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A mathematical model of cybercrime contagion: impact of employment on youth offending

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Abstract

The growing incidence of youth cybercrime, driven by unemployment and the precarity of temporary employment, constitutes a major social crisis that has received limited mathematical analysis. This study develops a deterministic four-compartment UCTR model comprising unemployed (U), cybercriminal (C), temporarily employed (T), and regularly employed (R) youth populations to examine interactions among unemployment, temporary employment, stable employment, and illicit digital activity. Using the next-generation matrix, we derive the cybercrime generation number, N_0, and establish that the cybercrime-free equilibrium is globally stable when $N_0 < 1$. Global sensitivity analysis based on Latin hypercube sampling (LHS) and partial rank correlation coefficients (PRCCs) identifies peer influence and recruitment into cybercrime from temporary employment as the strongest drivers of cybercrime. Conversely, transition to stable employment and legal prosecution are the principal deterrents. Numerical simulations show that high job-loss rates and continued entry into temporary employment sustain the cybercriminal population, whereas consistent transition to permanent employment substantially reduces cybercrime over time. These findings indicate that cybercrime control requires structural policies that strengthen long-term employment security rather than short-term interventions that do not provide sustained socioeconomic stability.

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1. Introduction

Youth unemployment has become a major socioeconomic problem in the 21st century worldwide [1]. Economic disenfranchisement of youth has attracted considerable debate globally since the late 20th century, particularly regarding the long-term effects of a generation left without stable means of livelihood [2]. This is particularly acute in developing economies, as demographic changes and macroeconomic insecurity have left the younger generation in a fluctuating environment [2]. Unemployment as a social problem is a triggering element that causes social unrest, which contributes to social vices, including substance abuse, vandalism, and violent crimes [3, 4]. The effect of unemployment on crime is even more significant in Africa, where economic marginalisation can frequently lead to the inclination of young people to illegal practices as the only means of livelihood [4].

There is considerable evidence that youth unemployment and non-employment, including unstable temporary employ-

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ment and those not in education, employment, or training (NEET), are associated with a high propensity for criminal activity [3]. Although conventional physical crime remains a crucial issue, over the past few years it has shifted toward cybercrime [5]. This shift is conditioned by the rapid development of digital infrastructure, alongside disparities in education, leaving technically proficient young people with no lawful economic means to apply their skills [2]. The internet has turned into a digital haven of unemployed young people in Nigeria who have moved from offline crime to more advanced cyber-based offences [6]. Such actions include fraudulent emails, spam, ATM fraud, and phishing, all of which endanger the privacy of online banking and undermine trust in online financial systems among the population [6, 7]. This surge in activity not only puts the general public on alert but also generates global panic regarding digital transactions with domestic organisations [6].

It is possible to better understand economic hardship and its relationship with cybercrime by examining factors affecting youth engagement. Studies show that low socioeconomic status and unemployment are directly associated with cybercrime engagement [8]. The internet, although illegal, offers a profitable alternative to earning income in areas with high unemployment [9]. For many "Yahoo Boys", a term used for young people who engage in cyber fraud, their involvement is sometimes driven by the fact that they are well-educated undergraduates with advanced digital skills but no formal employment [9].

Moreover, peer influence is reported as a very important predictor of internet fraud tendency, and it tends to prevail over other demographic factors such as age or gender [10]. Substantial evidence links youth unemployment and non-employment (including temporary employment) to increased crime rates [11, 12]. Unemployment significantly impacts both young people and society, with its adverse effects closely associated with rising crime, particularly cybercrime. Contributing factors include demographic shifts, changes in micro and macroeconomic conditions, educational and training disparities, rural-urban migration, and increased high school enrollment, all of which have played a significant role in the alarming rise in cybercrime [13, 14]. Also, unemployment is a significant contributor to cybercrime in Nigeria. Current estimates indicate that over 20 million graduates in the country lack gainful employment, which has led to an increase in their participation in criminal activities as a means of survival [15]. Prasanthi [16] reported that, as of 2003, the United States and South Korea had the highest rates of cyberattacks, accounting for 35.4% and 12.8%, respectively. Based on the 2006 census, Nigeria's population was estimated at 160 million, and recent statistics indicate that approximately 28.9% of Nigerians have internet access [15].

Furthermore, it has been established that 39.6% of African internet users are Nigerian, which correlates with the significant increase in internet crime rates in Nigeria. [15].

Also, available data demonstrate that Nigeria's population has exhibited sustained growth. From 2020 to 2024, Nigeria's population growth rate reached a low of 2.10% in 2024, while the urban population grew by 3.8% over the same period. The national unemployment rate increased from 4.1% in the first quarter of 2023 to 5.0% in the third quarter of 2023. Youth unemployment rose from 6.9% to 8.6%, and urban unemployment increased from 5.4% to 6.0%. Furthermore, Nigeria ranks fifth globally in cybercrime, with an impact rate of 8.25, a professionalism rate of 6.49, and cybercrime comprising 52.17% of reported cases [17]. Additionally, official statistics from the Nigeria Police Force National Cybercrime Centre show that reported cybercrime incidents increased from approximately 3,800 in 2018 to over 16,500 in 2024, marking a more than fourfold rise over seven years. However, these figures likely underreport the true scale of cybercrime, as victimisation surveys conducted by NOIPolls (a public opinion and research organization) in collaboration with Cybersecurity firms consistently indicate that fewer than one in five, Cybercrime victims report their experiences to law enforcement authorities [18]. Recent data indicate that, as of the first quarter of 2025, Nigeria ranks fifth globally in cybercrime prevalence, with an average of 4,388 weekly cyberattacks and estimated annual economic losses of approximately $500 million [19].

On the one hand, internet services offer unlimited possibilities for commercial development, but on the other hand, they threaten the system, especially among vulnerable and unemployed youth [20]. To manage these complexities, mathematical modelling has become an essential means of understanding the interplay between economic factors and crime [21, 22].

Mathematical models are characterised by equations that provide an understanding of real-life phenomena and enable prediction. This strategy has been used by various authors to address social problems, such as the nature of poverty and imprisonment [1]. In the labour market, researchers have investigated the relationship between job openings and unemployment and have observed that previous unemployment rates can delay job creation [23]. Still, numerous models of the labour market in its initial stage did not consider the specific impact of cybercrime or the subtle effects of temporary work [23].

A critical analysis of the available literature on modelling reveals a significant gap in contemporary methods. For example, the findings reported by Soemarsono *et al.* [24] used models to examine the correlation between unemployment growth rates and general crime rates and found a proportional relationship. Although these models implied that system controls could be maximised to reduce crime, they did not account for the behavioural and transmission characteristics of the cybercrime industry. Likewise, other research has examined unemployment using counts of the unemployed, employed, and available vacancies, but it did not consider the presence of a separate cybercrime compartment or the issue of non-employment due to the instability of temporary jobs [25]. Although other models have highlighted the significance of vocational training and skill acquisition in reducing unemployment, they have failed to account for the specific effects of cybercriminal contagion within peer networks [25, 26]. Moreover, recent developments in epidemiological modelling have begun to treat crime as a social epidemic [27]. Such deterministic models describe transitions among states, including susceptible, cybercriminal, and incarcerated [27, 28]. Researchers have successfully applied next-generation matrices to obtain criminal reproduction numbers, enabling them to determine the conditions for local and global stability in society. [27]. Chikore *et al.* [5] adopted a deterministic approach to examine the effects of criminals' access to cyber-networks on crime spread and found that network characteristics significantly affect participation rates. The dynamics of social contagions such as digital piracy and financial fraud have been studied using SIR-like models by other researchers [29]. Although these models incorporate population-based statistics and steady-state analysis, they may not capture the detailed interplay between employment stability and the unique stress of non-employment among youth [29]. The analysis of unemployment in the context of financial crises has demonstrated that government support for new businesses is an effective intervention, but studies often lack global sensitivity analyses to identify the most influential policy engines in the digital environment [30]. Other models have distinguished between regular and temporary work but have not examined cybercrime as a distinct population group that dynamically interacts with these types of labour [22, 31]. The urgent requirement is a model that captures the transition from unemployment to cybercrime, the recovery into temporary or permanent employment, and the probability of recidivism or relapse into criminality due to job loss [1]. Existing models have typically examined employment, unemployment, cybercrime, or available vacancies as separate phenomena. In contrast, the present study jointly evaluates unemployment, cybercrime, temporary employment, and full employment to assess the impact of each factor on the dynamics of unemployment, temporary employment, and full employment in relation to cybercrime. This integrated approach aims to provide a broader understanding and inform more effective strategies for addressing cybercrime in society.

This paper will attempt to fill these gaps by constructing a deterministic mathematical model explicitly splitting the population of the youth into four dynamical compartments: unemployed members of the population $U(t)$, cybercrime members of the population $C(t)$, temporarily employed (or non-employed) members of the population $T(t)$, and fully employed members of the population $R(t)$. This study evaluates the stability of a society without cybercrime by calculating the cybercrime generation number N_0, which estimates the number of new cybercrime cases arising from a single case introduced into the population [1, 27]. We also conduct a global sensitivity analysis using Latin Hypercube Sampling and Partial Rank Correlation Coefficients to identify the most critical parameters influencing these dynamics [28]. This strategy offers a holistic approach to explaining how the stability of employment and policy responses can be used to suppress the development of cybercrime among the young generation.

The remainder of this paper is structured as follows: Section 2 elaborates on the model formulation and qualitative analysis, including the calculation of the invariant region and the equilibrium states. The sensitivity analysis and stability results are given in Section 3. Section 4 presents the numerical simulations and research findings, and Section 5 provides concluding remarks and policy recommendations to stakeholders and government agencies.

2. Methodology

In this section, the model formulation and its quantitative analysis are carried out. The quantitative analysis includes the invariant region, the reproduction number, and the existence and stability of the equilibrium state. Additionally, sensitivity analysis and numerical simulations are implemented.

2.1. Description and formulation of the model

This study is based on a deterministic mathematical framework to explore how different employment states and youth propensity to commit cybercrime interrelate dynamically. By conceptualising criminal behaviour as a social epidemic, we model the flow of individuals as a system of non-linear ordinary differential equations across four mutually exclusive population compartments.

The overall youth population, denoted by $N(t)$, at any time, t, is divided into four compartments, which are dynamic according to their employment status and criminality:

- Unemployed ($U(t)$): The people who are not working at present but who are trying to be employed or are vulnerable to the external factors.
- Cybercrime ($C(t)$): The individuals who have moved to illegal digital behaviours (phishing, ATM fraud, or spamming) due to economic pressure or peer influence [6].
- Temporarily Employed ($T(t)$): people who work in fluctuating conditions, that is, the group of underemployed people who are still susceptible to cybercriminal activity [3].
- Fully Employed ($R(t)$): Those who have obtained long-term, stable employment, which provides a form of recovery or protection against recruitment into crime [2].

The model describes how people move between these compartments at a given rate. New youths are recruited to the unemployed pool at a rate of Λ, and natural mortality is exerted on every compartment at the same rate of μ. The rate of transition from unemployment to cybercrime is governed by the influence rate β, which reflects the contagion effect of already criminal individuals on the unemployed. The mobility from state of employment is governed by the rates θ_1 and θ_2 (job placement rates) for temporary and full-time employment, respectively, whereas the rates γ_1 and γ_2 (job loss rates) return people to the unemployed population. Besides, the change of individuals from temporary and full employment into cybercriminals is considered with parameters of α_1 and α_2, respectively, and the rehabilitation of criminals into the workforce, temporary or full employment, is explained by the rates δ_1 and δ_2, respectively [27].

Model assumptions

In order to make sure that the model is both mathematically justified and sociologically related, there are the following assumptions:

- Uniform Population: It is assumed that the youth population is homogeneously mixed, implying that any unemployed person has an equal probability of being affected by an active cybercriminal [5].

- Directed Recruitment: Recruitment of all new entrants into the system (school leavers and graduates) is initially assumed to be unemployed ($U(t)$) [1].

- Variable Influence: The transition rate to the cybercrime of the entity is proportional to the population of active criminals, which indicates the impact of social networks on the criminal recruitment process [24].

- Employment Instability: There is no employment security, and thus both T and R can return to U upon an economic shock, the end of a contract, or the full-time employed retire at rate, μ_R [25].

- Repercussion Costs: The members of the C compartment can leave the active population as a result of law enforcement forcing them out, or imprisoning them at a cost of d [1, 27].

A graphical representation of the model formulation is presented in Figure 1, while state variables and parameter descriptions are detailed in Tables 1 and 2, respectively.

