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Fuzzy semi-Markovian stochastic model for single-unit system with repairman arrival delay under Lindley lifetime distribution using bell-shaped membership function

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Abstract

The key objective of the present analysis is to analyze the impact of fuzziness and repairman arrival delay on the mean time to system failure (MTSF) and availability of single-unit systems. Using a semi-Markovian approach, two stochastic models were developed, and recursive relations were obtained for the reliability measures. The lifetime, failure time, and repairman arrival time are considered Lindley-distributed random variables, with a parameter represented as a fuzzy number defined by a bell-shaped membership function. The switching devices and repairs are assumed to be as good as new. The numerical results show that increasing the α-cut reduces fuzziness and improves the estimates, while immediate arrival of the repairman increases system availability. The importance of the results is highlighted using a numerical example generated from a Lindley-distributed random sample of size 1000 by considering $\varepsilon = 0.1$ and dynamic δ in RStudio.

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Keywords: Fuzzy availability, Lindley distribution, Single-unit system, Bell-shaped membership function, Regenerative point technique.

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1. Introduction

Reliability and availability are among the most important performance indicators for engineering products and systems. Researchers have developed several reliability-evaluation and enhancement techniques, including Markovian approaches, semi-Markovian approaches, fault-tree analysis, and reliability block diagrams. Several mechanical, industrial, and process industries demonstrate the applicability of Markov models for reliability evaluation. A semi-Markov process is a generalization of a Markov process in which the future state depends on the present as well as the past; that is, the memoryless property does not hold. Many researchers have developed semi-Markov models for single- and multiple-unit systems to evaluate reliability measures such as MTSF and availability. Loganathan *et al*. applied a semi-Markovian approach to investigate the availability of manufacturing systems [1], and Gitanjali used the approach for complex industrial redundant systems [2]. Grabski developed semi-Markov models for a non-identical-unit cold standby system [3]. Rani *et al*. used the regenerative point technique and stochastic models for the performance evaluation of heat exchangers [4]. Kumar *et al*. analyzed the concepts of arrival time, cold standby redundancy, and imperfect coverage for standby-system performance [5]. Singhal and Kumar used a semi-Markovian approach to assess a pumping system [6].

Cost reduction remains a primary focus for manufacturers seeking maximum benefit. It is often achieved by adopting

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reliability-enhancement policies. Among these techniques, the use of a spare unit is well known; however, the provision of an additional unit increases equipment cost. Because of budget restrictions or complexity, it is not always feasible to use additional units. In such situations, single-unit systems are recommended. Nandal and Malik analyzed a single-unit system with increasing failure rates using the gamma distribution [7]. Nandal developed several reliability models under various lifetime distributions [8].

A common assumption is that failure and repair times are exponentially distributed. However, as systems age because of wear, corrosion, and fatigue, their hazard rates can increase with time, indicating non-constant hazard behavior and declining performance. Medical devices, water pumps, and mechanical equipment are examples of such systems. This flexibility and tractability make the Lindley distribution a suitable alternative to the exponential distribution for modeling system lifetime in such contexts. Ghitany *et al*. developed the Lindley distribution and its application [9]. Malik and Nandal explored the use of the Lindley distribution in lifetime modeling of multiple-unit systems [10]. Qayoom *et al*. highlighted the effectiveness of a Lindley-distribution class in system reliability analysis, simulation, and applications [11]. Safari *et al*. showed the robustness of the Lindley distribution in reliability estimation [12]. Alotaibi and Elshahhat proposed a new discrete model using the Lindley family and presented reliability inferences and applications [13]. Yaghoubi used Lindley-distributed random variables for repairable-system availability modeling [14]. Tyagi and Poonia used a weighted exponential Lindley distribution in the availability prediction of a repairable gas station [15]. Pundir and Patawa investigated the stochastic behavior of cold standby systems under waiting time for repair [16]. The availability of an immediate repair facility is another reliability-enhancement technique frequently adopted in small-scale industries. Sharma and Kumar estimated the reliability of a stochastic model under hot standby redundancy [17]. Varshney *et al*. investigated unreliable repairmen and multiple failure modes in multi-unit systems [18].

