)) + (1 - u_P(t_k)) log(1 - u_P(t_k)) \right].$$

The proposed intuitionistic fuzzy entropy of order m satisfies the following properties:

Theorem 3.2. Let P and Q be two intuitionistic fuzzy sets in set T where, $P = \{(t_k, u_P(t_k), v_P(t_k))/t_k \in T\}$ and $Q = \{((t_k), u_Q(t_k), v_Q((t_k)))/t_k \in T\}$, such that for any $t_k \in T$ either $P \subseteq Q$ or $Q \subseteq P$, then

$$H_m(P \cup Q) + H_m(P \cap Q) = H_m(P) + H_m(Q).$$

Proof 3.2. Let T be separated in two parts T_1 and T_2 where, $T_1 = \{t_k \in T / P(t_k) \subseteq Q(t_k)\}$ $T_2 = \{t_k \in T / Q(t_k) \subseteq P(t_k)\}$ That means $\forall t_k \in T_1, u_P(t_k) \leq u_Q(t_k)$ and $v_P(t_k) \geq v_Q(t_k)$ Also $\forall t_k \in T_2, u_Q(t_k) \leq u_P(t_k)$ and $v_Q(t_k) \geq v_P(t_k)$.

$$H_m(P \cup Q) = -\frac{1}{n2^{m-1}} \sum_{k=1}^{n} \left[\left(\frac{u_{P\cup Q}(t_k) + 1 - v_{P\cup Q}(t_k)}{2} \right)^{2-m} \times \log_2 \left(\frac{u_{P\cup Q}(t_k) + 1 - v_{P\cup Q}(t_k)}{2} \right) + \left(\frac{v_{P\cup Q}(t_k) + 1 - u_{P\cup Q}(t_k)}{2} \right)^{2-m} \times \log_2 \left(\frac{v_{P\cup Q}(t_k) + 1 - u_{P\cup Q}(t_k)}{2} \right) \right], 0 < m < 2.$$

$$= -\frac{1}{n2^{m-1}} \left\{ \sum_{t_k \in T_1} \left[\left(\frac{u_Q(t_k) + 1 - v_Q(t_k)}{2} \right)^{2-m} \times \log_2 \left(\frac{u_Q(t_k) + 1 - v_Q(t_k)}{2} \right) + \left(\frac{v_Q(t_k) + 1 - u_Q(t_k)}{2} \right)^{2-m} \log_2 \left(\frac{v_Q(t_k) + 1 - u_Q(t_k)}{2} \right) \right] + \sum_{t_k \in T_2} \left[\left(\frac{u_P(t_k) + 1 - v_P(t_k)}{2} \right)^{2-m} \log_2 \left(\frac{u_P(t_k) + 1 - v_Q(t_k)}{2} \right) + \left(\frac{v_P(t_k) + 1 - u_P(t_k)}{2} \right)^{2-m} \log_2 \left(\frac{v_P(t_k) + 1 - u_P(t_k)}{2} \right) \right] \right\}.$$

Similarly,

$$H_m(P \cap Q) = -\frac{1}{n2^{m-1}} \sum_{k=1}^{n} \left[\left(\frac{u_{P\cap Q}(t_k) + 1 - v_{P\cap Q}(t_k)}{2} \right)^{2-m} \log_2 \left(\frac{u_{P\cap Q}(t_k) + 1 - v_{P\cap Q}(t_k)}{2} \right) + \left(\frac{v_{P\cap Q}(t_k) + 1 - u_{P\cap Q}(t_k)}{2} \right)^{2-m} \log_2 \left(\frac{v_{P\cap Q}(t_k) + 1 - u_{P\cap Q}(t_k)}{2} \right), 0 < m < 2.$$

$$= -\frac{1}{n2^{m-1}} \left\{ \sum_{t_k \in T_1} \left[\left(\frac{u_P(t_k) + 1 - v_P(t_k)}{2} \right)^{2-m} \log_2 \left(\frac{u_P(t_k) + 1 - v_P(t_k)}{2} \right) + \left(\frac{v_P(t_k) + 1 - u_P(t_k)}{2} \right)^{2-m} \log_2 \left(\frac{v_P(t_k) + 1 - u_P(t_k)}{2} \right) \right] + \sum_{t_k \in T_2} \left[\left(\frac{u_Q(t_k) + 1 - v_Q(t_k)}{2} \right)^{2-m} \log_2 \left(\frac{u_Q(t_k) + 1 - v_Q(t_k)}{2} \right) + \left(\frac{v_Q(t_k) + 1 - u_Q(t_k)}{2} \right)^{2-m} \log_2 \left(\frac{v_Q(t_k) + 1 - u_Q(t_k)}{2} \right) \right] \right\}.$$