The following system of non-linear differential equations is presented based on the model's assumptions, along with the parameters listed in Tables 1 and 2.

$$\left.\begin{aligned}
\frac{dU}{dt} &= \Lambda - \beta UC + \gamma_1 T + \gamma_2 R - (\theta_1 + \theta_2 + \mu)U,\\
\frac{dC}{dt} &= \beta UC + \alpha_1 T - (\delta_1 + \delta_2 + d + \mu)C,\\
\frac{dT}{dt} &= \theta_1 U + \delta_1 C - (\alpha_1 + \gamma_1 + \alpha_2 + \mu)T,\\
\frac{dR}{dt} &= \theta_2 U + \alpha_2 T + \delta_2 C - (\gamma_2 + \mu_R + \mu)R,
\end{aligned}\right\} \tag{1}$$

with non-negative initial conditions, $U(0)$, $C(0)$, $T(0)$, and $R(0)$. The model parameters are assumed to be non-negative except for human recruitment, which is strictly positive.

This mathematical framework provides the necessary basis for establishing the threshold for a cybercrime-free environment and assessing the long-term impact of intervention through employment.

2.2. Invariant region

To demonstrate the Invariant Region, we must show that the mathematical model (1) is mathematically and sociologically well-posed. This also includes demonstrating that the population is nonnegative and lies within a practicable space, making the solutions physically relevant over time. We state the following theorem:

Theorem 2.1. The solution to the system is feasible for all $t > 0$ if it enters the invariant region defined as

$$\Phi = \Big\{(U, C, T, R) \in \mathbb{R}^4_+ : U(0) > 0,\ C(0) \geq 0,\ T(0) \geq 0,\ R(0) \geq 0,\ N \leq \frac{\Lambda}{\mu}\Big\}.$$

where $N(t) = U(t) + C(t) + T(t) + R(t)$ represents the total youth population.

Proof. With $N(t) = U(t) + C(t) + T(t) + R(t)$, we have the following rate of change of the total youth population as

$$\frac{dN}{dt} = \Lambda - dC - \mu_R R - \mu N. \tag{2}$$

Because all parameters, as well as the population variables, are non-negative, Eq. (2) becomes

$$\frac{dN}{dt} \leq \Lambda - \mu N. \tag{3}$$

Using the standard theory of differential inequalities, specifically Gronwall's inequality, it follows that

$$N \leq \frac{\Lambda}{\mu} + \left(N_0 - \frac{\Lambda}{\mu}\right)e^{-\mu t},$$

with N_0 as the initial condition of $N(t)$.

This proves that the total youth population $N(t)$ does not increase indefinitely and is instead bounded by $0 \leq N \leq \frac{\Lambda}{\mu}$ as $t \to \infty$. As a result, the model has the feasible region as follows:

$$\Phi = \Big\{(U, C, T, R) \in \mathbb{R}^4_+ : U(0) > 0,\ C(0) \geq 0,\ T(0) \geq 0,\ R(0) \geq 0,\ N \leq \frac{\Lambda}{\mu}\Big\}.$$

This implies that any model solution whose initial conditions are in the region of Φ will be in the region of Φ at all times at $t > 0$. This proves that the model is mathematically and sociologically well-posed, and that the state variables remain within a sociologically realistic range, with no population compartment taking negative values. Hence, the model can be used to analyse the dynamics of cybercrime. □

2.3. Existence of steady states and computation of the cybercrime generation number

Determining the existence of steady states of a deterministic model is achieved by setting the rates of change of all population compartments to zero. In the case of the model (1), two main steady states are explored: the cybercrime-free equilibrium and the cybercrime-presence equilibrium.

- Cybercrime-free equilibrium (CFE): The CFE, $E_0 = (U^0, 0, T^0, R^0)$, indicates a situation in which the young population is completely cybercriminal-free ($C =$

0 and $\alpha_1 = 0$). This steady state implies that the population is partitioned into the unemployed, temporarily employed, and fully employed compartments, determined by the recruitment, job placement, and job-loss rates.

One can find the coordinates of E_0 by substituting $C = 0$ in the set of equations of model (1) and solving the rest of the variables to obtain

$$E_0 = (U^0, C^0, T^0, R^0) = \left(\frac{\Lambda f_3 f_4}{A_4},\ 0,\ \frac{\theta_1 U^0}{f_3},\ \frac{A_2 U^0}{f_3 f_4}\right).$$

- The cybercrime-presence equilibrium (CPE): CPE, $E^* = (U^*, C^*, T^*, R^*)$, is a condition in which cybercrime is chronic in the population. By solving the system of equations (1) at equilibrium state, we have for $C \neq 0$ that

$$\begin{aligned} U^* &= \frac{\Lambda f_3 f_4 - A_5 C^*}{A_4}, \\ T^* &= \frac{\theta_1 U^* + \delta_1 C^*}{f_3}, \\ R^* &= \frac{A_2 U^* + A_3 C^*}{f_3 f_4}, \end{aligned}$$

where $C^* \neq 0$ is the root of the polynomial

$$B_1 C^2 + B_2 C - B_3 = 0, \tag{4}$$

with

$$\begin{aligned} B_1 &= A_5 \beta f_3, \\ B_2 &= \alpha_1 (A_5 \theta_1 - A_4 \delta_1) + A_4 f_2 f_3 - \Lambda \beta f_3^2 f_4, \\ B_3 &= \Lambda \alpha_1 f_3 f_4 \theta_1. \end{aligned}$$

Here,

$$\begin{aligned} f_1 &= \theta_1 + \theta_2 + \mu, \\ f_2 &= \delta_1 + \mu + d + \delta_2, \\ f_3 &= \alpha_1 + \gamma_1 + \mu + \alpha_2, \\ f_4 &= \gamma_2 + \mu_R + \mu, \\ A_1 &= f_2 f_3 - \alpha_1 \delta_1 > 0, \\ A_2 &= f_3 \theta_2 + \alpha_2 \theta_1, \\ A_3 &= \alpha_2 \delta_1 + f_3 \delta_2, \\ A_4 &= (\mu_R + \mu)[\theta_1(\mu + \alpha_2) + (\theta_2 + \mu) f_3] \\ &\quad + \gamma_2 \mu (f_3 + \theta_1), \\ A_5 &= A_1 f_4 - \delta_1 f_4 \gamma_1 - A_3 \gamma_2 > 0. \end{aligned} \tag{5}$$

The possible positive roots determined using Descartes' rule of signs are summarised in Table 3.

Hence, the system has one unique positive cybercrime-presence equilibrium state, given as

$$C^* = \frac{-B_2 + \sqrt{B_2^2 + 4 B_1 B_3}}{2 B_1} > 0.$$

This positive equilibrium illustrates the continued presence of cybercrime in society.

2.3.1. Computation of the cybercrime generation number, N_0

Using the concept of disease modelling of the definition of basic reproduction number, R_0 by Diekmann *et al.* [32] and Dreessche and Watmough [33] as adopted by Tijani *et al.* [34] and Lawal *et al.* [35], the threshold parameter, which defines the stability of the equilibrium points, is the Cybercrime Generation Number, which is denoted by N_0. It is characterised as the number of second-degree cybercrime cases that a single active offender will produce within a vulnerable population during the interval of their social contagion [1, 36]. When $N_0 < 1$, it implies that the average number of new cybercriminals recruited by one active offender is less than one, and cybercrime will be eliminated, meaning that an unemployed person can either be temporarily employed or fully employed. Conversely, if $N_0 > 1$, it shows that cybercrime will remain in society, leading to more unemployed persons. The cybercrime compartment (C) is the infected group in this model. N_0 is computed at CFE, E_0, as the spectral radius of the matrix, FV^{-1}, where $F = \frac{\partial F_i(E_0)}{\partial x_i}$ is the rate of new infection (the transition into cybercrime) and $V = \frac{\partial V_i(E_0)}{\partial x_i}$ is the exit of the system (mortality, rehabilitation, and legal consequences) [27, 36]. The new recruitment rate presented by the cybercrime compartment is as follows:

$$F = [\beta U^0].$$

The exit rate of the criminal compartment is provided by:

$$V = (\delta_1 + \delta_2 + d + \mu) \text{ and } V^{-1} = \frac{1}{(\delta_1 + \delta_2 + d + \mu)}.$$

The cybercrime generation number, N_0, is the spectral radius (largest eigenvalue) of the next-generation matrix FV^{-1} and is given by

$$N_0 = \frac{\beta \Lambda f_3 f_4}{f_2 A_4},$$

$$\begin{aligned} N_0 &= \frac{\beta \Lambda (\alpha_1 + \gamma_1 + \mu + \alpha_2)(\gamma_2 + \mu_R + \mu)}{(\delta_1 + \mu + d + \delta_2) D}, \\ D &= (\mu_R + \mu)[\theta_1(\mu + \alpha_2) + (\theta_2 + \mu)(\alpha_1 + \gamma_1 + \mu + \alpha_2)] \\ &\quad + \gamma_2 \mu [(\alpha_1 + \gamma_1 + \mu + \alpha_2) + \theta_1]. \end{aligned}$$

The expression defines the cybercrime generation number, N_0, determined by the influence rate, β, on susceptible unemployed individuals. This threshold increases with the recruitment rate, Λ, within a population of size $\frac{\Lambda}{\mu}$, over a time interval of $\dfrac{1}{(\delta_1 + \mu + d + \delta_2)}$.

Threshold interpretation

- In case of $N_0 < 1$: CFE is locally asymptotically stable. It means that when a limited number of cybercriminals infiltrate society, they will fail to find enough followers to continue the practice, and the cybercrime will ultimately go extinct.

- When $N_0 > 1$: An average number of recruits per cybercriminal is above 1. CFE goes out of control, and the system shifts to the cybercrime-presence equilibrium (CPE), meaning that cybercrime will continue to increase and remain concentrated among youth.

This threshold analysis shows that it is important to reduce the influence rate of the parameter of the β (with the help of awareness and monitoring) or to increase the rate of rehabilitation and exit, d (with the help of more powerful legal systems and socio-economic support) to push the value of the parameter of the N_0 to the level that would be less than unity.

2.4. Stability analysis of the crime-free equilibrium state

The model of eliminating illicit activities among youth through cybercrime is incomplete without an analysis of the stability of the equilibrium in a cybercrime-free environment, which is a key element of the model. The analysis will be performed from two perspectives: local asymptotic stability and global asymptotic stability.

2.4.1. Local stability of the crime-free equilibrium state

Local stability of the CFE, which is denoted by $E_0 = (U^0, C^0, T^0, R^0) = \left(\frac{\Lambda f_3 f_4}{A_4},\ 0,\ \frac{\theta_1 U^0}{f_3},\ \frac{A_2 U^0}{f_3 f_4}\right)$, is examined by linearizing the system of differential equations in terms of the Jacobian matrix at E_0 for $\alpha_1 = 0$. We present the theorem regarding the stability of the equilibrium E_0. of the equilibrium E_0 as follows.

Theorem 2.2. The cybercrime-free equilibrium state of the model is locally asymptotically stable if $N_0 < 1$ and unstable if $N_0 > 1$ for $\alpha_1 = 0$..