Information about failure and repair times is often imperfect and uncertain. In such situations, fuzzy set theory can outperform traditional reliability-evaluation techniques. Fuzzy set theory has several applications in expert-system performance evaluation. Researchers have developed several membership functions, including triangular, bell-shaped, and trapezoidal functions, to model parameter fuzziness. Because of its smoothness, compatibility with statistical estimation, realistic representation of uncertainty, and compatibility with probability distributions, the bell-shaped membership function is generally recommended in semi-Markovian models. Maan *et al*. investigated the impact of Weibull-distributed failure laws with fuzzy parameters on single-unit systems [19]. Savoia used fuzzy methodology for structural reliability analysis [20]. Maturo and Fortuna developed a membership function associated with the normal curve [21]. Dutta and Limboo and Ranjan and Dash showed the applications of bell-shaped fuzzy sets in decision making and medical diagnosis [22, 23]. Zadeh introduced the concept of fuzzy set theory [24]. Kadyan *et al*. explored applications of picture fuzzy theory for reliability evaluation of complex industrial systems [25]. Gupta *et al*. studied a magnetic bearing system using various fuzzy membership functions with modeling and simulation [26]. Temraz and Saini and Kumar obtained MTSF values for repairable systems in fuzzy environments [27, 28]. Liu *et al*. studied unrepairable systems with fuzzy lifetimes [29]. Zhang and Zhang investigated a parallel system by developing stochastic models under the provision of a single standby unit [30]. Kumari and Sharma investigated a retrial machine repair problem using a Gaussian membership function [31]. Hooda and Barak showed the application of fuzzy set theory in the design of experiments and estimation of missing information [32, 33].

Several studies have considered reliability modeling of single-unit systems using either the Lindley distribution or a semi-Markov approach. However, to the best of our knowledge, no study has reported the development of fuzzy semi-Markov models under the Lindley distribution with repairman arrival delay. The novelty of the proposed work therefore lies in the development of a fuzzy semi-Markov model.

Based on these facts, the present analysis focuses on the impact of fuzziness and repairman arrival delay on MTSF and availability in single-unit systems. Using a semi-Markovian approach, two stochastic models are developed, and recursive relations are derived for reliability measures. The lifetime, failure time, and repairman arrival time are considered Lindley-distributed random variables with parameters treated as fuzzy numbers defined by a bell-shaped membership function. The switching devices and repairs are assumed to be as good as new. The theoretical and practical implications of the proposed work lie in the development of a fuzzy semi-Markovian stochastic model that incorporates repairman arrival delay and Lindley-distributed random variables to obtain reliability measures. From a practical point of view, the proposed models and derived results can support maintenance scheduling and help determine the impact of repair and repairman arrival delay on system performance. The numerical results show that increasing the α-cut reduces fuzziness and improves the estimates, while immediate arrival of the repairman increases system availability. The importance of the results is highlighted by a numerical example based on a Lindley-distributed random sample of size 1000, generated by considering $\varepsilon = 0.1$ and dynamic δ in RStudio.

2. Materials and methods

2.1. Bell-shaped membership function

Because a binary-state system does not cover all system states and the uncertainty associated with the data is not handled efficiently by traditional reliability-evaluation techniques, fuzzy theory is useful in this context. Several types of membership functions exist in fuzzy set theory. Among these, the bell-shaped membership function captures data uncertainty in a smoother and more flexible way through its function parameters. It provides smoother α-cuts and represents uncertainty more realistically. It also improves numerical stability and continuity in the computation of MTSF and availability. Therefore,

Figure 1: Illustration of a single-unit system.

