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Weibull-based reliability and profitability analysis of an industrial multi-state repairable system

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Abstract

Reliability is a primary concern in industrial manufacturing because operational interruptions can reduce production, cause substantial economic losses, and create workplace-safety risks. This study applies the two-parameter Weibull distribution and the regenerative point technique (RPT) to a repairable system comprising two non-identical units: a primary unit operating in parallel with a secondary unit arranged in cold standby. The model incorporates inspection, repair, and replacement activities and is used to derive transition probabilities, mean sojourn times, mean time to system failure (MTSF), steady-state availability, maintenance busy periods, and a profit function. Numerical results show that system profitability decreases as the failure rate and Weibull shape parameter increase, whereas a higher repair rate improves profitability. For the reported parameter set, profit increases from 7325.258 to 8726.043 when the repair rate rises from 0.1 to 1.0 at $\alpha = 0.01$, and from 4879.802 to 6970.602 at $\alpha = 0.05$. The analysis also identifies reported zero-profit thresholds near $\alpha = 1.45$, 1.38, and 1.21 for $k = 0.8$, 1.0, and 2.0, respectively. The framework supports maintenance planning and economic assessment of multi-state repairable systems.

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1. Introduction

Industrial manufacturing systems must operate reliably to minimize downtime and economic losses while maintaining safe production environments. Reliability analysis provides mathematical and statistical tools for studying system failure and repair processes, thereby helping engineers design architectures that maximize availability and minimize maintenance costs. Real-world repairable systems commonly comprise several components operating in parallel or standby modes, are subject to random failures, and depend on limited repair facilities.

The Weibull distribution is a flexible model that can represent increasing, decreasing, or constant failure rates. This flexibility has made it a widely used distribution in modern reliability analysis.

Previous studies have extended reliability modelling by applying the Weibull distribution and other non-exponential distributions to complex systems. Jayatilleka *et al.* [1] used accelerated life tests to predict product life and identify robust designs. Cui [2] and Zhang and Geng [3] considered exponential failure and repair rates. Although this assumption simplifies the derivation of reliability measures, it does not represent age-dependent failure rates. Charruau *et al.* [4] highlighted the use of accelerated testing for reliability estimation [5].

Statistical inference and Bayesian estimation for acceler-

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ated life testing have been examined by Casella and Berger [6] and Voiculescu *et al.* [7]. Jung *et al.* [8] investigated crankshaft fatigue-life reliability. Zacks [9] and Scott [10] presented probability and stochastic-process frameworks relevant to repairable-system analysis. Gupta and Gupta [11] studied post-inspection, repair, preventive maintenance, and replacement. Berrade *et al.* [12] examined imperfect inspection and replacement, and Gupta and Gupta [13] subsequently considered maintenance-cost and replacement optimization.

The Weibull distribution has also been applied to energy systems and mechanical components. Azad *et al.* [14] analysed wind-energy conversion, while Kishan and Jain [15] developed a two-unit parallel-system model with Weibull failure and repair laws. Doostparast *et al.* [16] incorporated preventive-maintenance scheduling into coherent-system reliability models, and Singh and Taneja [17] analysed a turbine system with random inspection.

Kakkar *et al.* [18] applied availability analysis to a system with unit replacement. Salomon *et al.* [19] considered imprecision in complex-system reliability analysis. Bhatti *et al.* [20–23] analysed standby and redundant systems under different failure and repair assumptions. Reliability, availability, maintainability, and dependability (RAMD) analysis has been applied to photovoltaic systems by Maihulla and Yusuf [24]. Mellal and Salhi [25] and Pang *et al.* [26] examined reliability under allocation, inspection, repair, and dynamic-loading policies. Alamri and Mo [27] considered preventive-maintenance optimization. Kaur and Sharma [28], Malhotra [29], Pankaj *et al.* [30], and Khanday *et al.* [31] further studied reliability and availability in standby or technical systems. Bhardwaj *et al.* [32] integrated preventive maintenance and server-arrival time into the estimation of reliability characteristics for a repairable system.

This study investigates the reliability and profitability of a system comprising two non-identical units with distinct failure rates and repair mechanisms. The regenerative point technique is used to derive system-performance measures under Weibull-distributed transition times.

2. System model description

To model time-dependent degradation, wear-out, and maintenance-restoration behaviour in the multi-state repairable system, component operating lifetimes (times to failure) and maintenance durations (times to repair or replacement) are represented by two-parameter Weibull distributions.

For a continuous Weibull-distributed random variable T, the probability density function (PDF), $f(t)$, and survival function, $R(t)$, are

$$f(t) = k\lambda^k t^{k-1} e^{-(\lambda t)^k}, \quad t \geq 0, \quad \lambda > 0, \quad k > 0, \tag{1}$$

and

$$R(t) = e^{-(\lambda t)^k}, \quad t \geq 0, \tag{2}$$

where k is the shape parameter and λ is the Weibull rate parameter.

The model is based on the following assumptions:

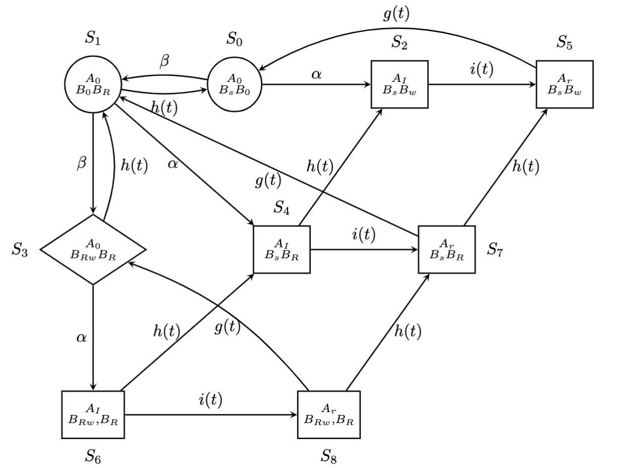

Figure 1: State-transition diagram of a two-unit non-identical cold-standby redundant system incorporating inspection, repair, and replacement policies.

- The system contains two non-identical units, A and B, with failure rates α and β, respectively.
- Complete system failure occurs only when both units are non-operational.
- One maintenance facility performs inspections, repairs, and replacements at rates i, r, and R, respectively.
- Repaired or replaced units are considered as good as new; consequently, each relevant state functions as a regenerative point.
- Component times to failure follow Weibull distributions, while repair, replacement, and inspection durations follow their respective distributions.

The nine-state system shown in Figure 1 is categorized as follows:

- Fully operational upstates (S_0, S_1):
  - State S_0 represents the baseline configuration in which unit A is fully operational, one unit B is operating, and the second unit B is idle in cold standby.
  - State S_1 is an upstate because unit A remains online and full production continues while the standby unit B replaces the failed unit B. The system has no redundancy in this state because the failed unit B is undergoing replacement.
- Degraded upstate (S_3):
  - In state S_3, unit A remains operational. Unit B has failed and is waiting for the repairman; therefore, the system operates in a non-redundant, degraded condition.
- Non-operative downstates ($S_2, S_4, S_5, S_6, S_7, S_8$):

Table 1: Symbols and abbreviations used in the model.