Proof. The Jacobian matrix of the four-compartment system at E_0, i.e. $J(E_0)$ has the following structure:

$$J(E_0) = \begin{pmatrix} -f_1 & -\frac{\beta\Lambda f_3 f_4}{A_4} & \gamma_1 & \gamma_2 \\ 0 & \frac{\beta\Lambda f_3 f_4}{A_4} - f_2 & \alpha_1 & 0 \\ \theta_1 & \delta_1 & -f_3 & 0 \\ \theta_2 & \delta_2 & \alpha_2 & -f_4 \end{pmatrix}.$$

At $\alpha_1 = 0$,

$$J(E_0) = \begin{pmatrix} -f_1 & -\frac{\beta\Lambda f_3 f_4}{A_4} & \gamma_1 & \gamma_2 \\ 0 & f_2(N_0 - 1) & 0 & 0 \\ \theta_1 & \delta_1 & -f_3 & 0 \\ \theta_2 & \delta_2 & \alpha_2 & -f_4 \end{pmatrix}. \tag{6}$$

The characteristic equation of $J(E_0)$ is given by

$$|J(E_0) - \lambda I| = 0,$$

where λ is the eigenvalue of the Jacobian matrix, and I is a 4×4 identity matrix. The Jacobian matrix $J(E_0)$ has the eigenvalues $\lambda_1 = f_2(N_0 - 1)$ and the root of the characteristic equation of $J(E_0)$ given as

$$\lambda^3 + A\lambda^2 + B\lambda + C = 0, \tag{7}$$

where

$$\begin{aligned} A &= f_1 + f_3 + f_4, \\ B &= f_1 f_3 + f_1 f_4 + f_3 f_4 - \gamma_1\theta_1 - \gamma_2\theta_2 > 0, \\ C &= f_1 f_3 f_4 - \gamma_1\theta_1 f_4 - \gamma_2(\theta_2 f_3 + \alpha_2\theta_1) = A_4 > 0. \end{aligned} \tag{8}$$

To ensure that the CFE is locally asymptotically stable, all the eigenvalues of the characteristic equation of the CFE, i.e. the roots of the characteristic equation of E_0, must have negative real parts; otherwise, it is considered unstable. Adopting the concept of the Routh-Hurwitz criterion for the cubic polynomial (7), it implies that negative solutions exist if the following conditions $A > 0,\ B > 0,\ C > 0,\ AB > C$ are satisfied. Clearly, $A = f_1 + f_3 + f_4 > 0$. Furthermore, substituting the definitions of f_1, f_3 and f_4 in (5) yields

$$\begin{aligned} B = {} & \alpha_2\gamma_2 + 2\alpha_2\mu + \alpha_2\mu_R + \alpha_2\theta_1 + \alpha_2\theta_2 \\ & + \gamma_1\gamma_2 + 2\gamma_1\mu + \gamma_1\mu_R + \gamma_1\theta_2 \\ & + 2\gamma_2\mu + \gamma_2\theta_1 + 3\mu^2 + 2\mu\mu_R \\ & + 2\mu\theta_1 + 2\mu\theta_2 + \mu_R\theta_1 + \mu_R\theta_2 > 0. \end{aligned}$$

Also, $C = A_4 > 0$. Since all model parameters are non-negative and $\Lambda, \mu > 0$, direct expansion of $AB - C$ gives a sum of positive parameter products. Hence, $AB - C > 0$. Thus, all roots of the cubic polynomial have negative real parts. Moreover, $\lambda_1 = f_2(N_0 - 1) < 0$ whenever $N_0 < 1$. Therefore, all eigenvalues of $J(E_0)$ have negative real parts when $N_0 < 1$ and $\alpha_1 = 0$. Hence, the crime-free equilibrium E_0 is locally asymptotically stable when $N_0 < 1$ and $\alpha_1 = 0$. When $N_0 > 1$, $\lambda_1 = f_2(N_0 - 1) > 0$, and therefore the crime-free equilibrium is unstable. This completes the proof. □

In a scenario where N_0 is less than 1, the number of new cybercriminals recruited is negligible relative to the number of criminals leaving the system through rehabilitation or prosecution. The minor fluctuations in the size of the cybercriminal population will ultimately converge to zero, provided there are no temporarily employed individuals committing crimes due to economic instability, thereby restoring society to its pre-crime state. Generally, when $N_0 > 1$, the system is destabilised and approaches a cybercrime-presence equilibrium, in which cybercrime is maintained.

2.4.2. Global stability of the crime-free equilibrium state

Although local stability guarantees the system will reach the state of E_0 once perturbed slightly, global stability guarantees that the society will finally stabilise in the state of cybercrime-free, irrespective of the size of the population of criminals at the time of the initial perturbation. Using the comparison method in Ref. [37], as adopted in Ref. [38], the E_0 global stability is derived by redrafting the model equations in Eq. (1) as follows:

$$\frac{\mathrm{d}X^f}{\mathrm{d}t} = \Pi(X^f, Y^f), \tag{9}$$

$$\frac{\mathrm{d}Y^f}{\mathrm{d}t} = \Omega(X^f, Y^f),\ \Omega(X^f, 0) = 0, \tag{10}$$

where $X^f = (U, T, R) \in \mathbb{R}^3_+$ is the sub-classes that are cybercrime free, $Y^f = (C) \in \mathbb{R}^1_+$ refers to the group of individuals engaged in cybercrime. With X^f_0 as E_0, we show that the following conditions are satisfied;

Γ_1: For

$$\frac{dX^f}{dt} = \Pi(X^f_0, 0),$$

X_0^f is globally asymptotically stable.

Γ_2:
$$\Omega(X^f, Y^f) = AY^f - \hat{\Omega}(X^f, Y^f),$$
$$\hat{\Omega}(X^f, Y^f) \geq 0.$$

where $\left(X^f, Y^f\right) \in \Phi$ is the feasible solution region of the model and $AD_y\left[\Omega\left(X_0^f, 0\right)\right]$ is an M-matrix.

Proving the first condition, we state global stability of E_0 as follows:

Theorem 2.3. If $N_0 < 1$ and the rate at which temporarily employed individuals become cybercrime participants is zero, then the crime-free equilibrium state is given as

$$E_0 = (U^0, C^0, T^0, R^0) = \left(\frac{\Lambda f_3 f_4}{A_4},\ 0,\ \frac{\theta_1 U^0}{f_3},\ \frac{A_2 U^0}{f_3 f_4}\right)$$

is globally asymptotically stable; otherwise, it is unstable.

Proof. For the first condition. Γ_1, the cybercrime-free classes in the sub-model could be presented as

$$\frac{dX^f}{dt} = \Pi(X^f, 0),$$

where

$$\Pi(X^f, 0) = \begin{pmatrix} \Lambda + \gamma_1 T + \gamma_2 R - (\theta_1 + \theta_2 + \mu)U, \\ \theta_1 U - (\alpha_1 + \gamma_1 + \alpha_2 + \mu)T, \\ \theta_2 U + \alpha_2 T - (\gamma_2 + \mu_R + \mu)R. \end{pmatrix}.$$

With $X^f = \{U, T, R\}$ as the crime-free classes of the model under this regard, and the crime-present class $[C]$ is zero. The Jacobian matrix of $\Pi(X^f, 0)$ is established as follows

$$\Pi(X^f, 0) = \begin{pmatrix} -f_1 & \gamma_1 & \gamma_2 \\ \theta_1 & -f_3 & 0 \\ \theta_2 & \alpha_2 & -f_4 \end{pmatrix}, \tag{11}$$

where, $f_1,\ f_3,\ f_4$ are defined in (5). We need to establish that $\Pi(X^f, 0)$ (11) has eigenvalues with negative real parts. Hence, we show that the characteristic equation of the Jacobian matrix (11) as

$$a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0, \tag{12}$$

where

$$\begin{aligned} a_1 &= 1 > 0, \qquad a_2 = f_1 + f_2 + f_3 > 0, \\ a_3 &= f_3 f_4 + (f_4 + \gamma_1)(\theta_1 + \mu) \\ &\quad + f_1(\alpha_1 + \alpha_2 + \mu) + \theta_2(\mu_R + \mu) > 0, \\ a_4 &= \mu f_3 f_4 + \theta_2 f_3(\mu_R + \mu) \\ &\quad + \theta_1 f_4(\alpha_1 + \alpha_2 + \mu). \end{aligned}$$

According to Heffernan *et al.* [39], the coefficients of the characteristic equation of Eq. (12) $a_1, a_2, a_3, a_4 > 0$ implies that the Jacobian Matrix (11) has negative real eigenvalues. Thus, it is globally asymptotically stable. For Γ_2 condition, we have from the model system that $\Omega\left(X^f, Y^f\right) = AY^f - \hat{\Omega}\left(X^f, Y^f\right)$ where

$$\Omega(X^f, Y^f) = \begin{bmatrix} \beta UC + \alpha_1 T - (\delta_1 + \delta_2 + d + \mu)C \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$A = \begin{bmatrix} \beta U^0 - (\delta_1 + \delta_2 + d + \mu) \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\hat{\Omega}\left(X^f, Y^f\right) = \begin{bmatrix} \beta(U^0 - U)C - \alpha_1 T \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

With $\alpha_1 = 0$, it is clear that $\hat{\Omega}\left(X^f, Y^f\right) \geq 0$ since $U^0 \geq U$.

As a result, E_0 is globally asymptotically stable when $N_0 < 1$, and the rate at which temporarily employed individuals become involved in cybercrime is zero. □

The stability results indicate that eliminating cybercrime requires keeping N_0 below 1. This can be achieved by reducing interaction rates through public awareness or by increasing exit rates through improved legal enforcement and job creation. If these factors are left unmanaged, the system risks shifting from a state of no crime to one in which cybercrime becomes the norm.

We present a graphical illustration of global stability in Figure 2 to support Theorem 2.3, using $\alpha_1 = 0$ and $N_0 = 0.50 < 1$ to simulate various initial conditions of the cybercrime population. Regardless of the initial populations, all trajectories converge to the crime-free equilibrium $C^0 = 0$ over time. This shows that when $N_0 < 1$ and $\alpha_1 = 0$, the long-term behaviour of the system leads to the elimination of cybercrime, confirming the global asymptotic stability of the crime-free equilibrium.

2.4.3. Bifurcation analysis

Bifurcation analysis is another vital part of qualitative modelling, which is used to explore shifts in system stability as the cybercrime generation Number (N_0) crosses unity. This analysis identifies whether a simple decrease in the value of N_0 below one is enough to eradicate cybercrime or social complexities, including imitation by criminals or its impact on peers (forward bifurcation), or give rise to the situation in which the crime will not be eliminated by the favorable economic indicators even if the value of N_0 remains below one (backward bifurcation) [40], implying that $N_0 < 1$ is insufficient to control cybercrime, indicating needs for additional measures to reduce it.

In an attempt to assess the direction of bifurcation at the $N_0 = 1$, we apply the center manifold theory [41] as applied by Tijani *et al.* [34]. It is related to linearising the system of differential equations about the cybercrime-free equilibrium and computing the signs of two bifurcation coefficients, a and b, obtained from the second-order partial derivatives and eigenvectors of the Jacobian matrix with respect to the bifurcation parameter.

The bifurcation parameter is selected as a number denoted by $\beta = \beta^*$ at the cybercrime generation number, (N_0). It is derived as:

$$\beta^* = \frac{f_2 A_4}{\Lambda f_3 f_4}.$$

From the Jacobian matrix (13), the eigenvalues exist and have a simple zero eigenvalue at $N_0 = 1$ and $\alpha_1 = 0$. Therefore, we compute the left and right eigenvalues of the Jacobian matrix

$$J(E_0,\beta^*) = \begin{pmatrix} -f_1 & -\dfrac{\beta\Lambda f_3 f_4}{A_4} & \gamma_1 & \gamma_2 \\ 0 & \dfrac{\beta\Lambda f_3 f_4}{A_4} - f_2 & \alpha_1 & 0 \\ \theta_1 & \delta_1 & -f_3 & 0 \\ \theta_2 & \delta_2 & \alpha_2 & -f_4 \end{pmatrix} \tag{13}$$

$$= \begin{pmatrix} -f_1 & -f_2 & \gamma_1 & \gamma_2 \\ 0 & 0 & 0 & 0 \\ \theta_1 & \delta_1 & -f_3 & 0 \\ \theta_2 & \delta_2 & \alpha_2 & -f_4 \end{pmatrix}.$$

at $N_0 = \frac{\beta\Lambda f_3 f_4}{A_4 f_2} = 1$, $\alpha_1 = 0$. For left eigenvalues, $\mathbf{w} = (w_1, w_2, w_3, w_4)$, we multiply the $J(E_0,\beta^*)$ with $\mathbf{w}$ and equate to zero, to get

$$\begin{aligned} w_1 &= -\frac{c_1}{c_2} w_2, \qquad w_2 > 0, \\ w_3 &= \frac{\theta_1}{f_3} w_1 + \frac{\delta_1}{f_3} w_2, \\ w_4 &= \left(\frac{\theta_2 f_3 + \alpha_2\theta_1}{f_3 f_4}\right) w_1 + \left(\frac{\delta_1 f_3 + \alpha_2\delta_2}{f_3 f_4}\right) w_2. \end{aligned} \tag{14}$$

where $c_1 = f_1 - \frac{\gamma_1\theta_1}{f_3} - \gamma_2\left(\frac{\theta_2 f_3 + \alpha_2\theta_1}{f_3 f_4}\right) > 0$, $c_2 = f_2 - \frac{\gamma_1\delta_1}{f_3} - \gamma_2\left(\frac{\delta_1 f_3 + \alpha_2\delta_2}{f_3 f_4}\right) > 0$ using the definitions of f_i for $i = 1, \ldots, 4$ in (5).