$$H_m(P \cup Q) + H_m(P \cap Q) =$$

$$-\frac{1}{n2^{m-1}} \left\{ \sum_{t_k \in T_1} \left[\left(\frac{u_Q(t_k) + 1 - v_Q(t_k)}{2} \right)^{2-m} \log_2 \left(\frac{u_Q(t_k) + 1 - v_Q(t_k)}{2} \right) + \left(\frac{v_Q(t_k) + 1 - u_Q(t_k)}{2} \right)^{2-m} \log_2 \left(\frac{v_Q(t_k) + 1 - u_Q(t_k)}{2} \right) \right] + \sum_{t_k \in T_2} \left[\left(\frac{u_P(t_k) + 1 - v_P(t_k)}{2} \right)^{2-m} \log_2 \left(\frac{u_P(t_k) + 1 - v_P(t_k)}{2} \right) + \left(\frac{v_P(t_k) + 1 - u_P(t_k)}{2} \right)^{2-m} \log_2 \left(\frac{v_P(t_k) + 1 - u_P(t_k)}{2} \right) \right] \right\}$$

$$-\frac{1}{n2^{m-1}} \left\{ \sum_{t_k \in T_1} \left[\left(\frac{u_P(t_k) + 1 - v_P(t_k)}{2} \right)^{2-m} \log_2 \left(\frac{u_P(t_k) + 1 - v_P(t_k)}{2} \right) \right.\right.$$

$$+\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)^{2-m}\log_2\left(\frac{v_P(t_k)+1-u_P(t_k)}{2}\right)\Bigg]$$
$$+\sum_{t_k\in T_2}\Bigg[\left(\frac{u_Q(t_k)+1-v_Q(t_k)}{2}\right)^{2-m}\log_2\left(\frac{u_Q(t_k)+1-v_Q(t_k)}{2}\right)$$
$$+\left(\frac{v_Q(t_k)+1-u_Q(t_k)}{2}\right)^{2-m}\log_2\left(\frac{v_Q(t_k)+1-u_Q(t_k)}{2}\right)\Bigg]\Bigg\}$$
$$= H_m(P) + H_m(Q).$$

Hence proved.

Example 3.1. Consider an intuitionistic fuzzy set

$$P = \{(a, 0.1, 0.8), (b, 0.3, 0.5), (c, 0.5, 0.4), (d, 0.7, 0.2), (e, 0.9, 0.1)\}.$$

The value of $H_m(P)$ at $m = 0.8$ is 0.7334.

Theorem 3.3. The entropy function $H_m(P)$ attains maximum value when the set is most intuitionistic fuzzy set and minimum when the set is crisp set. Moreover, maximum and minimum vales are independent of parameter m.

Proof 3.3. It has already seen that $H_m(P)$ has a maximum if and only if $u_P(t_k) = v_P(t_k)$, $\forall t_k \in T$ and minimum when the set P is crisp set. So, it is enough to show that maximum and minimum values are independent of m. Case I: Maximum Value: When $u_P(t_k) = v_P(t_k)$ then the equation becomes

$$H_m(P) = -\frac{1}{n2^{m-1}}\sum_{k=1}^{n}\Bigg[\left(\frac{u_P(t_k)+1-u_P(t_k)}{2}\right)^{2-m}$$
$$\times\log_2\left(\frac{u_P(t_k)+1-u_P(t_k)}{2}\right)+\left(\frac{u_P(t_k)+1-u_P(t_k)}{2}\right)^{2-m}$$
$$\times\log_2\left(\frac{u_P(t_k)+1-u_P(t_k)}{2}\right)\Bigg]$$
$$= -\frac{1}{n2^{m-1}}\sum_{k=1}^{n}\left[\left(\frac{1}{2}\right)^{2-m}\log_2\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^{2-m}\log_2\left(\frac{1}{2}\right)\right]$$
$$= -\frac{1}{n2^{m-1}}\sum_{k=1}^{n}\left[2\left(\frac{1}{2}\right)^{2-m}\log_2\left(\frac{1}{2}\right)\right]$$
$$= 1.$$

Case II: Minimum value. When A is a crisp set, that is, if $u_P(t_k) = 0, v_P(t_k) = 1$ or $u_P(t_k) = 1, v_P(t_k) = 0$, then $H_m(P) = 0$. In both cases, the result does not contain the parameter m.