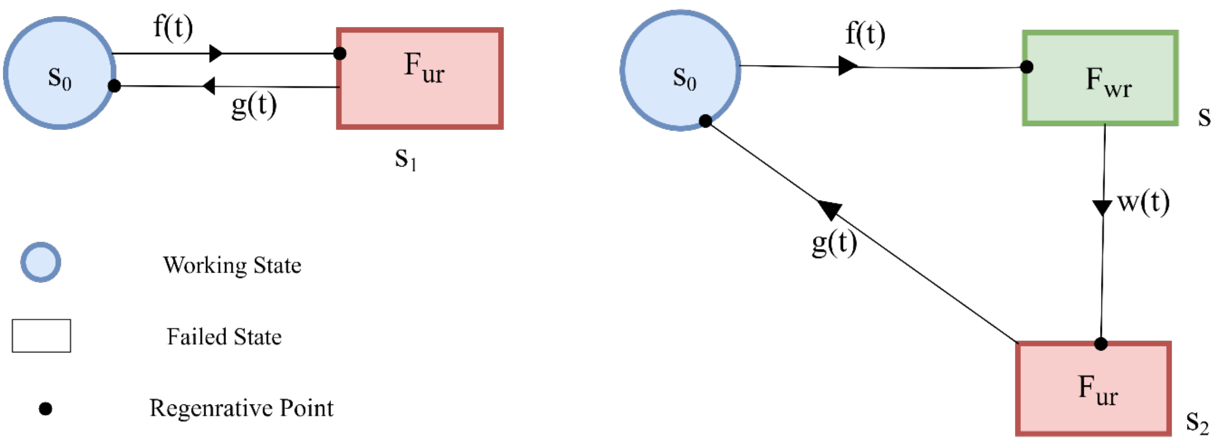

Figure 2: State-transition diagrams of the single-unit system.

all parameters of failure, arrival, and repair times are considered fuzzy numbers with a bell-shaped membership function. Mathematically, the membership function is defined as

$$\gamma(\theta) = e^{-\left(\frac{\theta-\mu}{\epsilon}\right)^2}, \quad \mu - \delta \leq \theta \leq \mu + \delta. \tag{1}$$

For arbitrary values of δ and ε, the intervals of the fuzzy parameters of random variables associated with failure, repair, and arrival are given by $\gamma(\theta) \geq \alpha$ for all $\alpha \in (0, 1)$. Thus,

$$\begin{aligned} & e^{-\left(\frac{\theta-\widehat{\theta}}{\varepsilon}\right)^2} \geq \alpha, \\ & \left[\widehat{\theta}_L, \widehat{\theta}_U\right] = \left\{\widehat{\theta} - \varepsilon\sqrt{-\ln(\alpha)} \leq \theta \leq \widehat{\theta} + \varepsilon\sqrt{-\ln(\alpha)}\right\}. \end{aligned} \tag{2}$$

Here, $\widehat{\theta}$ is the estimated value of the parameter θ.

2.2. Notations

The following notation is used:

- $f(t)$: probability density function of failure time between states S_i and S_j;
- $g(t)$: probability density function of repair time between states S_i and S_j;
- $w(t)$: probability density function of repairman arrival time between states S_i and S_j;
- θ: failure-time distribution parameter;
- γ: repair-time distribution parameter;
- β: arrival-time distribution parameter;
- O: operative state;
- Fur: system under repair;
- Fwr: system waiting for repair.

Table 1: Random sample data generated for the single-unit systems.