Also, obtaining the right eigenvalues $\mathbf{v} = (v_1, v_2, v_3, v_4)$, we transpose the Jacobian matrix, $J(E_0,\beta^*)$ (13) and multiply with $\mathbf{v}$ and equate to zero. This yields

$$\begin{aligned} v_1 &= 0, \qquad v_2 > 0, \\ v_3 &= \left(\frac{\gamma_1 f_4 + \alpha_2\gamma_2}{f_3 f_4}\right) v_1, \\ v_4 &= \frac{\gamma_2}{f_4} v_1. \end{aligned}$$

as we have $v_1\left(-f_1 + \frac{\gamma_1 f_4 + \alpha_2\gamma_2}{f_3 f_4} + \frac{\gamma_2}{f_4}\right) = 0$ and $-f_1 + \frac{\gamma_1 f_4 + \alpha_2\gamma_2}{f_3 f_4} + \frac{\gamma_2}{f_4} \neq 0$. Next, we derive the bifurcation coefficients, a and b by finding the non-zero second partial derivatives of the model system (1) at cybercrime-free equilibrium state using the first and second equations. Denoting the state variables: U, C, T, R, as: x_1, x_2, x_3, x_4, we have

$$\begin{aligned} \frac{\partial^2 f_1(E_0)}{\partial x_1 \partial x_2} &= -\beta, & \frac{\partial^2 f_2(E_0)}{\partial x_1 \partial x_2} &= \beta, \\ \frac{\partial^2 f_1(E_0)}{\partial x_2 \partial \beta} &= -U^0 = -\frac{\Lambda f_3 f_4}{A_4}, & \frac{\partial^2 f_2(E_0)}{\partial x_2 \partial \beta} &= U^0 = \frac{\Lambda f_3 f_4}{A_4}. \end{aligned}$$

With these derivatives and the right and left eigenvectors, v and w, the coefficients a and b are obtained:

$$a = -\beta v_2 w_2^2 \frac{c_1}{c_2} < 0, \quad b = v_2 w_2 \frac{\Lambda f_3 f_4}{A_4} > 0.$$

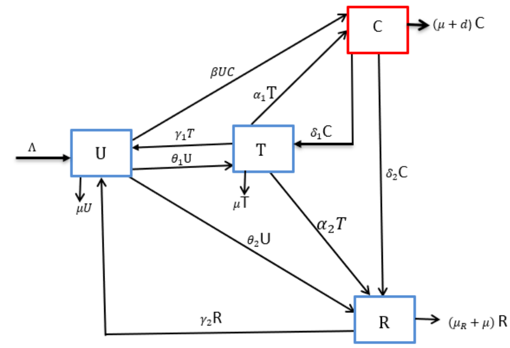


Figure 1: Schematic diagram of the cybercrime model.

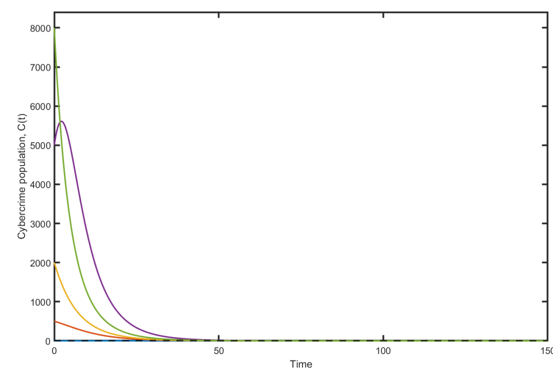


Figure 2: Numerical illustration of the global stability of the crime-free equilibrium for $\alpha_1 = 0$ and $N_0 = 0.5 < 1$, simulated from different initial conditions. The trajectories converge to the crime-free equilibrium, $C = 0$ (dotted line).

So, we have a forward bifurcation at the cybercrime generation number $N_0 = 1$ since $a < 0$ and $b > 0$ for $\alpha_1 = 0$, meaning that no backward bifurcation exists. This is further illustrated in Figure 3 using the definition of f_i and A_j in (5), where $i = 1, \ldots, 4$ and $j = 1, \ldots, 4$ and the quadratic equation (4) such that $B_2 = \alpha_1(A_5\theta_1 - A_4\delta_1) + A_4 f_2 f_3 - \Lambda\beta f_3^2 f_4 = \alpha_1(A_5\theta_1 - A_4\delta_1) + A_4 f_2 f_3(1 - N_0)$.

The findings suggest that, regardless of the prevailing rate of cybercrime associated with unemployment, its incidence can be significantly reduced if $N_0 < 1$ and $\alpha_1 = 0$, provided that measures such as the establishment of empowerment programs, skill acquisition initiatives, and comprehensive awareness campaigns regarding the consequences of cybercrime are implemented by the government and relevant stakeholders. These results are consistent with the findings of [11, 14]. Conversely, if $N_0 > 1$, cybercrime is likely to become prevalent among unemployed individuals, particularly youth, potentially resulting in economic instability and posing a significant threat to society, as demonstrated by Ref. [15].

3. Sensitivity analysis

Table 1: State variables of the model.

State variable	Interpretation	Unit
$U(t)$	Number of unemployed persons at time t	Human population
$C(t)$	Number of persons engaged in cybercrime at time t	Human population
$T(t)$	Number of temporarily employed persons at time t	Human population
$R(t)$	Number of fully employed persons at time t	Human population

Table 2: Description and parameter values of the model.

Parameter	Description	Unit	Value	Source
Λ	Human recruitment rate	Human population/time	500	Assumed
β	Cybercrime influence rate	$time^{-1}$	0.00022	Assumed
θ_1	Rate at which unemployed persons obtain temporary employment	$time^{-1}$	0.048	Assumed
θ_2	Rate at which unemployed persons obtain full employment	$time^{-1}$	0.05	Ref. [36]
γ_1	Rate at which temporarily employed persons lose employment	$time^{-1}$	0.00009	Assumed
γ_2	Rate at which fully employed persons lose employment	$time^{-1}$	0.00009	Assumed
δ_1	Rate at which persons engaged in cybercrime obtain temporary employment	$time^{-1}$	0.1125	Assumed
δ_2	Rate at which persons engaged in cybercrime obtain full employment	$time^{-1}$	0.05	Ref. [36]
α_1	Rate at which temporarily employed persons become cybercrime participants because of employment instability	$time^{-1}$	0.04	Ref. [46]
α_2	Rate at which temporarily employed persons become fully employed	$time^{-1}$	0.04	Ref. [46]
μ_R	Retirement rate for fully employed persons	$time^{-1}$	0.007	Assumed
d	Legal-repercussion rate for persons engaged in cybercrime	$time^{-1}$	0.066	Assumed
μ	Natural human death rate	$time^{-1}$	0.00247	Ref. [47]

Table 3: Possible real roots of the polynomial in Eq. (4).

Case	B_1	B_2	B_3	Possible real roots
1	+	−	−	1
2	+	+	−	1

Table 4: Sensitivity indices of selected model parameters.

Parameter	Index sign	Sensitivity index
β	+	0.9999
δ_1	−	0.6870
δ_2	−	0.2164
θ_1	−	0.3200
θ_2	−	0.6476
γ_1	+	0.00035
γ_2	+	0.008933
d	−	0.2857
α_1	+	0.1550
α_2	−	0.1461

Sensitivity analysis examines uncertainty in model parameters and in the properties of observable outputs (model outputs) [42]. It is primarily used to assess the accuracy of mathematical model predictions across a range of parameter values. It can also be used to identify model parameters that significantly affect the cybercrime generation Number (N_0) and the model state variables, thereby informing intervention targeting [43].

Sensitivity analysis techniques include Local Sensitivity Analysis (LSA), which evaluates input effects at a single point, and Global Sensitivity Analysis (GSA), which assesses multiple points for average sensitivities [44]. In this paper, the concept of global and local sensitivity analysis, as used by Ref. [45] and Ref. [35], is applied. The global sensitivity analysis is deployed using the Partial rank correlation coefficient (PRCC) with Latin hypercube sampling (LHS) techniques of 1000 runs. It is noteworthy to mention that the signs (+ and -) of PRCC show the definite qualitative relationship between the parameters and output on N_0, C, T, and R. The parameters with positive PRCC mean that increasing them increases the value of N_0, C, T, and R, which, by implication, increases the spread of cybercrime in terms of N_0, and C but increases the number of employed population (temporary and permanent) via T, and R. At the same time, parameters with negative PRCC values indicate that N_0, C, T, and R decrease as their values increase, implying reduced influence of cybercrime dynamics and the unemployment rate, respectively. Additionally, the most significant parameters are those with $|PRCC| \geq 0.5$. However, those

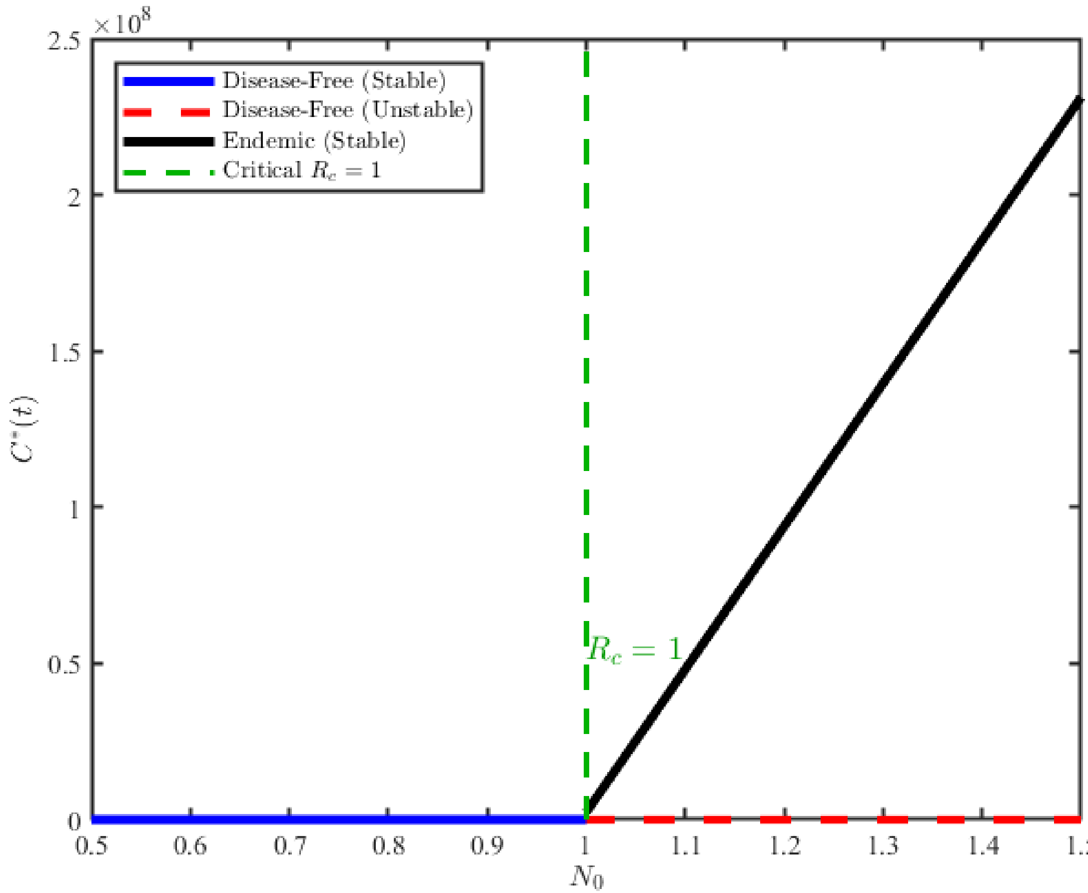


Figure 3: Forward bifurcation plot of the cybercrime model for $\alpha_1 = 0$.

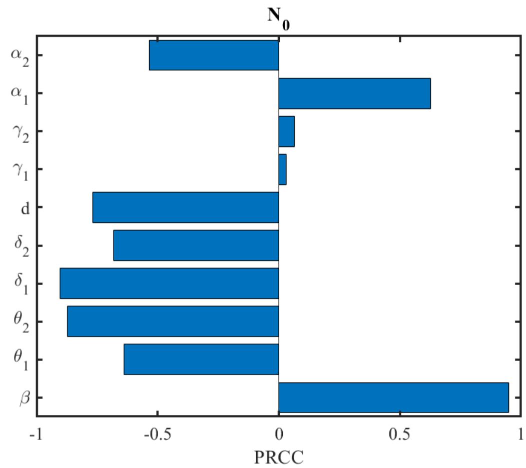
Table 5: Descriptive statistics from the model sensitivity analysis.