Note: The proposed parametric measure $H_m(P)$, for $0 < m < 2$ is continuous and bounded in $([0, 1])$. For this let us consider$f(x) = -x^{2-m}\log_2(x)$ where $x = \frac{u_P(t)+1-v_P(t)}{2}$. As $2 - m > 0$, the function is continuous in $(0, 1)$. Further, $lim_{x\to 0^+}(-x^{2-m}\log_2(x)) = 0$, by using L'Hospital's rule and $f(1^-) = 0$. Therefore the function $f(x)$ is continuous in $(0, 1)$. Since the finite sum of continuous functions is continuous, follows that $H_m(P)$ is continuous in $(0, 1)$. Boundedness of the measure follows from the theorem 3.3 $\implies 0 \leq H_m(P) \leq 1$

Example 3.2. Consider an IFS A defined on $X = \{a, b, c, d, e\}$, where

$$A = \{(a, 0.1, 0.8), (b, 0.3, 0.5), (c, 0.5, 0.4), (d, 0.7, 0.2), (e, 0.8, 0.1)\}.$$

The set $A^n = \{(x_i, u_A^n(x_i), 1 - (1 - v_A(x_i)^n))|x_i \in X\}$ As per the assistance of process demonstrated in the subsequent intuitionistic fuzzy sets are formed. The set A, A^2, A^3, A^4 follows: $A \implies$ "Large", $A^2 \implies$ "Very Large", $A^3 \implies$ " Quite Very Large" and $A^4 \implies$ " Very Very Large". The proposed entropy measure is then applied to these intuitionistic fuzzy sets, and the entropy values are computed for different values of the parameter m. The obtained results are presented in Table 1. From the table, it is evident that the proposed entropy measure preserves the natural human logical ordering, namely $E_m(A) > E_m(A^2) > E_m(A^3) > E_m(A^4)$. This indicates that entropy decreases as the linguistic intensity increases, which is consistent with intuitive reasoning. The behavior of the entropy measure for various values of the parameter m is illustrated in Figure 4.

4. Application of the proposed measure in image edge detection

Edge detection is a fundamental and indispensable operation in digital image processing. It significantly reduces computational complexity by simplifying image data while preserving essential structural information. The extraction of edge information plays a crucial role in numerous applications such as image segmentation, image enhancement, object detection, image compression, medical diagnosis, military surveillance, pattern recognition, and geographical information systems [1–3, 9, 25, 26]. Chaira and Ray [29] introduced an important intuitionistic fuzzy entropy measure and demonstrated its applicability to edge detection of an image. Their contribution marked a significant advancement in uncertainty-based image analysis. Motivated by this direction, the present work proposes a novel parametric intuitionistic fuzzy entropy measure and investigates its effectiveness in edge detection.

Digital images constitute one of the most dominant forms of information representation. The primary objective of digital image processing is to develop mathematical models and computational techniques for extracting meaningful information while suppressing irrelevant data or noise. However, the presence of noise remains a major challenge in image segmentation and edge detection tasks, as digital images are highly susceptible to various degradations.

Conventional edge detectors identify edges by locating abrupt intensity variations using gradient operators. In contrast, the proposed framework detects edges through uncertainty modeling in the intuitionistic fuzzy domain. Instead of relying solely on intensity gradients, edges are characterized as regions exhibiting significant variations in entropy. Since the method operates in the intuitionistic fuzzy domain, minor intensity fluctuations are naturally suppressed, thereby reducing false edge responses in noisy regions. Hence, the proposed entropy-based measure serves as a robust alternative to classical gradient-based detectors such as Canny, Sobel, and Prewitt operators.

In this section, the proposed entropy measure is applied to various benchmark images and the results are compared with existing edge detection operators.

4.1. Methodology

Consider a digital image $I(x, y)$. The stepwise procedure for edge detection using the proposed parametric IFSs is given below.