Model-I		Model-II		
Repair time (γ)	Failure time (θ)	Repair time (γ)	Failure time (θ)	Arrival time (β)
0.852106	5.441602	0.852106	5.441602	4.924607
0.379701	1.428145	0.379701	1.428145	1.269778
1.287982	9.004296	1.287982	9.004296	1.274591
1.116480	0.843358	1.116480	0.843358	9.314322
0.240051	10.227920	0.240051	10.227920	5.466874
2.267365	2.915825	2.267365	2.915825	8.348403
0.486638	12.736540	0.486638	12.736540	3.145466
0.715310	2.906078	0.715310	2.906078	1.293159
0.769564	13.375680	0.769564	13.375680	5.846267
0.256227	18.632830	0.256227	18.632830	1.878826
2.086431	3.259359	2.086431	3.259359	0.287463
0.136789	6.804117	0.136789	6.804117	2.915553
0.349682	2.097447	0.349682	2.097447	7.475553
0.540077	17.909620	0.540077	17.909620	0.338908
0.055841	2.955635	0.055841	2.955635	0.996271
0.887649	6.450767	0.887649	6.450767	5.503351
2.809438	12.343850	2.809438	12.343850	5.456524
0.962730	4.240610	0.962730	4.240610	1.144787
1.244853	4.181991	1.244853	4.181991	2.598127

Table 2: Descriptive statistics of the sample data.

Statistic	Min.	1st Quart.	Median	Mean	3rd Quart.	Max.
Model-I (failure time)	0.02617	2.73239	4.95610	6.03874	8.53248	31.68728
Model-I (repair time)	0.003704	0.322131	0.703782	0.954482	1.368324	7.030173
Model-II (repair time)	0.003704	0.322131	0.703782	0.954482	1.368324	7.030173
Model-II (failure time)	0.02617	2.73239	4.95610	6.03874	8.53248	31.68728
Model-II (arrival time)	0.008379	1.313551	2.724745	3.529141	5.045381	22.374840

Table 3: Estimated parameters of the random variables for the single-unit system.

Model-I		Model-II		
θ	γ	θ	γ	β
0.3	1.5	0.3	1.5	0.5
$\widehat{\theta}$	$\widehat{\gamma}$	$\widehat{\theta}$	$\widehat{\gamma}$	$\widehat{\beta}$
0.2936096	1.471583	0.2936096	1.471583	0.4754072

3. System model description

Two stochastic models of a single-unit system are developed under the concept of repairman arrival time. Figure 1 illustrates the single-unit system, and Figure 2 shows the corresponding state-transition diagrams. The system assumptions are as follows:

- In both models, at reference time $t = 0$, the system is in fully operational mode.
- After failure, the system in Model-I undergoes repair, whereas the system in Model-II waits for repair because the repair facility is not immediately available.
- In Model-II, the system undergoes repair after the repairman arrives.
- Repairs are perfect.
- In both models, the system becomes operative after repair.

Table 4: Comparative analysis of data parameters between Model-I and Model-II.

Variable	Model-I		Model-II	
	Param 1	Param 2	Param 1	Param 2
0.1	0.079013	0.508206	0.260811	0.690004
0.2	0.114197	0.473022	0.295995	0.654819
0.3	0.138434	0.448785	0.320232	0.630583
0.4	0.158237	0.428982	0.340034	0.610780
0.5	0.175869	0.411351	0.357666	0.593148
0.6	0.192533	0.394686	0.374330	0.576484
0.7	0.209150	0.378070	0.390947	0.559867
0.8	0.226805	0.360414	0.408602	0.542212
0.9	0.247705	0.339514	0.429503	0.521312
1.0	0.293610	0.293610	0.475407	0.475407

Table 5: Fuzzy repair rate associated with Model-I and Model-II.

α-cut	Model-I		Model-II	
	$\widehat{\gamma}_L$	$\widehat{\gamma}_U$	$\widehat{\gamma}_L$	$\widehat{\gamma}_U$
0.1	1.256986	1.686179	1.256986	1.686179
0.2	1.292170	1.650995	1.292170	1.650995
0.3	1.316407	1.626758	1.316407	1.626758
0.4	1.336210	1.606956	1.336210	1.606956
0.5	1.353842	1.589324	1.353842	1.589324
0.6	1.370506	1.572659	1.370506	1.572659
0.7	1.387123	1.556043	1.387123	1.556043
0.8	1.404778	1.538387	1.404778	1.538387
0.9	1.425678	1.517487	1.425678	1.517487
1.0	1.471583	1.471583	1.471583	1.471583

Table 6: Fuzzy arrival rate associated with model-II.