Symbol/ abbreviation	Description
PDF	Probability density function
CDF	Cumulative distribution function
RPT	Regenerative point technique
MTSF/MTBF	Mean time to system failure/mean time between failures
$A(t)$	System availability at time t
$R_i(t)$	Reliability of the system at time t when starting from state S_i
$A_i(t)$	Probability that the system is operative in state S_i at epoch t
$I_i(t)$	Probability that inspection is in progress in state S_i at epoch t
$\phi_i(t)$	Probability that the repairman is busy in state S_i at epoch t
μ_i	Mean sojourn time in state S_i
$P(t)$	Profit incurred by the system during $(0,t)$
α, β	Failure rates of units A and B, respectively
r, R	Repair and replacement rates of units A and B, respectively
k	Shape parameter of the Weibull distribution
$\Gamma(\cdot)$	Gamma function
$q_{ij}(t)$	Probability density of transition from state S_i to S_j in time t
$q^*_{ij}(s)$	Laplace transform of $q_{ij}(t)$
$Z_i(t)$	Probability that the system remains in state S_i up to time t
$g(t)$	Transition-rate function from repair of unit A to the operative state
$h(t)$	Transition-rate function from replacement to the operative state
$i(t)$	Inspection-rate function before repair

- These states represent system downtime, during which production is zero because of maintenance interventions or multiple-unit failures.
- State S_2 represents an inspection period initiated from S_0, during which operational activity is suspended for diagnostic assessment of unit A or B; unit B is in standby mode.
- States S_4 and S_6 represent inspection, waiting, or queuing configurations in which failed units await inspection, repair, or replacement.
- States S_5, S_7, and S_8 represent active repair of unit A or B by the maintenance staff.

2.1. Notation and description of states

The states in Figure 1 are described as follows:

A_0: Operative state—Unit A is fully operational and performing its intended function.

A_I: Inspection state—Unit A is under inspection at rate i to determine its condition before repair.

A_r: Repair state—Unit A is undergoing active repair at rate r.

B_0: Operative state—Unit B is functioning normally and supporting the system.

B_R: Replacement state—Unit B is undergoing replacement at rate R after failure.

B_s: Standby state—Unit B is in standby mode and is ready to operate when required.

B_{Rw}: Waiting state—Unit B has failed and is waiting for repair or replacement.

2.2. Abbreviations and acronyms

The symbols and abbreviations used in the analysis are listed in Table 1.

3. Transition probabilities and mean sojourn times

3.1. Transition probabilities

The transition probability p_{ij} is the probability that the system leaves state S_i and next enters state S_j.

For example, p_{01} is obtained from

$$p_{01} = [\text{probability that unit B fails during } (t, t+\mathrm{d}t) \text{ and unit A survives up to time } t]\,\mathrm{d}t.$$

Integrating over all possible values of t gives

$$p_{01} = \int_0^\infty f_B(t) R_A(t)\,\mathrm{d}t, \tag{3}$$

where $f_B(t)$ is the PDF of the failure time of unit B and $R_A(t)$ is the survival function of unit A. Substitution of the Weibull PDF for unit B, with rate parameter β, and the survival function for unit A, with rate parameter α, gives

$$p_{01} = \int_0^\infty \left(k\beta^k t^{k-1} e^{-(\beta t)^k}\right)\left(e^{-(\alpha t)^k}\right)\mathrm{d}t,$$

and therefore

$$p_{01} = \int_0^\infty k\beta^k t^{k-1} e^{-(\alpha^k+\beta^k)t^k}\,\mathrm{d}t.$$

Let $u = (\alpha^k + \beta^k)t^k$. Then

$$\mathrm{d}u = k(\alpha^k+\beta^k)t^{k-1}\mathrm{d}t, \qquad kt^{k-1}\mathrm{d}t = \frac{\mathrm{d}u}{\alpha^k+\beta^k}.$$

Using the limits $t = 0 \Rightarrow u = 0$ and $t \to \infty \Rightarrow u \to \infty$ gives

$$p_{01} = \frac{\beta^k}{\alpha^k+\beta^k}\int_0^\infty e^{-u}\,\mathrm{d}u.$$

Since $\int_0^\infty e^{-u}\,\mathrm{d}u = 1$,

$$p_{01} = \frac{\beta^k}{\alpha^k + \beta^k}.$$

Similarly, using the Weibull density functions for the failure and maintenance transitions shown in Figure 1, the non-zero transition probabilities are

$$\begin{aligned}
p_{01} &= \frac{\beta^k}{\alpha^k+\beta^k}, & p_{02} &= \frac{\alpha^k}{\alpha^k+\beta^k},\\
p_{10} &= \frac{R^k}{\alpha^k+\beta^k+R^k}, & p_{13} &= \frac{\beta^k}{\alpha^k+\beta^k+R^k},\\
p_{14} &= \frac{\alpha^k}{\alpha^k+\beta^k+R^k}, & p_{25} &= 1,\\
p_{31} &= \frac{R^k}{\alpha^k+R^k}, & p_{36} &= \frac{\alpha^k}{\alpha^k+R^k}, \qquad (4)\\
p_{42} = p_{64} &= \frac{R^k}{i^k+R^k}, & p_{47} = p_{68} &= \frac{i^k}{i^k+R^k},\\
p_{50} &= 1, & p_{71} = p_{83} &= \frac{r^k}{R^k+r^k},\\
p_{75} = p_{87} &= \frac{R^k}{R^k+r^k}.
\end{aligned}$$

These probabilities satisfy

$$\begin{aligned}
&p_{01}+p_{02}=1, \quad p_{10}+p_{13}+p_{14}=1, \quad p_{25}=1,\\
&p_{31}+p_{36}=1, \quad p_{42}+p_{47}=1, \quad p_{50}=1,\\
&p_{64}+p_{68}=1, \quad p_{71}+p_{75}=1, \quad p_{83}+p_{87}=1.
\end{aligned}$$

For $k = 1$, the Weibull distribution reduces to the exponential distribution; for $k \neq 1$, it represents time-dependent ageing effects.

3.2. Mean sojourn times

The mean sojourn time μ_i in state S_i is the expected time spent in that state before a transition to another state. If $f_i(t)$ is the Weibull PDF in state i, then

$$\mu_i = \int_0^\infty t f_i(t)\,\mathrm{d}t = \int_0^\infty P(T_i > t)\,\mathrm{d}t. \qquad (5)$$

For state S_i, this quantity can be interpreted as the probability that the operating units survive up to time t, integrated over time. The submitted formulation gives

$$\boxed{\mu_i = \frac{\Gamma\left(1+\frac{1}{k}\right)}{\Lambda_i^{1/k}}},$$

where Λ_i can be explicitly defined as $\Lambda_i = \sum_j (\lambda_{ij})^k$, and λ_{ij} denotes the total transition rate factor out of state Si.