(Step 1) Image Acquisition and Preprocessing:

1. Consider a digital image $I(x, y)$.
2. If the input image is a color image, convert it into its grayscale representation $I_g(x, y)$.
3. Convert pixel intensity values into double precision format to ensure numerical stability.
4. Compute the statistical parameters:

$$I_{\max} = \max(I_g), \quad I_{\min} = \min(I_g), \tag{21}$$

$$I_{\text{mid}} = \frac{I_{\max} + I_{\min}}{2}. \tag{22}$$

(Step 2) Construction of Intuitionistic Fuzzy Representation:

The grayscale image is transformed into the intuitionistic fuzzy domain by defining the membership function $u(x, y)$ using an S-shaped nonlinear function derived from image intensity statistics. This ensures smooth transition between dark and bright regions. The non-membership function is defined as:

$$v(x, y) = \lambda(1 - u(x, y)), \quad 0 < \lambda < 1, \tag{23}$$

where λ is a regulating parameter. The hesitation degree is computed as:

$$\pi(x, y) = 1 - u(x, y) - v(x, y). \tag{24}$$

(Step 3) Intuitionistic fuzzy entropy and Local Dispersion:

The proposed intuitionistic fuzzy entropy is computed at each pixel to generate an entropy image. Then a 3×3 sliding window is applied to evaluate local entropy variation. Edge strength is calculated using local entropy dispersion. Regions exhibiting high entropy dispersion correspond to significant uncertainty transitions and are identified as potential edge locations.

(Step 4) Adaptive Thresholding:

A threshold value calculated by using the mean and standard deviation of the entropy variation image as follows:

$$T = \mu_D + \sigma_D, \tag{25}$$

where μ_D and σ_D denote the mean, standard deviation of the dispersion image, respectively. All these image pixels which satisfy $D(x, y) > T$ are classified as edge pixels which results in final binary edge map.

4.2. Experimental setup

All experimental results were obtained using MATLAB (R2021a) software (MathWorks Inc., USA). The input images are converted to grayscale images using Image Processing Toolbox available in MATLAB. Grayscale conversion is performed because it has a single intensity channel, whereas identifying true edges in color images can be more difficult The experiments were performed on a Windows 10 system, Intel Core i3 processor and RAM 6 GB.

The proposed methodology generates several intermediate representations, including the grayscale image, membership image, non-membership image, entropy image, and the final edge map. The membership image is obtained by applying the S-shaped membership function to the grayscale image, whereas the non-membership image is computed as a scaled complement of the corresponding membership values. The performance of the proposed parametric intuitionistic fuzzy entropy measure was evaluated using a variety of benchmark and real-world photographic images. Representative results for the Cameraman, Lena, Cell, Rice Grain, Man, Saturn, Coin, Cake, and Butterfly images are presented in Figures 5 to 13. The experimental results demonstrate that the proposed entropy measure effectively detects significant and well-defined edges. The images are not normalized to $[0, 1]$. All the pixel values are maintained in the standard 8-bit integer range $[0, 255]$. To assess the performance of the proposed measure quantitatively, the Mean Squared Error (MSE), Peak Signal-to-Noise Ratio (PSNR) Precision, Recall, F measure and Pratt's Figure of Merit (FOM) were computed. These metrics provide an objective evaluation of the quality and accuracy of the detected edges. The mathematical expressions used for calculating performance measures are given below:

$$\text{Peak Signal} - \text{Noise Ratio (PSNR)} = 10 \, \log\left(\frac{255^2}{MSE}\right), \tag{26}$$

where MSE is calculated between the original grayscale image x (pixel range $[0, 255]$), the corresponding edge detected image y (pixel range [0,255]) by applying proposed measure and $M \times N$ is the size of an image

$$\text{Mean Square Error (MSE)} = \frac{\sum_{j=1}^{N} (\sum_{i=1}^{M} (x_{ij} - y_{ij})^2)}{MN}. \tag{27}$$

When a large proportion of pixels differ between the grayscale image and the corresponding binary edge map, the Mean Squared Error (MSE) becomes a significant fraction of (255^2), resulting in relatively low PSNR values, typically in the range of (3)–(13) dB. Unlike image compression or reconstruction tasks, where PSNR is used to evaluate the accuracy of reconstruction with respect to the original image, in edge detection it serves primarily as a relative performance metric for comparing different algorithms.

Table 2 represents the quantitative comparison in terms of MSE and PSNR. The results indicate that the proposed method achieves competitive performance across all test images. In particular, for the Cell, Cake and Butterfly images, the proposed approach produces lower MSE and higher PSNR values than the Canny operator, suggesting improved edge preservation and enhanced image quality. Furthermore, its performance is comparable to that of the Sobel and Prewitt operators. The proposed method does not assume the existence of ground-truth binary edge maps, since it operates on real photographic images (Person Photo, Cake, Coin, Saturn, Butterfly) for which standard ground-truth annotations are not available. MSE and PSNR are used here as *image-fidelity measures* that quantify how closely the edge map preserves the structural information of the original image. A lower MSE and higher PSNR for the proposed method imply that the extracted edges introduce less distortion relative to the image content.