Compartment	Mean	Median	Minimum	Maximum	Variance
$C(t)$	3783.2	3769.7	2990.5	4540.4	79522
$T(t)$	3822.3	3822.2	3088	4479	78160
$R(t)$	4157.5	4161.5	3552.2	4770.5	57062

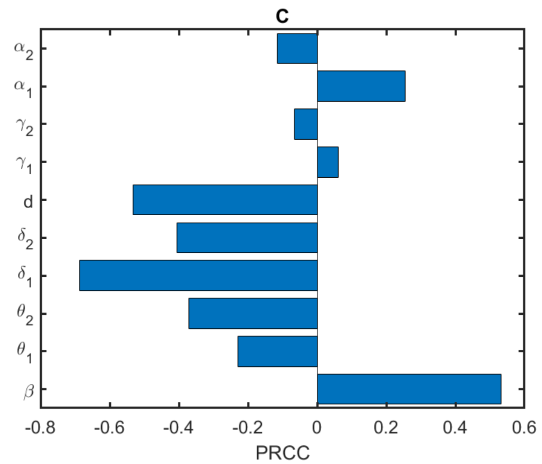
with $|PRCC| < 0.5$ can also contribute to the dynamics of cybercrime in society.

Figure 4 presents a visual representation of the PRCCs for the parameters as drivers and inhibitors of the cybercrime epidemic. Based on the analysis presented in Figure 4a and 4b, the most influential factors of the social contagion are the rate of influence (β) and the rate of being recruited into crime through temporary work (α_1), and they are positively correlated with both N_0 and the criminal compartment $C(t)$. These results suggest that the non-employed sector is an important entry point for illegal online practices. However, the legal repercussion rate (d), the transition rate between temporary and full employment (α_2), and the rehabilitation rates (δ_1, δ_2) exhibit a strong negative correlation with both N_0 and the cybercriminal compartment $C(t)$. This means that the most effective socio-economic interventions to eliminate cybercrime are increasing the employment rate and strengthening legal enforcement against criminal activities [27]. Furthermore, Figure 4c reveals that the population of temporarily employed individuals ($T(t)$) increases with higher values of $\beta, \delta_1, \theta_1$ while in Figure 4d, the population of fully employed individuals ($R(t)$) rises with increasing $\alpha_2, \delta_2, \theta_2$, making these the most influential parameters as their PRCCs exceed 0.5. To reduce cybercrime in society, improving full employment is crucial, driven by the transition of temporary workers to full employment and efforts to integrate unemployed individuals.

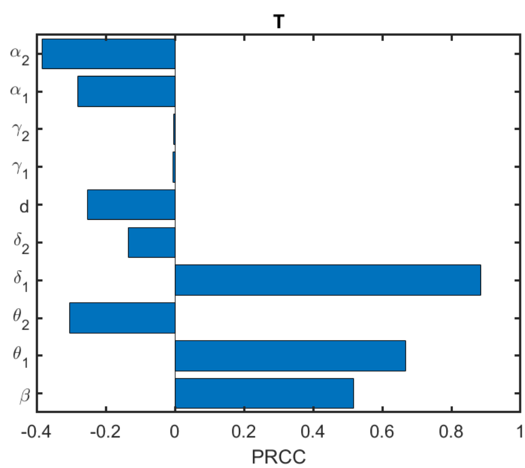
Local sensitivity analysis quantifies the impact of a small influence around a nominal parameter value on the model output. This approach evaluates one parameter at a time by calculating the first-order partial derivative of the model output with respect to that parameter [48]. The analysis is implemented using the



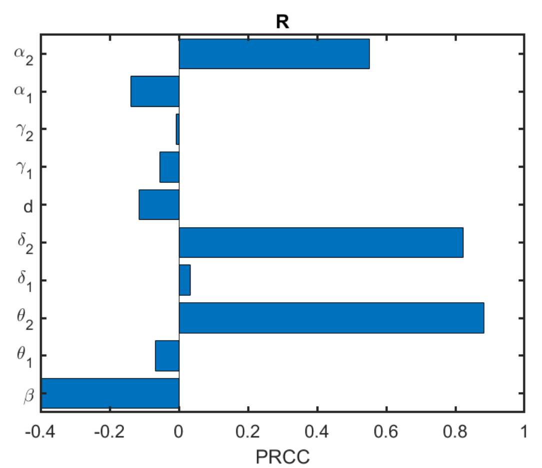
(a) PRCCs for selected parameters of N_0.



(b) PRCCs for selected parameters of $C(t)$.



(c) PRCCs for selected parameters of $T(t)$.



(d) PRCCs for selected parameters of $R(t)$.

Figure 4: Partial rank correlation coefficients (PRCCs) showing the effects of selected model parameters on N_0, $C(t)$, $T(t)$, and $R(t)$. The results were generated from 1000 runs using parameter values ranging from 0% to 140% of the values in Table 2.

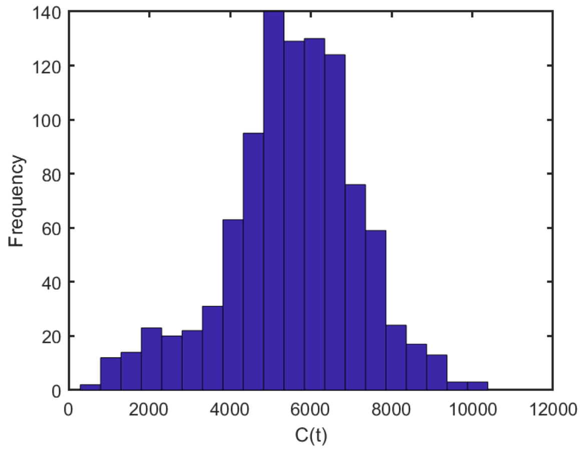
normalised forward index method as described below:

$$S_p^{N0} = \frac{\partial N_0}{\partial p} \times \frac{p}{N_0},$$

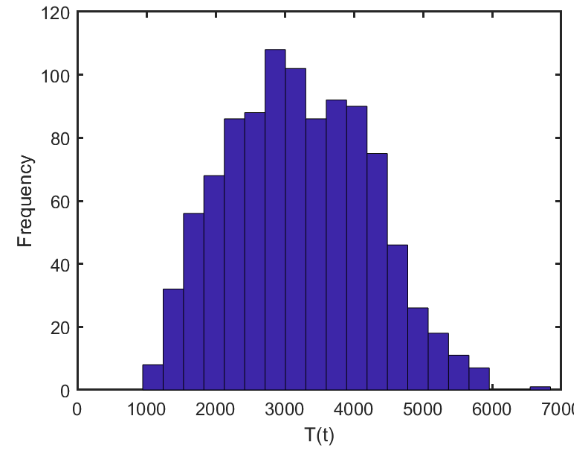
where S_p^{N0} is the sensitivity index of parameter p and N_0 and is the basic generation number (reproduction number). A local sensitivity analysis was conducted on N_0 using normalised forward sensitivity indices to determine which parameters influence cybercrime. The sensitivity indices for the parameters of N_0 are presented in Table 4, which are consistent with the results of the global sensitivity analysis. The indices in Table 4 indicate that the cybercrime generation number, N_0, decreases as the legal repercussion rate (d), the transition rate between temporary and full employment (α_2), the rehabilitation rates (δ_1, δ_2), and the rate at which unemployed persons obtain temporary employment (θ_1) increase. An increase in the rate at which unemployed persons obtain full employment (θ_2) also contributes to this decrease, with δ_1 and θ_2 identified as the most influential parameters. These findings suggest that enhancing these parameters may reduce cybercrime in the society.

An increase in the rate of influence (β), the rate of recruitment into crime through temporary work (α_1), and the rate at which temporarily employed individuals lose employment (γ_1) all contribute to an increase in the cybercrime generation number, N_0. Additionally, the rate at which fully employed individuals lose employment (γ_2) further increases N_0. Among these, (β) and (α_1) are identified as the most influential parameters. Consequently, increases in these parameters are associated with a higher prevalence of cybercrime within the population.

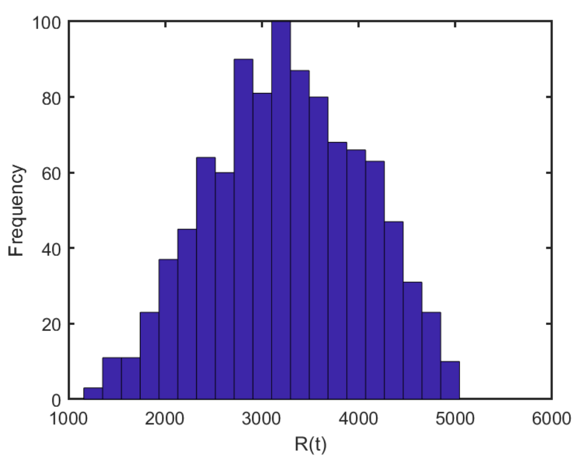
The overall effect of these parameters at the end of a 10-year simulated period is shown in the descriptive statistics in Table 5. The data indicates that the population of cybercriminals (C) has a predicted mean and median of 3783.2 and 3769.7, respectively, and relatively small population sizes of temporarily employed (T) and fully employed (R) indicate the means of 3822.3 and 4157.5, respectively. The closeness of the mean and median values across all compartments indicates that the model results are nearly symmetric and not predominantly skewed by outliers, supporting the model's reliability for prediction. The large variance observed, that is, 79522 in the $C(t)$ compartment,



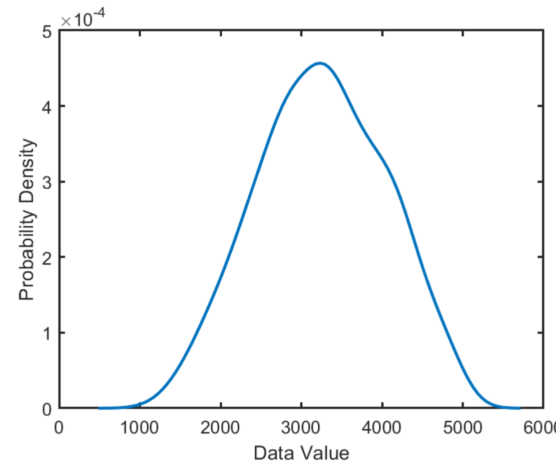
(a) Frequency distribution for $C(t)$.



(b) Frequency distribution for $T(t)$.

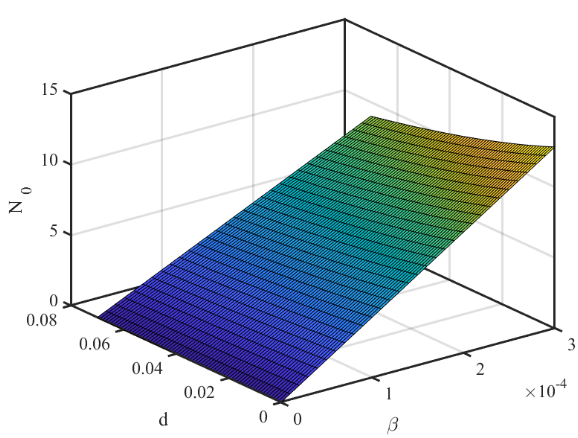


(c) Frequency distribution for $R(t)$.

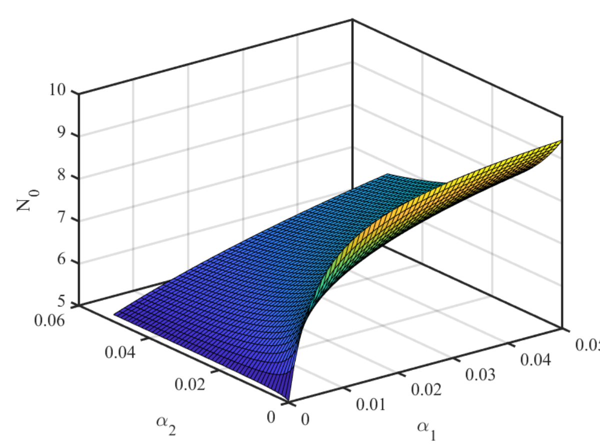


(d) Kernel density estimate for 1000 runs.

Figure 5: Frequency distributions of cumulative employment and crime cases after 10 years and the kernel density estimate generated from 1000 runs using parameter values ranging from 0% to 140% of the values in Table 2.

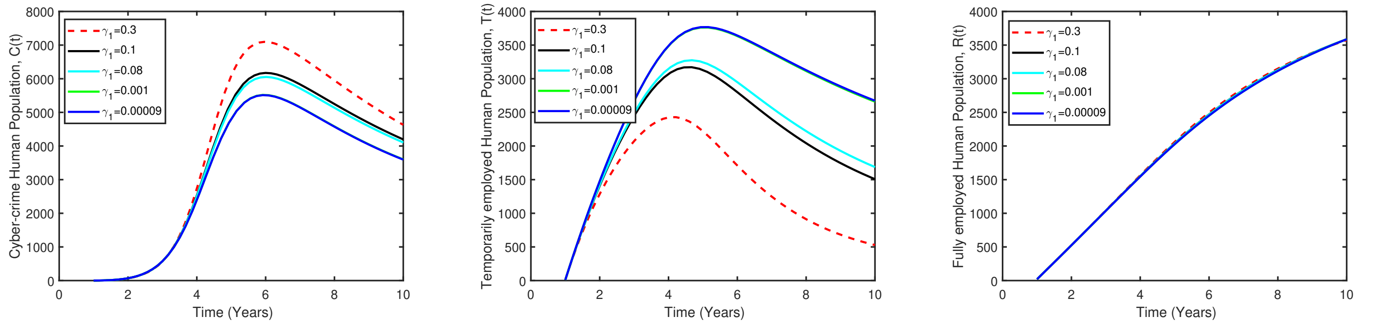


(a) Three-dimensional plot of the effects of β and d on N_0.