Hence,

$$\begin{aligned}
\mu_0 &= \frac{\Gamma\left(1+\frac{1}{k}\right)}{(\alpha^k+\beta^k)^{1/k}}, & \mu_1 &= \frac{\Gamma\left(1+\frac{1}{k}\right)}{(\alpha^k+\beta^k+R^k)^{1/k}},\\
\mu_2 &= \frac{\Gamma\left(1+\frac{1}{k}\right)}{i^{1/k}}, & \mu_3 &= \frac{\Gamma\left(1+\frac{1}{k}\right)}{(\alpha^k+R^k)^{1/k}},
\end{aligned}$$

Table 2: Reliability metrics and profit for $k = 0.8$ at different failure rates α (K_1 = 10,000, K_2 = 5,000, K_3 = 500, K_4 = 400, and K_5 = 600).

α	MTSF	A_0	P_0	I_0	B_0	ϕ_0	Profit
0.01	293.86	0.923	0.012	0.02	0.03	0.05	9240.573
0.10	83.99	0.614	0.055	0.14	0.18	0.24	6129.964
0.20	31.82	0.439	0.069	0.21	0.27	0.33	4325.827
0.30	16.94	0.343	0.071	0.26	0.32	0.38	3300.551
0.40	10.81	0.281	0.069	0.28	0.35	0.40	2635.284
0.50	7.66	0.239	0.066	0.31	0.38	0.41	2164.179

$$\begin{aligned}
\mu_4 &= \frac{\Gamma\left(1+\frac{1}{k}\right)}{(i^k+R^k)^{1/k}}, & \mu_5 &= \frac{\Gamma\left(1+\frac{1}{k}\right)}{r^{1/k}},\\
\mu_6 &= \frac{\Gamma\left(1+\frac{1}{k}\right)}{(i^k+R^k)^{1/k}}, & \mu_7 &= \frac{\Gamma\left(1+\frac{1}{k}\right)}{(R^k+r^k)^{1/k}},\\
\mu_8 &= \frac{\Gamma\left(1+\frac{1}{k}\right)}{(R^k+r^k)^{1/k}}.
\end{aligned}$$

4. Reliability and mean time to system failure

The mean time to system failure (MTSF) is the expected operating time from the initial state to the first system failure. Let T_i be the time to system failure when the system starts in state S_i. The reliability is

$$R_i(t) = P(T_i > t). \qquad (6)$$

To determine reliability, failed states are treated as absorbing states. In the recursive formulation submitted by the authors, $M_i(t)$ denotes the reliability quantity associated with state S_i. The relations for $i = 0, 1, 3$ are

$$M_0(t) = Z_0(t) + q_{01}(t) * M_1(t) + q_{02}(t), \qquad (7)$$

$$M_1(t) = Z_1(t) + q_{10}(t) * M_0(t) + q_{13}(t) * M_3(t) + q_{14}(t), \qquad (8)$$

$$M_3(t) = Z_3(t) + q_{31}(t) * M_1(t) + q_{36}(t). \qquad (9)$$

The MTSF is obtained in the transform domain as

$$\text{MTSF} = E(T_0) = \lim_{s\to 0} M_0^*(s), \qquad (10)$$

which yields

$$M_0 = \frac{\mu_0(1 - p_{13}p_{31}) + p_{01}\mu_1 + p_{01}p_{13}\mu_3}{1 - p_{10}p_{01} - p_{13}p_{31}}. \qquad (11)$$

5. Availability analysis

Availability is the probability that a system is operating at a specified time under stated environmental conditions.

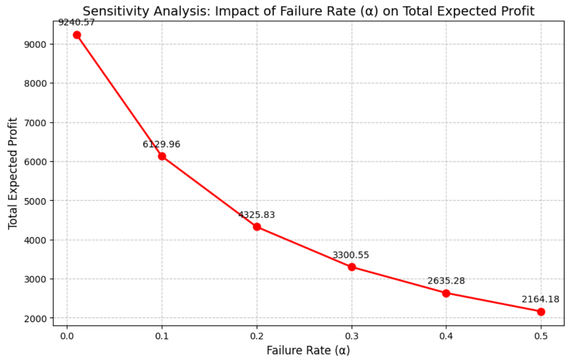

Figure 2: Sensitivity of total expected profit to the failure rate α.

Table 3: Reliability metrics and profit at different Weibull shape parameters k.

k	A_0	P_0	I_0	B_0	ϕ_0	Profit
0.5	0.993	0.001	0.002	0.003	0.003	9934.982
1.0	0.838	0.026	0.062	0.073	0.114	8386.884
1.50	0.632	0.058	0.146	0.162	0.280	6309.213
2.0	0.504	0.077	0.201	0.216	0.390	5013.841
2.50	0.431	0.087	0.233	0.247	0.457	4260.347
3.0	0.385	0.093	0.254	0.266	0.499	3792.162

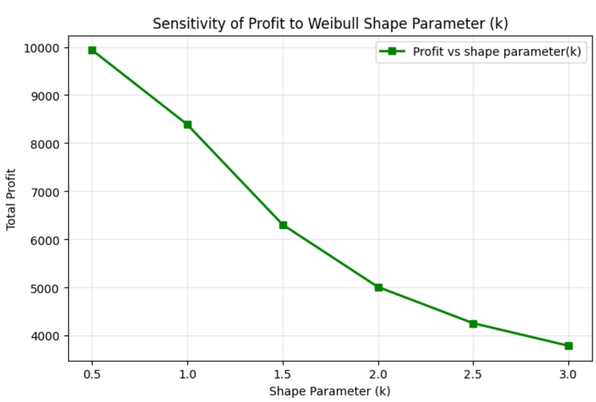

Figure 3: Effect of the Weibull shape parameter k on system profitability.

Table 4: System profit at different repair rates r for $\alpha = 0.01$.

r	A_0	P_0	I_0	B_0	ϕ_0	Profit
0.1	0.7381	0.0234	0.0546	0.1839	0.1134	7325.258
0.2	0.7993	0.0254	0.0591	0.1162	0.1172	7971.869
0.4	0.8383	0.0266	0.0620	0.0731	0.1148	8386.884
0.6	0.8545	0.0271	0.0632	0.0552	0.1115	8559.512
0.8	0.8638	0.0274	0.0639	0.0448	0.1088	8658.327
1.0	0.8701	0.0276	0.0644	0.0379	0.1065	8726.043

5.1. Steady-state availability in the fully operational state

Let $A_i(t)$ be the probability that the system is in a fully operational upstate at time t, given that it started in state S_i at $t = 0$.

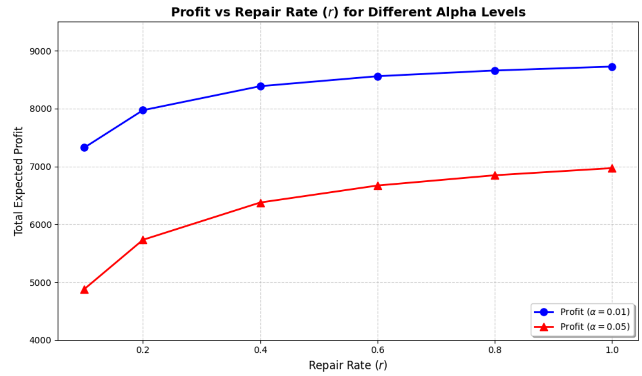

Figure 4: Sensitivity of system profit to the repair rate r at low and high failure rates.