This usage is consistent with prior works in fuzzy entropy-based edge detection that compare different operators in the absence of ground-truth data [29]. For standard benchmark images where ground-truth edge maps are available (Cameraman, Lena, and Cell images from the BSDS500 dataset), we have now computed Precision, Recall, and F-measure. These are defined as follows:

$$\text{Precision} = \frac{TP}{TP + FP}, \tag{28}$$

$$\text{Recall} = \frac{TP}{TP + FN}, \tag{29}$$

$$F\text{-measure} = \frac{2 \times \text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}, \tag{30}$$

where TP, FP, and FN denote true positives, false positives, and false negatives, respectively, with respect to the ground-truth edge map.

$$\text{Pratt's Figure of Merit (FOM)} = \frac{1}{\max(N_I, N_A)} \sum_{k=1}^{N_A} \frac{1}{1 + \alpha\, d_k^2}, \tag{31}$$

where N_I is the number of ideal (ground-truth) edge pixels, N_A is the number of actual detected edge pixels, d_k is the distance of the k-th detected edge pixel from the nearest ideal edge pixel, and $\alpha = 1/9$ is a scaling constant.

The updated quantitative results (with $m = 0.8$ for our proposed method) on the three benchmark images with available ground-truth are presented in Table 3.

These results confirm that the proposed parametric intuitionistic fuzzy entropy measure achieves superior or competitive performance across all four ground-truth-based metrics, particularly for the Cell image where structural boundary localization is critical. The proposed method consistently outperforms the Canny operator on all three images, and matches or exceeds Sobel and Prewitt on F-measure and FOM.

The parameter $m \in (0, 2)$ controls the *sensitivity* of the proposed entropy $H_m(P)$ to uncertainty in the intuitionistic fuzzy representation:

- For $m \to 0$: the term $(2 - m) \to 2$, placing greater weight on extreme membership values ($u \approx 1$ or $u \approx 0$), making the measure *more sensitive* and better suited to high-contrast images with clear intensity boundaries.
- For $m = 1$: the measure reduces to the Zhang–Jiang entropy (the standard intuitionistic fuzzy Shannon entropy), suitable for moderate-contrast images with balanced uncertainty.
- For $m \to 2$: the term $(2 - m) \to 0$, suppressing the power weighting and distributing entropy more uniformly, making the measure *less sensitive* and robust for low-contrast or highly textured images where strong hesitation persists across most pixels.

Based on the above analysis and empirical validation, we propose the following practical guideline for selecting m:

Sensitivity analysis : To quantitatively demonstrate the role of m, we obtain F-measure values across a range of m for the three benchmark images in Table 4.

The sensitivity results in Table 5 confirm the following:

1. For the Cameraman, Lena, and Cell images (moderate-contrast), $m = 0.8$ consistently yields the highest F-measure, validating the recommendation of $m \in [0.8, 1.0]$ for such images.
2. For the Saturn image (high-contrast with sharp rings against a black background), smaller $m = 0.2$ yields the best edge detection performance, consistent with the recommendation of $m \in [0.2, 0.6]$ for high-contrast images.
3. For the Rice Grain image (textured, low-contrast), larger $m = 1.2$ gives the best result, supporting the recommendation of $m \in [1.2, 1.6]$.

5. Conclusion

In this paper, we have proposed a novel parametric entropy measure for IFSs and established its mathematical validity. Several fundamental properties of the measure were examined, numerical and graphical illustrations were provided to demonstrate the flexibility of proposed measure with respect to parameter variations. Comparative study for the proposed measures with some existing measures in IFSs is carried out. The effectiveness of the proposed method has been demonstrated using some benchmark and real photographic images. Experimental results reveal that the proposed entropy-based framework is capable of accurately identifying significant edge structures while effectively suppressing noise-induced distortions and preserving structural continuity. A comparative analysis with conventional edge detection techniques, namely Canny, Sobel, and Prewitt operators, indicates that the proposed method provides competitive performance and, for several images, exhibits superior edge localization characteristics. Quantitative evaluation based on MSE and PSNR further confirms the robustness and effectiveness of the proposed approach. Therefore, the proposed parametric intuitionistic fuzzy entropy measure constitutes a promising alternative to conventional gradient-based edge detection methods. Future research may focus on extending the proposed framework to color images, medical image analysis, remote sensing applications, and edge detection under highly noisy environments. Furthermore, the integration of the proposed entropy measure with deep learning and optimization techniques may provide enhanced performance for complex image processing applications.

Data availability

All the data used in the work are already contained herein. There is no other data about this work to share.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this manuscript.

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