α-cut	Model-II	
	$\widehat{\beta}_L$	$\widehat{\beta}_U$
0.1	0.260811	0.690004
0.2	0.295995	0.654819
0.3	0.320232	0.630583
0.4	0.340034	0.610780
0.5	0.357666	0.593148
0.6	0.374330	0.576484
0.7	0.390947	0.559867
0.8	0.408602	0.542212
0.9	0.429503	0.521312
1.0	0.475407	0.475407

- The lifetime, repair time, and repairman arrival time follow the Lindley distribution.

Table 7: Fuzzy MTSF associated with Model-I & II.

Variable	Model-I		Model-II	
	Param 1	Param 2	Param 1	Param 2
0.1	3.272371	24.385530	3.272372	24.385533
0.2	3.549258	16.616040	3.549258	16.616043
0.3	3.766242	13.568920	3.766242	13.568920
0.4	3.962397	11.775910	3.962397	11.775913
0.5	4.153492	10.521690	4.153492	10.521695
0.6	4.350308	9.549290	4.350308	9.549290
0.7	4.564379	8.735510	4.564379	8.735510
0.8	4.814100	8.003029	4.814100	8.003029
0.9	5.144235	7.272643	5.144235	7.272643
1.0	6.038737	6.038737	6.038737	6.038737

Table 8: Fuzzy availability associated with Model-I & II.

Variable	Model-I		Model-II	
	Param 1	Param 2	Param 1	Param 2
0.1	0.740287	0.967704	0.289701	0.886547
0.2	0.761517	0.952197	0.333388	0.834969
0.3	0.775933	0.941132	0.364174	0.799298
0.4	0.787577	0.931866	0.389745	0.770141
0.5	0.797838	0.923451	0.412824	0.744212
0.6	0.807441	0.915356	0.434900	0.719761
0.7	0.816921	0.907153	0.457163	0.695459
0.8	0.826885	0.898294	0.481084	0.669748
0.9	0.838531	0.887623	0.509743	0.639492
1.0	0.863513	0.863513	0.544521	0.544521

4. Reliability measures

4.1. Transition probabilities

Using simple probabilistic arguments,

$$p_{ij} = \lim_{t\to\infty} Q_{ij}(t) = \int_0^\infty q_{ij}(t)\,dt. \tag{3}$$

Based on the state-transition diagrams and using the Laplace–Stieltjes transform, the transition-probability expressions using equation (3) are as follows. For Model-I,

$$\begin{aligned} Q_{01}^{**} &= \int_0^\infty e^{-st} q_{01}(t)\,dt \\ &= \int_0^\infty \frac{\theta^2}{1+\theta}(1+t)e^{-(s+\theta)t}\,dt, \\ Q_{10}^{**} &= \int_0^\infty e^{-st} q_{10}(t)\,dt \\ &= \int_0^\infty \frac{\gamma^2}{1+\gamma}(1+t)e^{-(s+\gamma)t}\,dt. \end{aligned}$$

For Model-II,

$$Q_{01}^{**} = \int_0^\infty e^{-st} q_{01}(t)\, dt = \int_0^\infty \frac{\theta^2}{1+\theta}(1+t)e^{-(s+\theta)t}\, dt,$$

$$Q_{10}^{**} = \int_0^\infty e^{-st} q_{10}(t)\, dt = \int_0^\infty \frac{\beta^2}{1+\beta}(1+t)e^{-(s+\beta)t}\, dt,$$

$$Q_{20}^{**} = \int_0^\infty e^{-st} q_{20}(t)\, dt = \int_0^\infty \frac{\gamma^2}{1+\gamma}(1+t)e^{-(s+\gamma)t}\, dt.$$

4.2. Mean sojourn times

The mean sojourn time is the average time spent by the system in a particular state before transition to another state. By simple probabilistic arguments,

$$\mu_i = \int_0^\infty \Pr(T_i > t)\, dt = \sum_j m_{ij}, \qquad m_{ij} = -\frac{d}{ds}\left[Q_{ij}^{**}(s)\right]_{s=0}. \tag{4}$$

Hence, the mean sojourn times at various states using equation (4) are as follows.