Table 5: System profit at different repair rates r for $\alpha = 0.05$.

r	A_0	P_0	I_0	B_0	ϕ_0	Profit
0.1	0.499	0.042	0.104	0.353	0.212	4879.802
0.2	0.585	0.049	0.123	0.241	0.237	5731.104
0.4	0.648	0.054	0.136	0.160	0.245	6376.001
0.6	0.676	0.057	0.142	0.124	0.243	6669.703
0.8	0.693	0.058	0.145	0.102	0.240	6847.205
1.0	0.705	0.059	0.148	0.087	0.237	6970.602

The recursive relations for the fully operational states S_0 and S_1 in Figure 1 are

$$A_0(t) = Z_0(t) + q_{01}(t) * A_1(t) + q_{02}(t) * A_2(t), \tag{12}$$

$$A_1(t) = Z_1(t) + q_{10}(t) * A_0(t) + q_{13}(t) * A_3(t) + q_{14}(t) * A_4(t), \tag{13}$$

$$A_2(t) = q_{25}(t) * A_5(t), \tag{14}$$

$$A_3(t) = q_{31}(t) * A_1(t) + q_{36}(t) * A_6(t), \tag{15}$$

$$A_4(t) = q_{42}(t) * A_2(t) + q_{47}(t) * A_7(t), \tag{16}$$

$$A_5(t) = q_{50}(t) * A_0(t), \tag{17}$$

$$A_6(t) = q_{64}(t) * A_4(t) + q_{68}(t) * A_8(t), \tag{18}$$

$$A_7(t) = q_{71}(t) * A_1(t) + q_{75}(t) * A_5(t), \tag{19}$$

$$A_8(t) = q_{83}(t) * A_3(t) + q_{87}(t) * A_7(t). \tag{20}$$

Let $A_i^*(s) = \mathcal{L}\{A_i(t)\}$ and $q_{ij}^*(s) = \mathcal{L}\{q_{ij}(t)\}$. In the steady state, $s \to 0$, $q_{ij}^*(0) = p_{ij}$, and $Z_i^*(0) = \mu_i$. The steady-state availability is

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_0}{D_0}, \tag{21}$$

where the submitted numerator is

$$\begin{aligned} N_0 = {} & \mu_0 + \mu_1 p_{01} - \mu_0 p_{13} p_{31} \\ & - \mu_0 p_{14} p_{47} p_{71} - \mu_0 p_{36} p_{68} p_{83} \\ & - \mu_1 p_{01} p_{36} p_{68} p_{83} \\ & - \mu_0 p_{13} p_{36} p_{47} p_{64} p_{71} \\ & - \mu_0 p_{13} p_{36} p_{68} p_{71} p_{87} \end{aligned}$$

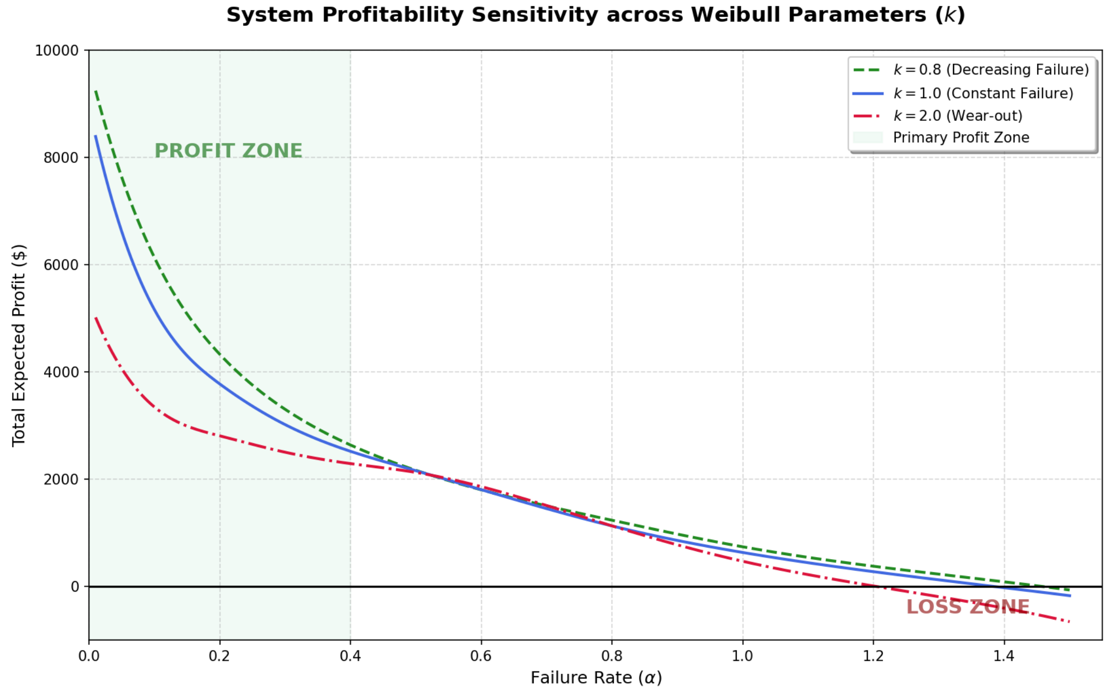

Figure 5: Profit and loss zones in the Weibull profitability model, with the reported cutoff points.