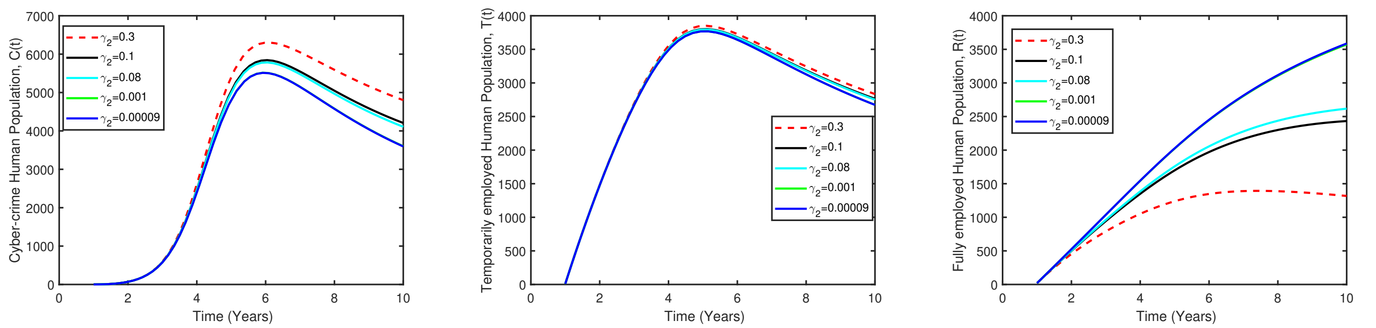
(b) Three-dimensional plot of the effects of α_1 and α_2 on N_0.

Figure 6: Three-dimensional plots showing the effects of selected model parameters on N_0.



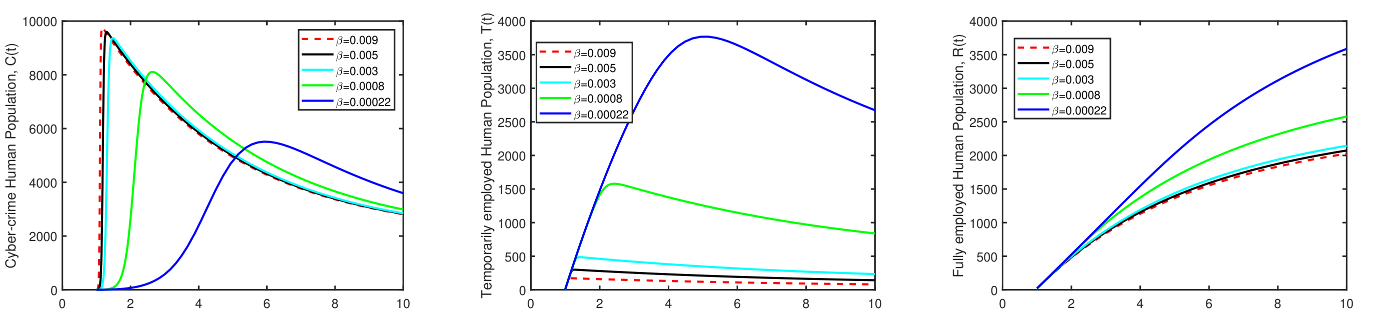
(a) Effect of γ_1 on $C(t)$. (b) Effect of γ_1 on $T(t)$. (c) Effect of γ_1 on $R(t)$.

Figure 7: Effect of the rate at which temporarily employed persons lose employment, γ_1, on $C(t)$, $T(t)$, and $R(t)$, using the parameter values in Table 2.



(a) Effect of γ_2 on $C(t)$. (b) Effect of γ_2 on $T(t)$. (c) Effect of γ_2 on $R(t)$.

Figure 8: Effect of the rate at which fully employed persons lose employment, γ_2, on $C(t)$, $T(t)$, and $R(t)$, using the parameter values in Table 2.



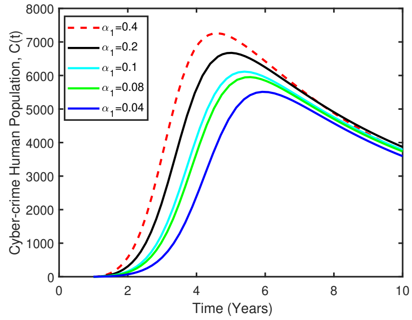
(a) Effect of β on $C(t)$. (b) Effect of β on $T(t)$. (c) Effect of β on $R(t)$.

Figure 9: Effect of the influence rate, β, on $C(t)$, $T(t)$, and $R(t)$, using the parameter values in Table 2.

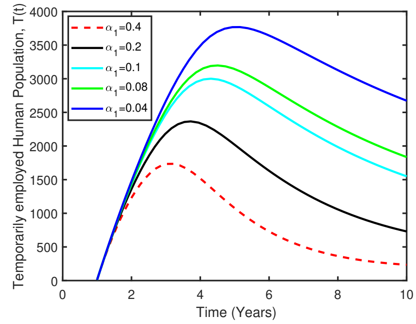
however, is an indication of a wide confidence interval in the forecast, which demonstrates that the system is quite sensitive to external factors of the labour market and social peer pressure.

Figure 5 supports these statistical observations using frequency histograms and Kernel Density Estimates, which provide a visualisation of the uncertainty in the model's 10-year projections. The cybercriminal and employment histograms are nearly symmetrical, and the mean is nearly equal to the median, indicating that the system consistently yields a predictable performance over 1000 runs. The frequency distribution indicates that, by year 10, there is a high probability that more than 4, 540 individuals will commit cybercrime, and approximately 9, 249 individuals will be temporarily or fully employed. The kernel pattern also reveals the likelihood of such cases and shows that the dual-pronged approach that maximises movement toward the fully employed category and breaks pathways of peer-to-peer influence that lead to the recruitment of cybercriminals is the only way to establish a stable society. The statistical simulations were performed using a 95% confidence interval.

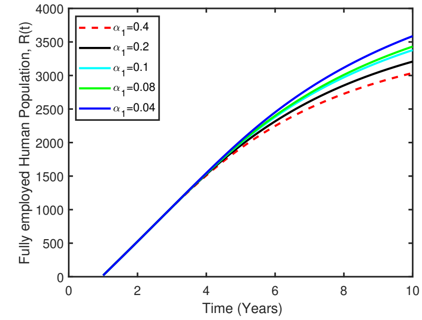
The sensitivity analysis reveals that the most significant



(a) Effect of α_1 on $C(t)$.

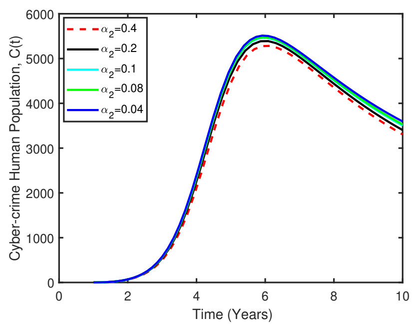


(b) Effect of α_1 on $T(t)$.

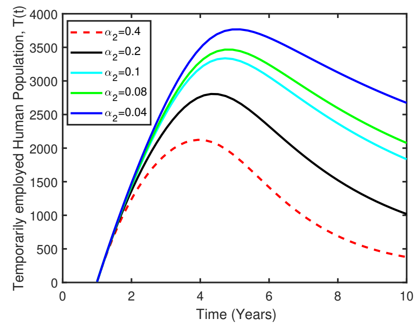


(c) Effect of α_1 on $R(t)$.

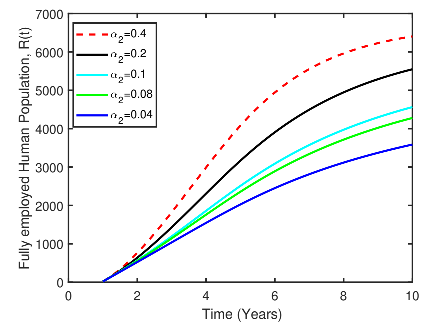
Figure 10: Effect of the rate at which temporarily employed persons become cybercriminals, α_1, on $C(t)$, $T(t)$, and $R(t)$, using the parameter values in Table 2.



(a) Effect of α_2 on $C(t)$.

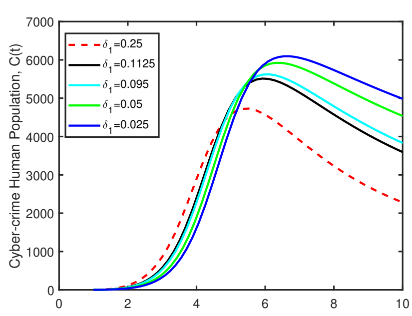


(b) Effect of α_2 on $T(t)$.

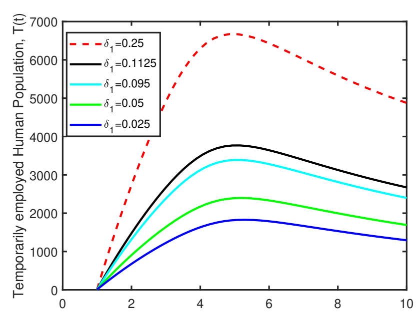


(c) Effect of α_2 on $R(t)$.

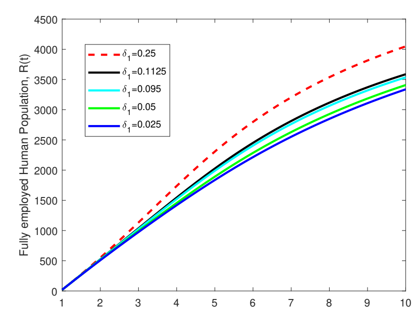
Figure 11: Effect of the rate at which temporarily employed persons become fully employed, α_2, on $C(t)$, $T(t)$, and $R(t)$, using the parameter values in Table 2.



(a) Effect of δ_1 on $C(t)$.



(b) Effect of δ_1 on $T(t)$.



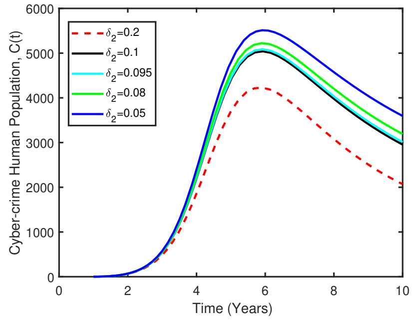
(c) Effect of δ_1 on $R(t)$.

Figure 12: Effect of the rate at which cybercriminals obtain temporary employment, δ_1, on $C(t)$, $T(t)$, and $R(t)$, using the parameter values in Table 2.

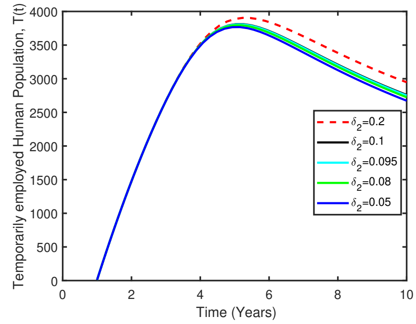
drivers of the cybercrime generation number (N_0) are peer influence (β) and unstable recruitment (α_1). The most effective inhibitors are full employment (α_2) and legal repercussions (d). Even though the $1,000$ simulations project means $3,783.2$ cybercriminals in 10 years, which could be higher when using real data, the high variance indicates that it is highly sensitive to socio-economic shocks. In turn, these findings indicate that eliminating cybercrime requires a structural policy change favouring permanent employment over temporary employment approaches to social security.

4. Numerical simulations and discussion

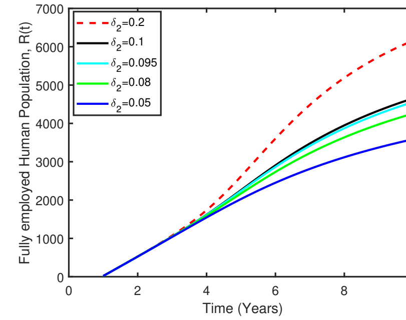
The model's analytical findings are confirmed by the numerical results presented in this section, including the stability of steady states and the threshold behaviour controlled by N_0. Within the framework of the fourth-order Runge-Kutta method, the development of the population compartments is simulated



(a) Effect of δ_2 on $C(t)$.