For Model-I,

$$\mu_0 = \frac{2+\theta}{\theta(\theta+1)}; \qquad \mu_1 = \frac{2+\gamma}{\gamma(\gamma+1)}.$$

For Model-II,

$$\mu_0 = \frac{2+\theta}{\theta(\theta+1)}; \qquad \mu_1 = \frac{2+\beta}{\beta(\beta+1)}; \qquad \mu_2 = \frac{2+\gamma}{\gamma(\gamma+1)}.$$

4.3. Mean time to system failure

Using the cumulative distribution function of first-passage time from the regenerative state to the failed state and considering the failed state as absorbing, recursive relations are derived based on the state-transition diagrams and simplified using the Laplace transform and L'Hospital's rule. Finally, the MTSF expression for Models I and II is derived as

$$\text{MTSF} = \mu_0 = \frac{2+\theta}{\theta(\theta+1)}. \tag{5}$$

4.4. Availability

By considering the probability density function of transition time from the operative state to the regenerative state at time $t = 0$, the recursive relations are derived for both models using the state-transition diagrams. Applying Laplace transformation and Cramer's rule gives the steady-state availability of the system in the presence and absence of the repairman as follows.

For Model-I,

$$A(\infty) = \lim_{s\to 0} \frac{sN}{D} = \frac{\mu_0}{\mu_0 p_{10} + \mu_1 p_{01}}. \tag{6}$$

For Model-II,

$$A(\infty) = \lim_{s\to 0} \frac{sN}{D} = \frac{\mu_0}{\mu_0 + \mu_1 + \mu_2}. \tag{7}$$

5. Sample generation and parameter estimation

5.1. Algorithm

The proposed investigation of the single-unit system is performed using the following methodology:

1. Generate random samples of size 1000 for failure time, repair time, and arrival time for both models from the Lindley distribution.
2. Obtain the point estimators of the failure rate, repair rate, and arrival rate for both models using maximum likelihood estimation (MLE).
3. Derive the α-cuts for the fuzzy numbers of the failure rate, repair rate, and arrival rate using the formulation in equation (2).
4. Obtain the fuzzy MTSF and fuzzy availability for both models against the α-cuts using equations (5)–(7).

5.2. Estimation

The lifetime of industrial systems can be modeled by several probability distributions, including the exponential, Weibull, gamma, and Lindley distributions. Some system lifetimes do not show constant behavior; in such cases, the Lindley distribution is an appropriate alternative. It has an increasing hazard rate, which makes it suitable for single-unit systems. In the present work, the lifetime, repairman arrival time, and repair time of the system follow the Lindley distribution with probability density function

$$f(t) = \frac{\theta^2}{1+\theta}(1+t)e^{-\theta t}, \qquad t > 0,$$

distribution function

$$F(t) = 1 - \left(1 + \frac{\theta t}{1+\theta}\right)e^{-\theta t},$$

and reliability function

$$R(t) = \left(1 + \frac{\theta t}{1+\theta}\right)e^{-\theta t}.$$

To estimate the parameter θ, the maximum likelihood estimation method is used because of its estimation accuracy. Let $t_1, t_2, t_3, \ldots, t_n$ be a random sample of repair time, waiting time, or lifetime. Then the likelihood function is

$$L(\theta) = \prod_{i=1}^n \frac{\theta^2}{1+\theta}(1+t_i)e^{-\theta t_i} = \left(\frac{\theta^2}{1+\theta}\right)^n \prod_{i=1}^n (1+t_i)e^{-\theta \sum_{i=1}^n t_i}.$$