$$+\mu_0 p_{14} p_{36} p_{47} p_{68} p_{71} p_{83}.$$

The submitted fully expanded denominator, representing the mean cycle time, is

$$
\begin{aligned}
D_0 = {} & \mu_0 p_{02} p_{13} p_{25} p_{31} p_{50} - \mu_1 p_{01} p_{10} - \mu_1 p_{13} p_{31} - \mu_3 p_{13} p_{31} \\
& - \mu_0 p_{02} p_{25} p_{50} - \mu_2 p_{02} p_{25} p_{50} - \mu_5 p_{02} p_{25} p_{50} \\
& - \mu_1 p_{14} p_{47} p_{71} - \mu_4 p_{14} p_{47} p_{71} \\
& - \mu_7 p_{14} p_{47} p_{71} - \mu_3 p_{36} p_{68} p_{83} - \mu_6 p_{36} p_{68} p_{83} \\
& - \mu_8 p_{36} p_{68} p_{83} - \mu_0 p_{01} p_{10} + \mu_1 p_{02} p_{13} p_{25} p_{31} p_{50} \\
& + \mu_2 p_{02} p_{13} p_{25} p_{31} p_{50} + \mu_3 p_{02} p_{13} p_{25} p_{31} p_{50} \\
& + \mu_5 p_{02} p_{13} p_{25} p_{31} p_{50} - \mu_0 p_{01} p_{14} p_{25} p_{42} p_{50} \\
& - \mu_1 p_{01} p_{14} p_{25} p_{42} p_{50} - \mu_2 p_{01} p_{14} p_{25} p_{42} p_{50} \\
& - \mu_4 p_{01} p_{14} p_{25} p_{42} p_{50} - \mu_5 p_{01} p_{14} p_{25} p_{42} p_{50} \\
& - \mu_0 p_{01} p_{14} p_{47} p_{50} p_{75} - \mu_1 p_{01} p_{14} p_{47} p_{50} p_{75} \\
& - \mu_4 p_{01} p_{14} p_{47} p_{50} p_{75} - \mu_5 p_{01} p_{14} p_{47} p_{50} p_{75} \\
& - \mu_7 p_{01} p_{14} p_{47} p_{50} p_{75} + \mu_0 p_{01} p_{10} p_{36} p_{68} p_{83} \\
& + \mu_1 p_{01} p_{10} p_{36} p_{68} p_{83} + \mu_3 p_{01} p_{10} p_{36} p_{68} p_{83} \\
& + \mu_6 p_{01} p_{10} p_{36} p_{68} p_{83} + \mu_8 p_{01} p_{10} p_{36} p_{68} p_{83} \\
& - \mu_1 p_{13} p_{36} p_{47} p_{64} p_{71} - \mu_3 p_{13} p_{36} p_{47} p_{64} p_{71} \\
& - \mu_4 p_{13} p_{36} p_{47} p_{64} p_{71} - \mu_6 p_{13} p_{36} p_{47} p_{64} p_{71} \\
& - \mu_7 p_{13} p_{36} p_{47} p_{64} p_{71} - \mu_1 p_{13} p_{36} p_{68} p_{71} p_{87} \\
& - \mu_3 p_{13} p_{36} p_{68} p_{71} p_{87} - \mu_6 p_{13} p_{36} p_{68} p_{71} p_{87} \\
& - \mu_7 p_{13} p_{36} p_{68} p_{71} p_{87} - \mu_8 p_{13} p_{36} p_{68} p_{71} p_{87} \\
& + \mu_0 p_{02} p_{14} p_{25} p_{47} p_{50} p_{71} + \mu_1 p_{02} p_{14} p_{25} p_{47} p_{50} p_{71} \\
& + \mu_2 p_{02} p_{14} p_{25} p_{47} p_{50} p_{71} + \mu_4 p_{02} p_{14} p_{25} p_{47} p_{50} p_{71} \\
& + \mu_5 p_{02} p_{14} p_{25} p_{47} p_{50} p_{71} + \mu_7 p_{02} p_{14} p_{25} p_{47} p_{50} p_{71} \\
& + \mu_0 p_{02} p_{25} p_{36} p_{50} p_{68} p_{83} + \mu_2 p_{02} p_{25} p_{36} p_{50} p_{68} p_{83} \\
& + \mu_3 p_{02} p_{25} p_{36} p_{50} p_{68} p_{83} + \mu_5 p_{02} p_{25} p_{36} p_{50} p_{68} p_{83} \\
& + \mu_6 p_{02} p_{25} p_{36} p_{50} p_{68} p_{83} + \mu_8 p_{02} p_{25} p_{36} p_{50} p_{68} p_{83} \\
& + \mu_1 p_{14} p_{36} p_{47} p_{68} p_{71} p_{83} + \mu_3 p_{14} p_{36} p_{47} p_{68} p_{71} p_{83} \\
& + \mu_4 p_{14} p_{36} p_{47} p_{68} p_{71} p_{83} + \mu_6 p_{14} p_{36} p_{47} p_{68} p_{71} p_{83} \\
& + \mu_7 p_{14} p_{36} p_{47} p_{68} p_{71} p_{83} + \mu_8 p_{14} p_{36} p_{47} p_{68} p_{71} p_{83} \\
& - \mu_0 p_{01} p_{13} p_{25} p_{36} p_{42} p_{50} p_{64} - \mu_1 p_{01} p_{13} p_{25} p_{36} p_{42} p_{50} p_{64} \\
& - \mu_2 p_{01} p_{13} p_{25} p_{36} p_{42} p_{50} p_{64} - \mu_3 p_{01} p_{13} p_{25} p_{36} p_{42} p_{50} p_{64} \\
& - \mu_4 p_{01} p_{13} p_{25} p_{36} p_{42} p_{50} p_{64} - \mu_5 p_{01} p_{13} p_{25} p_{36} p_{42} p_{50} p_{64} \\
& - \mu_6 p_{01} p_{13} p_{25} p_{36} p_{42} p_{50} p_{64} - \mu_0 p_{01} p_{13} p_{36} p_{47} p_{50} p_{64} p_{75} \\
& - \mu_1 p_{01} p_{13} p_{36} p_{47} p_{50} p_{64} p_{75} - \mu_3 p_{01} p_{13} p_{36} p_{47} p_{50} p_{64} p_{75} \\
& - \mu_4 p_{01} p_{13} p_{36} p_{47} p_{50} p_{64} p_{75} - \mu_5 p_{01} p_{13} p_{36} p_{47} p_{50} p_{64} p_{75} \\
& - \mu_6 p_{01} p_{13} p_{36} p_{47} p_{50} p_{64} p_{75} - \mu_7 p_{01} p_{13} p_{36} p_{47} p_{50} p_{64} p_{75} \\
& - \mu_0 p_{01} p_{13} p_{36} p_{50} p_{68} p_{75} p_{87} - \mu_1 p_{01} p_{13} p_{36} p_{50} p_{68} p_{75} p_{87} \\
& - \mu_3 p_{01} p_{13} p_{36} p_{50} p_{68} p_{75} p_{87} - \mu_5 p_{01} p_{13} p_{36} p_{50} p_{68} p_{75} p_{87} \\
& - \mu_6 p_{01} p_{13} p_{36} p_{50} p_{68} p_{75} p_{87} - \mu_7 p_{01} p_{13} p_{36} p_{50} p_{68} p_{75} p_{87} \\
& - \mu_8 p_{01} p_{13} p_{36} p_{50} p_{68} p_{75} p_{87} \\
& + \mu_0 p_{02} p_{13} p_{25} p_{36} p_{47} p_{50} p_{64} p_{71} \\
& + \mu_1 p_{02} p_{13} p_{25} p_{36} p_{47} p_{50} p_{64} p_{71} \\
& + \mu_2 p_{02} p_{13} p_{25} p_{36} p_{47} p_{50} p_{64} p_{71} \\
& + \mu_3 p_{02} p_{13} p_{25} p_{36} p_{47} p_{50} p_{64} p_{71} \\
& + \mu_4 p_{02} p_{13} p_{25} p_{36} p_{47} p_{50} p_{64} p_{71} \\
& + \mu_5 p_{02} p_{13} p_{25} p_{36} p_{47} p_{50} p_{64} p_{71} \\
& + \mu_6 p_{02} p_{13} p_{25} p_{36} p_{47} p_{50} p_{64} p_{71} \\
& + \mu_7 p_{02} p_{13} p_{25} p_{36} p_{47} p_{50} p_{64} p_{71}
\end{aligned}
$$

Table 6: Reliability and economic metrics at different shape parameters k and failure rates α.