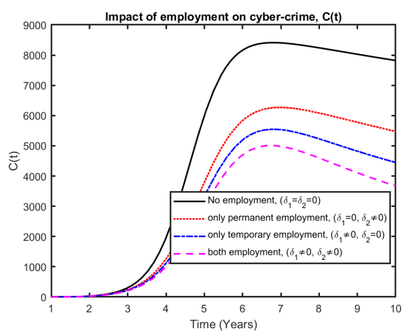


(b) Effect of δ_2 on $T(t)$.

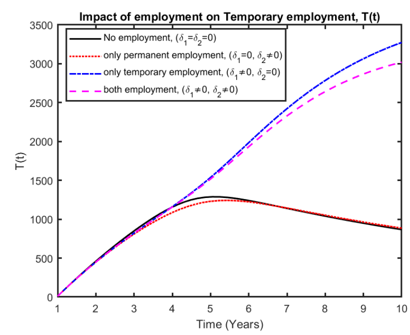


(c) Effect of δ_2 on $R(t)$.

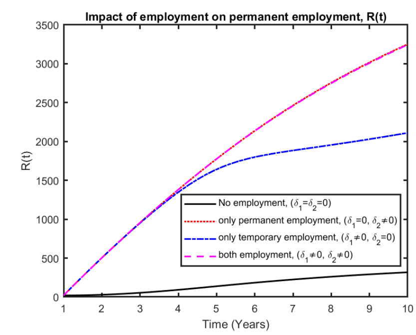
Figure 13: Effect of the rate at which cybercriminals obtain full employment, δ_2, on $C(t)$, $T(t)$, and $R(t)$, using the parameter values in Table 2.



(a) Employment scenarios for $C(t)$.



(b) Employment scenarios for $T(t)$.



(c) Employment scenarios for $R(t)$.

Figure 14: Effects of employment scenarios on $C(t)$, $T(t)$, and $R(t)$, using the parameter values in Table 2.

under the initial conditions for the human compartments, with $U = 10,000$, $C = 1$, $T = 10$, and $R = 20$, and with baseline parameter values listed in Table 2.

Figure 6 shows a 3D surface plot of the relationship between the model parameters and the Cybercrime Generation Number, N_0. In particular, as Figure 6a illustrates, N_0 is extremely sensitive to the interaction of the influence rate (β), although it is very sensitive to the legal exit rate (d). There is a sharp increase in N_0 with increasing β, indicating a high probability that cybercrime will become endemic. The increased exit rate (d), which can be equated to efficient legal enforcement and rehabilitation, on the other hand, dramatically reduces the value of N_0. Besides, Figure 6b shows the effects of transitions between the temporarily employed class (T) on N_0. Increasing the rate at which these individuals become cybercriminals (α_1) increases N_0, whereas increasing the transition rate to full employment (α_2) reduces N_0. This underscores that the transfer of people into stable, full-time jobs is essential for reducing the contagion potential of cybercriminal networks [24].

The outcomes of the simulations in Figures 7 and 8 show the relationship between the rate of job loss and the continuation of cybercriminal activity. Figure 7 investigates the effect of the rate of job loss among temporarily employed people. The findings show that as the value of γ_1 increases, the number of people committing cybercrime will further increase ($C(t)$), since individuals who lose interim sources of income tend to resort to illegal internet practices to survive. Equally, Figure 8 shows how the fully employed are affected by job loss among the fully employed (γ_2). As the value of γ_2 increases to 0.3, the number of cybercriminals is expected to peak at approximately 6,400 individuals in 6 years. Even at a low rate of 0.00009, the model indicates a base criminal population of approximately 3,500 and that instability in the formal labour market can sustain an endemic level of cybercrime. These results demonstrate the significance of both temporary and full employment in influencing the dynamics of cybercrime, highlighting the necessity for substantial societal intervention. These findings are consistent with the recommendations of Anyanwu [12] and Chen *et al.* [9].

Figure 9 shows the impact of peer interaction on the recruitment of criminals. The higher the influence rate (β), the greater the number of people committing cybercrime, as shown in Figure 9a. At the same time, it is evident that the smaller the value of β, the larger the population of temporarily employed people (Figure 9b), which implies that youths who have gained a work position are unlikely to be affected by criminal peer networks. Moreover, Figure 9c indicates that when the rate of influence drops, the number of the fully employed population (R) also grows, supporting the idea that social stability is maximised when the rate of being infected by cybercriminal behaviour is limited with the help of social monitoring and awareness. These outcomes are consistent with the findings of Gregory and Grace [10], who report that peer influence is a significant predictor of the tendency toward internet fraud. Furthermore, Ewuzie *et al.* [13] and Chen *et al.* [9] suggest that this influence can be mitigated through gainful employment.

Figures 10 and 11 examine how population changes as a function of the transition rates α_1 and α_2. Figure 10 indicates that with the rate of temporarily employed into the cybercriminal category, denoted as α_1, the population also increases sharply: at the rate of $\alpha_1 = 0.4$, the population of cybercriminals rises to about 7,400 by the end of year 4, whereas at the rate of $\alpha_1 = 0.04$, the population is about 5,200 at the end of year 6. This underscores that higher recruitment rates of the non-employed or temporary population lead to more rapid and severe outbreaks of cybercrime. On the other hand, Figure 11 shows that although the rate of shift to full employment, denoted by α_2, does not have a significant influence on the current cybercriminal population, denoted by $C(t)$, it has a strong influence on the workforce stability. An increase in the value of α_2 leads to a reduction in the number of those who are temporarily employed and an increase in the number of fully employed people, which happens sharply as depicted in Figure 11c. These findings conform with the results by Anyanwu [12]

Figures 12 and 13 show the effects of the change in cybercriminal compartment into temporary employment and direct full employment on the stability of society. Figure 12 indicates that the presence of temporary employment is inversely correlated with the prevalence of cybercrimes; the higher the temporary employment δ_1 is, the lower the number of cybercriminals, ($C(t)$) is. In particular, there is an optimal number of 6,800 cybercriminals at a transition rate, $\delta_1 = 0.25$. A low transition rate does not provide job opportunities for youth, leaving them stuck in cybercrime. Full employment levels also increase due to the rise in temporary employment, indicating that temporary jobs are essential for securing permanent jobs. A transition rate $\delta_1 > 0.25$ is advisable to effectively reduce cybercrime and increase permanent employment. This still demonstrates the impact of employment on the dynamics of cybercrime in society, and this outcome corroborates the findings and recommendations of Godslight [17].

Figure 13 also shows that direct transitions to full employment would have a strong effect on growth in the fully employed population (Figure 13c), compared with temporary employment (Figure 13b). The importance of direct pathways to stable jobs is essential for reducing the cybercriminal population in the long term. The cybercriminal population may reach an estimated peak of about 8,000 people without job opportunities, yet by optimising temporary and permanent employment opportunities, it could be reduced to an estimated 3,800 over a decade. This aligns with the outcomes of Ewuzie *et al.* [13].

Figure 14 provides a comparative analysis of how different employment strategies—temporary (δ_1) and permanent (δ_2) affect the trajectory of the cybercriminal population ($C(t)$) over a 10-year period. The simulation results underscore that providing no employment pathways leads to the highest prevalence of cybercrime, with the population of cybercriminals peaking at approximately 8,000 individuals before slightly receding to 7,800 by the tenth year as noted by Hassan *et al.* [15]. The introduction of a single-employment pathway leads to a notable reduction in criminal activity, though the reduction varies in magnitude. Specifically, implementing only temporary employment (δ_1) reduces the cybercriminal population to approximately 5,500 by year 10, whereas providing only permanent employment (δ_2) results in a more reduction to approximately 4,500. The most substantial impact, however, is observed when both temporary and permanent employment pathways are optimised simultaneously. By the end of the 10-year period, the population of cybercriminals is reduced to approximately 3,800. This synergy demonstrates that while both employment types are beneficial, a dual-pronged strategy that provides immediate temporary work as a stepping stone to stable, permanent employment is the most effective intervention for eradicating youth cybercrime and ensuring long-term societal stability. This conforms with the recommendation by Wangmo [11].

The numerical simulations indicate that a multi-layered approach is essential for creating a cybercrime-free environment. This strategy focuses on transforming temporary jobs into permanent positions and implementing strong legal and social penalties to diminish the influence of criminals.

5. Conclusion

In this study, we formulated and solved a deterministic mathematical model comprising four nonlinear ordinary differential equations to examine how unemployment, temporary non-employment, and diffusion of cybercrime among youth interact. By dividing the population into Unemployed ($U(t)$), Cybercriminal ($C(t)$), Temporarily Employed ($T(t)$), and Fully Employed ($R(t)$) compartments, we established two steady states: the cybercrime-free equilibrium (CFE) and the cybercrime-presence equilibrium (CPE).

Using the next-generation matrix method, our analytical findings indeed show that the Cybercrime Generation Number (N_0) is a threshold; below it, the CFE is both locally and globally stable, and thus, specific socio-economic interventions can indeed eradicate illicit digital activities among the youth population [27].

The numerical simulations emphasise the unstable character of youth employment and its direct effect on societal stability. In particular, the results indicate that an average rate of job loss among the fully employed (0.3) can cause a sharp increase in the cybercriminal population to about 6,400 people. Moreover, cybercriminal peaks were also evident in high recruitment rates in the non-permanent class (temporarily employed) of up to 7,400, indicating that young people in non-permanent or unstable jobs are most susceptible to criminal contagion. The sensitivity analysis through the PRCCs also highlights that the influencing rate (β) and the rate of legal exit (d) are the most crucial factors in driving the epidemic, implying that any policy that is effective should focus on both the creation of stable and safe employment and enhancing digital awareness and legal penalties [28].

Comparing the present study with previous mathematical models, it can be stated that it corresponds to the current state of knowledge in the field of crime dynamics and makes a significant contribution to it. Similar to the general crime models proposed by Mataru *et al.* [27] and Kazeem *et al.* [1], our model verifies the threshold behaviour of N_0 and why it is necessary to ensure that N_0 remains below 1 to maintain social stability. The current study, however, differs from these prior studies by introducing a more subtle category of Temporarily Employed (T), which gives a more realistic account of the contemporary industry of the so-called gig economy or informal labour market in which young people are frequently caught. Although the study by Soemarsono *et al.* [24] shows that unemployment growth correlates with crime overall, our study provides quantitative estimates of the so-called displacement effect of job loss.

Moreover, our findings on the rate of influence (β) are consistent with the dynamics of imitation studied by Gebrezabher and Eroglu [49], indicating that social contact is an even stronger driver of endemicity than natural recruitment alone. In contrast to the media-centric models of Goswami and Athithan [50], our results suggest that direct economic stabilisation, namely the transformation of temporary work into permanent work, is a more effective solution than social awareness alone. Although Kwofie *et al.* [28] described the importance of multi-pronged measures, our simulations indicate that temporary employees are the most vulnerable group for preventing rapid spikes in crime.

Although this deterministic model can offer important insights, it assumes that criminal networks are homogeneous in their mixing and that the complexity of space and the social aspects of criminal networks are overemphasised [49, 51]. Also, the model cannot accommodate random economic shocks or compute the most economically advantageous allocation of resources because it lacks stochastic parameters and an optimal control analysis [1]. To improve the assessment of the timing and effect of particular interventions, future studies will combine stochastic differential equations and optimal control theory [27, 49]. The strategies to stabilise the youth population and ensure a long-run crime-free society will be further refined by verifying these results with empirical data and accounting for the long-term factors of correctional rehabilitation [27, 28]. This study has limitations that may be addressed in future work. These limitations include the absence of a derivation of the global stability of the crime-endemic equilibrium, the lack of an application to real-life data, the assumption of a homogeneous population, and the omission of optimal-control and cost-effectiveness analyses, which are important for identifying the most suitable and cost-effective control strategies under limited resources.

Data availability

The parameter values used in this study are provided with citations in Table 2.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this manuscript.

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References

[1] A. B. Kazeem, S. A. Alimi & M. O. Ibrahim, "Threshold parameter for the control of unemployment in the society: mathematical model and analysis", Journal of Applied Mathematics and Physics **6** (2018) 2563. https://doi.org/10.4236/jamp.2018.612214.

[2] A. O. Olajuwon, "Youth unemployment syndrome in Nigeria: a case for review and political involvement", Annals of Spiru Haret University. Economic Series **22** (2022) 2246. https://doi.org/10.26458/2246.

[3] D. T. Anozi & B. Novianda, "Socio-economic factors and property crime rate in Indonesia", Economics Development Analysis Journal **12** (2023) 305. https://doi.org/10.15294/edaj.v12i3.68829.