After applying the logarithm on both sides and using the minima–maxima rule, we obtain

$$\widehat{\theta} = \frac{n - S + \sqrt{(S-n)^2 + 8nS}}{2S}, \qquad S = \sum_{i=1}^n t_i. \tag{8}$$

5.3. Sample generation algorithm

A random sample of size 1000 was generated through simulation for failure time, repair time, and arrival time of the single-unit system. All times follow the Lindley distribution. A subset of the Lindley-distributed sample observations is provided in Table 1. The descriptive information for the associated random data is given in Table 2. The average failure time is 6.03874, while the average repair time is 0.954482. The MLE method is used to estimate the parameters. The estimated parameters obtained using equation (8) are given in Table 3, together with the true values of the parameters. Corresponding to the crisp values of the random-variable parameters, the crisp MTSF for both models is 6.038737, while the crisp availability values are 0.8635132 and 0.5445206, respectively.

6. Fuzzy results and discussion

In this section, the effect of fuzziness on the MTSF and availability of the single-unit systems in the presence and absence of an immediate repairman is observed numerically. The numerical results for the fuzzy estimated parameters are derived using equations (1) and (2). Table 4 gives the fuzzy failure rates with respect to the α-cuts. It is observed that a smaller α-cut corresponds to higher uncertainty in the failure rates. Tables 5 and 6 present the fuzzy repair rates and fuzzy arrival time for the models. The numerical values show the same trend with respect to α-cuts. The fuzzy MTSF values are shown in Table 7. At $\alpha = 0.1$, the lower limit is 3.272371 and the upper limit is 24.38553, while at $\alpha = 0.9$, the lower limit is 5.144235 and the upper limit is 7.272643. The fuzzy MTSF is the same for both models.

Table 8 shows that the fuzzy availability of the system with an immediate repair facility is higher than that of the system that waits for repair because of repairman unavailability. Thus, Model-I outperforms Model-II. At $\alpha = 0.1$, the lower and upper limits of availability for Model-I are 0.740287 and 0.967704, respectively; at $\alpha = 0.9$, the corresponding limits are 0.838531 and 0.887623. For Model-II, the lower and upper limits at $\alpha = 0.1$ are 0.289701 and 0.886547, respectively; at $\alpha = 0.9$, the limits are 0.509743 and 0.639492. The availability of Model-I is higher for all α-cuts than that of Model-II. Therefore, the immediate arrival of the repairman in a single-unit system is beneficial for enhancing availability. From the estimated results, it is concluded that a higher α-cut corresponds to reduced fuzziness.

7. Conclusion

In the present study, two stochastic models were proposed to determine the impact of repairman arrival delay and Lindley-distributed random variables on the fuzzy availability of single-unit systems. The estimates of the fuzzy MTSF of such systems become more accurate with increasing α-cuts. The behavior of fuzzy availability shows the same trend for both models with respect to α-cuts, while the availability of the model with an immediate repair facility is always higher. The absence of an immediate repairman reduces service cost but increases downtime. The crisp availability values of Models I and II are the same as their fuzzy availability values at $\alpha = 1$, where no uncertainty is present in the data. Finally, for single-unit systems, the provision of an immediate repair facility is recommended. The proposed models apply to mechanical systems such as bearings, automobiles, wind-turbine gearboxes, and solar-powered tube wells, which have increasing failure rates due to aging effects. The approach is useful in industries such as power plants, solar power plants, and telecommunications, where a repairman is not always available on site. In future work, the model may be extended to imperfect repair and degradation for more realistic system behavior. System designers can use the derived fuzzy reliability measures, such as MTSF and availability, to identify the impact of delayed repair and repair rates. These measures are useful for decision making regarding maintenance strategies.

Data availability

Data are available from the corresponding author upon reasonable request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this manuscript.

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