α	A_0	P_0	I_0	B_0	ϕ_0	Profit
$k = 0.8$						
0.01	0.9236	0.0127	0.0285	0.0352	0.0510	9240.573
0.10	0.6146	0.0554	0.1474	0.1825	0.2439	6129.964
0.20	0.4398	0.0697	0.2191	0.2714	0.3376	4325.827
0.30	0.3430	0.0718	0.2615	0.3238	0.3803	3300.551
0.40	0.2816	0.0698	0.2898	0.3589	0.4021	2635.284
0.50	0.2390	0.0663	0.3104	0.3843	0.4140	2164.179
0.80	0.1600	0.0500	0.3500	0.4300	0.4500	1233.002
1.0	0.1200	0.0400	0.3800	0.4600	0.4800	738.004
1.3	0.0800	0.0300	0.4200	0.5000	0.5200	228.001
1.45	0.0645	0.0223	0.4433	0.5233	0.5433	0.000
1.5	0.0600	0.0200	0.4500	0.5300	0.5500	-67.003
$k = 1.0$						
0.01	0.8383	0.0266	0.0620	0.0731	0.1148	8386.884
0.10	0.5186	0.0706	0.1886	0.2223	0.3287	5158.562
0.20	0.3853	0.0803	0.2453	0.2891	0.4049	3773.268
0.30	0.3144	0.0808	0.2776	0.3272	0.4383	3015.341
0.40	0.2690	0.0784	0.2995	0.3530	0.4556	2517.689
0.50	0.2367	0.0752	0.3158	0.3722	0.4656	2156.857
0.80	0.1500	0.0550	0.3600	0.4200	0.5000	1127.001
1.0	0.1100	0.0450	0.3900	0.4500	0.5300	632.003
1.3	0.0700	0.0350	0.4300	0.4900	0.5700	122.002
1.38	0.0617	0.0309	0.4425	0.5025	0.5825	0.000
1.5	0.0500	0.0250	0.4600	0.5200	0.6000	-173.004
$k = 2.0$						
0.01	0.5048	0.0775	0.2010	0.2167	0.3908	5013.841
0.10	0.3416	0.0972	0.2700	0.2912	0.5113	3343.738
0.20	0.2911	0.0988	0.2936	0.3165	0.5424	2806.162
0.30	0.2633	0.0976	0.3075	0.3316	0.5564	2500.767
0.40	0.2445	0.0956	0.3175	0.3424	0.5643	2288.709
0.50	0.2302	0.0934	0.3254	0.3509	0.5693	2124.356
0.80	0.1500	0.0700	0.3800	0.4000	0.6200	1128.003
1.0	0.1000	0.0500	0.4200	0.4400	0.6600	468.002
1.21	0.0645	0.0358	0.4484	0.4684	0.6884	0.000
1.3	0.0500	0.0300	0.4600	0.4800	0.7000	-192.004
1.5	0.0200	0.0100	0.5000	0.5200	0.7500	-658.001

$$
\begin{aligned}
&+ \mu_0 p_{01} p_{14} p_{25} p_{36} p_{42} p_{50} p_{68} p_{83} \\
&+ \mu_1 p_{01} p_{14} p_{25} p_{36} p_{42} p_{50} p_{68} p_{83} \\
&+ \mu_2 p_{01} p_{14} p_{25} p_{36} p_{42} p_{50} p_{68} p_{83} \\
&+ \mu_3 p_{01} p_{14} p_{25} p_{36} p_{42} p_{50} p_{68} p_{83} \\
&+ \mu_4 p_{01} p_{14} p_{25} p_{36} p_{42} p_{50} p_{68} p_{83} \\
&+ \mu_5 p_{01} p_{14} p_{25} p_{36} p_{42} p_{50} p_{68} p_{83} \\
&+ \mu_6 p_{01} p_{14} p_{25} p_{36} p_{42} p_{50} p_{68} p_{83} \\
&+ \mu_8 p_{01} p_{14} p_{25} p_{36} p_{42} p_{50} p_{68} p_{83} \\
&+ \mu_0 p_{02} p_{13} p_{25} p_{36} p_{50} p_{68} p_{71} p_{87} \\
&+ \mu_1 p_{02} p_{13} p_{25} p_{36} p_{50} p_{68} p_{71} p_{87} \\
&+ \mu_2 p_{02} p_{13} p_{25} p_{36} p_{50} p_{68} p_{71} p_{87} \\
&+ \mu_3 p_{02} p_{13} p_{25} p_{36} p_{50} p_{68} p_{71} p_{87} \\
&+ \mu_5 p_{02} p_{13} p_{25} p_{36} p_{50} p_{68} p_{71} p_{87} \\
&+ \mu_6 p_{02} p_{13} p_{25} p_{36} p_{50} p_{68} p_{71} p_{87} \\
&+ \mu_7 p_{02} p_{13} p_{25} p_{36} p_{50} p_{68} p_{71} p_{87} \\
&+ \mu_8 p_{02} p_{13} p_{25} p_{36} p_{50} p_{68} p_{71} p_{87} \\
&+ \mu_0 p_{01} p_{14} p_{36} p_{47} p_{50} p_{68} p_{75} p_{83} \\
&+ \mu_1 p_{01} p_{14} p_{36} p_{47} p_{50} p_{68} p_{75} p_{83} \\
&+ \mu_3 p_{01} p_{14} p_{36} p_{47} p_{50} p_{68} p_{75} p_{83} \\
&+ \mu_4 p_{01} p_{14} p_{36} p_{47} p_{50} p_{68} p_{75} p_{83} \\
&+ \mu_5 p_{01} p_{14} p_{36} p_{47} p_{50} p_{68} p_{75} p_{83} \\
&+ \mu_6 p_{01} p_{14} p_{36} p_{47} p_{50} p_{68} p_{75} p_{83} \\
&+ \mu_7 p_{01} p_{14} p_{36} p_{47} p_{50} p_{68} p_{75} p_{83} \\
&+ \mu_8 p_{01} p_{14} p_{36} p_{47} p_{50} p_{68} p_{75} p_{83} \\
&- \mu_0 p_{02} p_{14} p_{25} p_{36} p_{47} p_{50} p_{68} p_{71} p_{83} \\
&- \mu_1 p_{02} p_{14} p_{25} p_{36} p_{47} p_{50} p_{68} p_{71} p_{83} \\
&- \mu_2 p_{02} p_{14} p_{25} p_{36} p_{47} p_{50} p_{68} p_{71} p_{83} \\
&- \mu_3 p_{02} p_{14} p_{25} p_{36} p_{47} p_{50} p_{68} p_{71} p_{83} \\
&- \mu_4 p_{02} p_{14} p_{25} p_{36} p_{47} p_{50} p_{68} p_{71} p_{83} \\
&- \mu_5 p_{02} p_{14} p_{25} p_{36} p_{47} p_{50} p_{68} p_{71} p_{83} \\
&- \mu_6 p_{02} p_{14} p_{25} p_{36} p_{47} p_{50} p_{68} p_{71} p_{83} \\
&- \mu_7 p_{02} p_{14} p_{25} p_{36} p_{47} p_{50} p_{68} p_{71} p_{83} \\
&- \mu_8 p_{02} p_{14} p_{25} p_{36} p_{47} p_{50} p_{68} p_{71} p_{83}
\end{aligned}
$$