[4] S. Timbi, A. Eric & T. M. Daniel, "Effects of unemployment on crime in selected sub-Saharan African countries", International Journal of Scientific Research and Management **9** (2021) 2061. https://doi.org/10.18535/ijsrm/v9i2.em03.

[5] T. Chikore, M. Nyirenda-Kayuni, Q. C. Chukwudum, Z. Chazuka, J. Mwaonanji, M. Ndlovu & K. A. W. White, "Exploring the impact of how criminals interact with cyber-networks—a mathematical modelling approach", Research in Mathematics **11** (2024) 2295059. https://doi.org/10.1080/27684830.2023.2295059.

[6] C. U. Ebelogu, S. D. Ojo, C. P. Andeh & E. O. Agu, "Cybercrime, its adherent negative effects on Nigerian youths and the society at large: possible solutions", International Journal of Advances in Scientific Research and Engineering **5** (2019) 155. https://doi.org/10.31695/IJASRE.2019.33658.

[7] K. G. Al-Hashedi & P. P. Magalingam, "Financial fraud detection applying data mining techniques: a comprehensive review from 2009 to 2019", Computer Science Review **40** (2021) 100402. https://doi.org/10.1016/j.cosrev.2021.100402.

[8] I. B. Ogunsakin, T. Babatimehin & A. R. A. Olawale-Jimoh, "Descriptive study of cybercrime engagement and contributing factors among secondary school students", International Journal of Research and Innovation in Social Science **9** (2025) 1291. https://doi.org/10.47772/IJRISS.2025.907000107.

[9] S. Chen, M. Hao, F. Ding, D. Jiang, J. Dong, S. Zhang, Q. Guo & C. Gao, "Exploring the global geography of cybercrime and its driving forces", Humanities and Social Sciences Communications **10** (2023) 71. https://doi.org/10.1057/s41599-023-01560-x.

[10] E. E. Gregory & A. A. Grace, "Psychodemographic factors predicting internet fraud tendency among youths in southwestern Nigeria", Journal of Educational and Social Research **5** (2015) 159. https://doi.org/10.5901/jesr.2015.v5n2p159.

[11] D. Wangmo, A study on youth unemployment and its consequences in Bhutan, M.P.P. thesis, KDI School of Public Policy and Management, Sejong, Republic of Korea, 2012. Available online: https://www.scribd.com/document/517493432/213853315.

[12] U. S. Anyanwu, "Youth unemployment and cybercrime in Nigeria", Canadian Social Science **20** (2024) 69. https://doi.org/10.3968/13574.

[13] O. C. Ewuzie, N. R. Obioma, U. O. Gift, E. I. Hayford, O. J. Chinaza & A. R. Udegbunam, "Youth unemployment and cybercrime in Nigeria", African Renaissance **20** (2023) 177. https://doi.org/10.31920/2516-5305/2023/20n2a9.

[14] S. C. Chikwendu, N. P. Oli & G. A. Ohazulike, "Youths' negotiation of livelihoods and cybercrimes in Nigeria", Greener Journal of Social Sciences **13** (2023) 4. https://doi.org/10.5281/zenodo.8141209.

[15] A. B. Hassan, F. D. Lass & J. Makinde, "Cybercrime in Nigeria: causes, effects and the way out", ARPN Journal of Science and Technology **2** (2012) 626. http://www.ejournalofscience.org/archive/vol2no7/vol2no7_11.pdf.

[16] M. M. L. Prasanthi & T. A. S. K. Ishwarya, "Cyber crime: prevention & detection", International Journal of Advanced Research in Computer and Communication Engineering **4** (2015) 45. https://doi.org/10.17148/IJARCCE.2015.4311.

[17] G. T. Sese, "Cybercrime: a threat to national security in Nigeria", International Journal of Basic Science and Technology **11** (2025) 197. https://doi.org/10.5281/zenodo.16729087.

[18] J. B. Anjorin, "A critical analysis of cybercrime trends in Nigeria (2018–2025): patterns, drivers, and policy responses", International Journal of Advanced Technology and Social Sciences **4** (2026) 197. https://doi.org/10.59890/ijatss.v4i2.182.

[19] A. Ochojila, "Stakeholders urge legal reforms to combat cybercrimes", The Guardian Nigeria (2025). Available online: https://guardian.ng/news/stakeholders-urge-legal-reforms-to-combat-cybercrimes/.

[20] F. Cremer, B. Sheehan, M. Fortmann, A. N. Kia, M. Mullins, F. Murphy & S. Materne, "Cyber risk and cybersecurity: a systematic review of data availability", The Geneva Papers on Risk and Insurance—Issues and Practice **47** (2022) 698. https://doi.org/10.1057/s41288-022-00266-6.

[21] M. T. Gençoğlu, "Mathematical modeling in cyber defense", International Journal of Engineering Science and Application **4**(4) (2020) 165–169. https://dergipark.org.tr/en/pub/ijesa/article/808438.

[22] L. Koss, "SIR models: differential equations that support the common good", CODEE Journal **12** (2019) 61. https://doi.org/10.5642/codee.201912.01.06.

[23] L. F. Vesa, "Dynamics of a simple mathematical model for unemployment with time delay", Annals of West University of Timisoara—Mathematics and Computer Science **58** (2022) 85. https://doi.org/10.2478/awutm-2022-0019.

[24] A. R. I. Soemarsono, I. Fitria, Nugraheni & N. Hanif, "Analysis of mathematical model on impact of unemployment growth to crime rates", Journal of Physics: Conference Series **1726** (2021) 012003. https://doi.org/10.1088/1742-6596/1726/1/012003.

[25] S. Al-Sheikh, R. Al-Maalwi & H. A. A. Ashi, "Mathematical model of unemployment with the effect of limited jobs", Comptes Rendus Mathématique **359** (2021) 283. https://doi.org/10.5802/crmath.164.

[26] A. K. Singh, P. K. Singh & A. K. Misra, "Combating unemployment through skill development", Nonlinear Analysis: Modelling and Control **25** (2020) 919. https://doi.org/10.15388/namc.2020.25.20598.

[27] B. Mataru, O. J. Abonyo & D. Malonza, "Mathematical model for crimes in developing countries with some control strategies", Journal of Applied Mathematics **2023** (2023) 8699882. https://doi.org/10.1155/2023/8699882.

[28] T. Kwofie, M. Dogbatsey & S. E. Moore, "Curtailing crime dynamics: a mathematical approach", Frontiers in Applied Mathematics and Statistics **8** (2023) 1086745. https://doi.org/10.3389/fams.2022.1086745.

[29] D. Mallick, K. Gaurav, S. Bhattacharya & S. Ghosh, "Quantitative analysis of social influence and digital piracy contagion with differential equations on networks", Scientific Reports **14** (2024) 10962. https://doi.org/10.1038/s41598-024-61531-3.

[30] E. R. Njike-Tchaptchet & C. Tadmon, "Mathematical modelling of the unemployment problem in a context of financial crisis", Mathematics and Computers in Simulation **211** (2023) 241. https://doi.org/10.1016/j.matcom.2023.04.014.

[31] A. K. Misra, M. Kumari & M. Sajid, "Role of informal sector to combat unemployment in developing economy: a modeling study", Heliyon **10** (2024) e33378. https://doi.org/10.1016/j.heliyon.2024.e33378.

[32] O. Diekmann, J. A. P. Heesterbeek & J. A. J. Metz, "On the definition and the computation of the basic reproduction ratio R_0 in models for infectious diseases in heterogeneous populations", Journal of Mathematical Biology **28** (1990) 365. https://doi.org/10.1007/BF00178324.

[33] P. van den Driessche & J. Watmough, "Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission", Mathematical Biosciences **180** (2002) 29. https://doi.org/10.1016/S0025-5564(02)00108-6.

[34] K. A. Tijani, C. E. Madubueze, R. I. Gweryina & T. Aboiyar, "Mathematical modelling and optimal-control strategies of schistosomiasis–typhoid fever co-infection: using the demographic setting of Makurdi, Benue State, Nigeria", Journal of the Nigerian Society of Physical Sciences **8** (2026) 3376. https://doi.org/10.46481/jnsps.2026.3376.

[35] O. F. Lawal, T. T. Yusuf & A. Abidemi, "On mathematical modelling of optimal control of typhoid fever with efficiency analysis", Journal of the Nigerian Society of Physical Sciences **6** (2024) 2057. https://doi.org/10.46481/jnsps.2024.2057.

[36] R. Al-Maalwi, H. Ashi & S. Al-Sheikh, "Unemployment model", Applied Mathematical Sciences **12** (2018) 989. https://doi.org/10.12988/ams.2018.87102.

[37] C. Castillo-Chavez, Z. Feng & W. Huang, "On the computation of R_0 and its role on global stability", in Mathematical Approaches for Emerging and Reemerging Infectious Diseases: An Introduction, C. Castillo-Chavez, S. Blower, P. van den Driessche, D. Kirschner & A.-A. Yakubu (Eds.), Springer, New York, USA, 2002, pp. 229–250. https://doi.org/10.1007/978-1-4757-3667-0_13.

[38] J. O. Odeh, B. C. Agbata, K. A. Tijani & C. E. Madubueze, "Optimal control strategies for the transmission dynamics of Zika virus: with the aid of Wolbachia-infected mosquitoes", Numerical and Computational Methods in Sciences and Engineering **5** (2024) 1. https://doi.org/10.18576/ncmse/050101.

[39] J. M. Heffernan, R. J. Smith & L. M. Wahl, "Perspectives on the basic reproductive ratio", Journal of the Royal Society Interface **2** (2005) 281. https://doi.org/10.1098/rsif.2005.0042.

[40] A. B. Gumel, "Causes of backward bifurcations in some epidemiological models", Journal of Mathematical Analysis and Applications **395** (2012) 355. https://doi.org/10.1016/j.jmaa.2012.04.077.

[41] C. Castillo-Chavez & B. Song, "Dynamical models of tuberculosis and their applications", Mathematical Biosciences and Engineering **1** (2004) 361. https://doi.org/10.3934/mbe.2004.1.361.

[42] A. Saltelli, M. Ratto, T. Andres, F. Campolongo, J. Cariboni, D. Gatelli, M. Saisana & S. Tarantola, Global Sensitivity Analysis: The Primer, 1st ed., John Wiley & Sons, Chichester, UK, 2008. https://doi.org/10.1002/9780470725184.

[43] H. S. Rodrigues, M. T. T. Monteiro & D. F. M. Torres, "Sensitivity analysis in a dengue epidemiological model", Conference Papers in Mathematics **2013** (2013) 721406. https://doi.org/10.1155/2013/721406.

[44] A. Charzyńska, A. Nałęcz, M. Rybiński & A. Gambin, "Sensitivity analysis of mathematical models of signaling pathways", BioTechnologia **93** (2012) 291. https://doi.org/10.5114/bta.2012.46584.

[45] C. E. Madubueze, K. A. Tijani & Fatmawati, "A deterministic mathematical model for optimal control of diphtheria disease with booster vaccination", Healthcare Analytics **4** (2023) 100281. https://doi.org/10.1016/j.health.2023.100281.

[46] A. K. Misra & A. K. Singh, "A delay mathematical model for the control of unemployment", Differential Equations and Dynamical Systems **21** (2013) 291. https://doi.org/10.1007/s12591-012-0153-3.

[47] G. T. Tilahun, O. D. Makinde & D. Malonza, "Modelling and optimal control of typhoid fever disease with cost-effective strategies", Computational and Mathematical Methods in Medicine **2017** (2017) 2324518. https://doi.org/10.1155/2017/2324518.

[48] P. U. Madueme & F. Chirove, "Understanding the transmission pathways of Lassa fever: a mathematical modeling approach", Infectious Disease Modelling **8** (2023) 27. https://doi.org/10.1016/j.idm.2022.11.010.

[49] Z. H. Gebrezabher & D. Eroglu, "Mathematical modeling of the role of imitation in crime dynamics", arXiv (2024). https://doi.org/10.48550/arXiv.2410.09989.

[50] N. K. Goswami & S. Athithan, "Modeling dynamics of the spread of crime with media impact", Research Square preprint (2022). https://doi.org/10.21203/rs.3.rs-2143173/v1.

[51] J. K. A. White, E. Campillo-Funollet, F. Nyabadza, D. Cusseddu, C. Kasumo, N. M. Imbusi & T. Marijani, "Towards understanding crime dynamics in a heterogeneous environment: a mathematical approach", Journal of Interdisciplinary Mathematics **24** (2021) 2139. https://doi.org/10.1080/09720502.2020.1860292.