5.2. *Availability in the degraded state*

Let $P_i(t)$ be the probability that the system is available and operating in a degraded capacity at time t, given that it started in regenerative state S_i at $t = 0$. Since S_3 is the degraded operational state, the recursive relations are

$$P_0(t) = q_{01}(t) * P_1(t) + q_{02}(t) * P_2(t), \tag{22}$$
$$P_1(t) = q_{10}(t) * P_0(t) + q_{13}(t) * P_3(t) + q_{14}(t) * P_4(t), \tag{23}$$
$$P_2(t) = q_{25}(t) * P_5(t), \tag{24}$$
$$P_3(t) = Z_3(t) + q_{31}(t) * P_1(t) + q_{36}(t) * P_6(t), \tag{25}$$
$$P_4(t) = q_{42}(t) * P_2(t) + q_{47}(t) * P_7(t), \tag{26}$$
$$P_5(t) = q_{50}(t) * P_0(t), \tag{27}$$
$$P_6(t) = q_{64}(t) * P_4(t) + q_{68}(t) * P_8(t), \tag{28}$$
$$P_7(t) = q_{71}(t) * P_1(t) + q_{75}(t) * P_5(t), \tag{29}$$
$$P_8(t) = q_{83}(t) * P_3(t) + q_{87}(t) * P_7(t). \tag{30}$$

Taking Laplace transforms of Eqs. (22)–(30) and solving the resulting system gives

$$P_0^*(s) = \frac{N_1(s)}{D_0(s)}, \tag{31}$$

and

$$P_0 = \lim_{s\to 0} sP_0^*(s) = \frac{N_1}{D_0}, \tag{32}$$

where

$$
\begin{aligned}
N_1 = {} & \mu_3 p_{01} p_{13} - \mu_3 p_{01} p_{13} p_{47} p_{71} - \mu_3 p_{01} p_{13} p_{47} p_{50} p_{75} p_{02} p_{25} \\
& + \mu_3 p_{01} p_{13} p_{42} p_{02} p_{25} p_{47} p_{50} p_{75} - \mu_3 p_{01} p_{13} p_{42} p_{02} p_{25}.
\end{aligned}
$$

6. Busy-period analysis

Busy-period analysis quantifies the time spent in downstates associated with inspection, repair, and replacement.

6.1. Busy period due to inspection

Let I_0 be the mean busy period due to inspection, and let $I_i(t)$ be the probability that inspection is in progress at time t, given that the process started in state S_i. For unit A in states S_2, S_4, and S_6 of Figure 1, the governing equations are

$$I_0(t) = q_{01}(t) * I_1(t) + q_{02}(t) * I_2(t), \tag{33}$$
$$I_1(t) = q_{10}(t) * I_0(t) + q_{13}(t) * I_3(t) + q_{14}(t) * I_4(t), \tag{34}$$
$$I_2(t) = Z_2(t) + q_{25}(t) * I_5(t), \tag{35}$$
$$I_3(t) = q_{31}(t) * I_1(t) + q_{36}(t) * I_6(t), \tag{36}$$
$$I_4(t) = Z_4(t) + q_{42}(t) * I_2(t) + q_{47}(t) * I_7(t), \tag{37}$$
$$I_5(t) = q_{50}(t) * I_0(t), \tag{38}$$
$$I_6(t) = Z_6(t) + q_{64}(t) * I_4(t) + q_{68}(t) * I_8(t), \tag{39}$$
$$I_7(t) = q_{71}(t) * I_1(t) + q_{75}(t) * I_5(t), \tag{40}$$
$$I_8(t) = q_{83}(t) * I_3(t) + q_{87}(t) * I_7(t). \tag{41}$$

The steady-state busy period due to inspection is

$$I_0 = \lim_{s\to 0} sI_0^*(s) = \frac{N_2}{D_0}, \tag{42}$$

where

$$\begin{aligned} N_2 = {} & \mu_2 p_{02}p_{25}p_{50}(p_{13}p_{31} - 1) \\ & + \mu_4 p_{01}p_{14}(p_{36}p_{68}p_{83} - 1) \\ & + \mu_6 p_{01}p_{14}p_{36}p_{64}(1 - p_{47}p_{71}) \\ & + \mu_4 p_{02}p_{14}p_{25}p_{42}p_{50}(1 - p_{13}p_{31}) \\ & + \mu_6 p_{02}p_{13}p_{25}p_{36}p_{42}p_{50}p_{64} \\ & + \mu_2 p_{01}p_{14}p_{25}p_{42}p_{50}(p_{36}p_{68}p_{83} - 1) \\ & + \mu_2 p_{01}p_{14}p_{47}p_{50}p_{75}(p_{36}p_{68}p_{83} - 1) \\ & + \mu_4 p_{02}p_{14}p_{25}p_{47}p_{50}p_{71}(p_{13}p_{31} - 1) \\ & + \mu_6 p_{02}p_{14}p_{25}p_{36}p_{47}p_{50}p_{68}p_{71}p_{83}. \end{aligned}$$

6.2. Busy period for repair of unit A

Let $B_i(t)$ denote the probability that the repairman is busy repairing unit A at time t, given that the process started in state S_i. For repair states S_5, S_7, and S_8,

$$B_0(t) = q_{01}(t) * B_1(t) + q_{02}(t) * B_2(t), \tag{43}$$
$$B_1(t) = q_{10}(t) * B_0(t) + q_{13}(t) * B_3(t) + q_{14}(t) * B_4(t), \tag{44}$$
$$B_2(t) = q_{25}(t) * B_5(t), \tag{45}$$
$$B_3(t) = q_{31}(t) * B_1(t) + q_{36}(t) * B_6(t), \tag{46}$$
$$B_4(t) = q_{42}(t) * B_2(t) + q_{47}(t) * B_7(t), \tag{47}$$
$$B_5(t) = Z_5(t) + q_{50}(t) * B_0(t), \tag{48}$$
$$B_6(t) = q_{64}(t) * B_4(t) + q_{68}(t) * B_8(t), \tag{49}$$
$$B_7(t) = Z_7(t) + q_{71}(t) * B_1(t) + q_{75}(t) * B_5(t), \tag{50}$$
$$B_8(t) = Z_8(t) + q_{83}(t) * B_3(t) + q_{87}(t) * B_7(t). \tag{51}$$

Applying the Laplace transform to the preceding recursive equations and using the final-value theorem gives

$$B_0 = \lim_{s\to 0} sB_0^*(s) = \frac{N_3}{D_0}, \tag{52}$$

where

$$\begin{aligned} N_3 = {} & \mu_5 p_{02}p_{25}(1 - p_{13}p_{31}) \\ & + \mu_7 p_{01}p_{14}p_{47}(1 - p_{36}p_{68}p_{83}) \\ & + \mu_8 p_{01}p_{14}p_{36}p_{68}(1 - p_{47}p_{71}) \\ & + \mu_7 p_{02}p_{14}p_{25}p_{42}p_{47}(p_{13}p_{31} - 1) \\ & + \mu_8 p_{02}p_{14}p_{25}p_{36}p_{42}p_{68}(p_{13}p_{31} - 1) \\ & + \mu_5 p_{01}p_{14}p_{47}p_{75}p_{02}p_{25}(p_{36}p_{68}p_{83} - 1) \\ & + \mu_8 p_{01}p_{14}p_{36}p_{64}p_{47}p_{83} \\ & + \mu_5 p_{01}p_{14}p_{36}p_{64}p_{42}p_{02}p_{25}p_{83} \\ & + \mu_7 p_{01}p_{13}p_{36}p_{68}p_{87} \\ & + \mu_5 p_{01}p_{13}p_{36}p_{68}p_{75}p_{02}p_{25}p_{87}. \end{aligned}$$

6.3. Busy period for repair or replacement of unit B

Let $\phi_i(t)$ denote the probability that the repairman is busy with unit B at time t, given that the process started in state S_i. For states S_1, S_3, S_4, S_6, S_7, and S_8,

$$\phi_0(t) = q_{01}(t) * \phi_1(t) + q_{02}(t) * \phi_2(t), \tag{53}$$
$$\phi_1(t) = Z_1(t) + q_{10}(t) * \phi_0(t) + q_{13}(t) * \phi_3(t) + q_{14}(t) * \phi_4(t), \tag{54}$$
$$\phi_2(t) = q_{25}(t) * \phi_5(t), \tag{55}$$
$$\phi_3(t) = Z_3(t) + q_{31}(t) * \phi_1(t) + q_{36}(t) * \phi_6(t), \tag{56}$$
$$\phi_4(t) = Z_4(t) + q_{42}(t) * \phi_2(t) + q_{47}(t) * \phi_7(t), \tag{57}$$
$$\phi_5(t) = q_{50}(t) * \phi_0(t), \tag{58}$$
$$\phi_6(t) = Z_6(t) + q_{64}(t) * \phi_4(t) + q_{68}(t) * \phi_8(t), \tag{59}$$
$$\phi_7(t) = Z_7(t) + q_{71}(t) * \phi_1(t) + q_{75}(t) * \phi_5(t), \tag{60}$$
$$\phi_8(t) = Z_8(t) + q_{83}(t) * \phi_3(t) + q_{87}(t) * \phi_7(t). \tag{61}$$

The corresponding steady-state busy period is

$$\phi_0 = \lim_{s\to 0} s\phi_0^*(s) = \frac{N_4}{D_0}, \tag{62}$$

where

$$\begin{aligned} N_4 = {} & \mu_1 p_{01}(1 - p_{36}p_{68}p_{83}) \\ & + \mu_3 p_{01}p_{13}(1 - p_{47}p_{71}) \\ & + \mu_4 p_{01}p_{14}(1 - p_{36}p_{68}p_{83}) \\ & + \mu_6 p_{01}p_{14}p_{36}p_{64}(1 - p_{47}p_{71}) \\ & + \mu_7 p_{01}p_{14}p_{47}(1 - p_{36}p_{68}p_{83}) \\ & + \mu_8 p_{01}p_{14}p_{36}p_{68}(1 - p_{47}p_{71}) \\ & + \mu_1 p_{02}p_{14}p_{25}p_{42}p_{50}(p_{36}p_{68}p_{83} - 1) \\ & + \mu_3 p_{02}p_{13}p_{25}p_{42}p_{50}p_{64}p_{36} \\ & + \mu_4 p_{02}p_{14}p_{25}p_{47}p_{50}p_{71}(p_{13}p_{31} - 1) \\ & + \mu_6 p_{02}p_{14}p_{25}p_{36}p_{47}p_{50}p_{68}p_{71}p_{83} \\ & + \mu_7 p_{02}p_{14}p_{25}p_{42}p_{47}(p_{13}p_{31} - 1) \end{aligned}$$

$$\begin{aligned}
&+ \mu_8 p_{02} p_{14} p_{25} p_{36} p_{42} p_{68} (p_{13} p_{31} - 1) \\
&+ \mu_3 p_{01} p_{14} p_{36} p_{64} p_{47} p_{83} \\
&+ \mu_3 p_{01} p_{14} p_{36} p_{64} p_{42} p_{02} p_{25} p_{83} \\
&+ \mu_7 p_{01} p_{13} p_{36} p_{68} p_{87} \\
&+ \mu_7 p_{01} p_{13} p_{36} p_{68} p_{75} p_{02} p_{25} p_{87}.
\end{aligned}$$

7. Results and discussion

The steady-state system profit function is

$$P = K_1 A_0 + K_2 P_0 - K_3 I_0 - K_4 B_0 - K_5 \phi_0, \tag{63}$$

where

- K_1 is the revenue generated during full availability;
- K_2 is the revenue generated during partial availability;
- K_3 is the cost incurred during inspection;
- K_4 is the cost incurred during maintenance or repair; and
- K_5 is the cost associated with replacement or other specified restoration activities.

The numerical analysis uses

$$K_1 = 10{,}000, \quad K_2 = 5{,}000, \quad K_3 = 500,$$
$$K_4 = 400, \quad K_5 = 600.$$

with the baseline transition rates

$$\beta = 0.02, \qquad i = 0.40, \qquad r = 0.50, \qquad R = 0.60.$$

Tables 2–6 and Figures 2–5 show how the reliability and economic measures change with the failure rate α, Weibull shape parameter k, and repair rate r.

Table 2 and Figure 2 show that increasing α from 0.01 to 0.50 reduces the reported profit from 9240.573 to 2164.179 and the MTSF from 293.86 to 7.66. Table 3 and Figure 3 show that the reported profit also decreases as the shape parameter increases from the infant-mortality regime ($k < 1$) to the wear-out regime ($k > 1$).

Figure 4 and Tables 4 and 5 indicate that increasing the repair rate mitigates economic losses. When $\alpha = 0.01$, the profit increases from 7325.258 at $r = 0.1$ to 8726.043 at $r = 1.0$. When $\alpha = 0.05$, it increases from 4879.802 to 6970.602 over the same repair-rate range. The marginal gains decrease as the system approaches its maximum feasible availability.

Finally, Table 6 and Figure 5 present the reported economic cutoff points. The profit reaches zero at approximately $\alpha = 1.45$ for $k = 0.8$, $\alpha = 1.38$ for $k = 1.0$, and $\alpha = 1.21$ for $k = 2.0$; beyond these values, the reported maintenance costs exceed revenue.

8. Conclusion

This study evaluated the reliability and economic performance of a two-unit non-identical cold-standby redundant industrial system using the regenerative point technique and Weibull distributions. The state-transition model was used to derive the MTSF, full and degraded availability, inspection and repair busy periods, and a steady-state profit function.

The reported numerical results show that higher failure rates and larger Weibull shape parameters reduce system availability and profitability, whereas more efficient repair improves both system performance and profit. The model therefore provides a mathematical basis for maintenance planning and economic decision-making in multi-state repairable systems. The Weibull shape parameter is particularly important in determining the reported threshold between long-term profitability and operational deficit.

Data availability

The authors state that no research data are available outside the submitted manuscript.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this manuscript.

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