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Application of the SEE transform with the Adomian decomposition method to a nonlinear tumor growth model

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Abstract

This study examines the combination of the Sadiq-Emad-Eman (SEE) integral transform and the Adomian decomposition method (ADM) for solving a nonlinear tumor growth model. The combined approach provides a direct analytical procedure for obtaining the exact solution of the model. The SEE transform is used because it generalizes several integral transforms and simplifies the required algebraic operations. The ADM is then applied to represent the nonlinear term as a series of Adomian polynomials. The resulting formulation yields the closed-form solution of the tumor growth model and illustrates how the model depends on the initial tumor-cell population, growth rate, and environmental carrying capacity. The results show that the combined SEE–ADM approach offers a simple analytical method for this nonlinear model while preserving its biological interpretation.

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1. Introduction

A tumor growth model describes how cancer cells multiply and how a tumor changes in size over time, denoted by t. Cancer is commonly classified into stages 0–4 according to tumor size and the extent of spread. Mathematical descriptions of tumor growth include exponential growth at an early stage and logistic growth at a later stage, with factors such as nutrient availability taken into account. Tumor growth is also studied using biological models, including genetically modified mice and subcutaneous tumor-culture models, to evaluate responses to treatment. Indicators such as tumor size, weight change, and other symptoms are monitored. These approaches are important for understanding the disease and developing treatments, as discussed in Refs. [1, 2].

Researchers have proposed many analytical and numerical methods for solving such models and related problems. Integral transforms and numerical methods are important tools for obtaining accurate solutions. Integral transforms have been used to solve problems in the physical and engineering sciences and in fields such as genetics and biotechnology; see Refs. [1, 3–12].

Sadiq, Emad, and Eman introduced the Sadiq-Emad-Eman (SEE) integral transform in Ref. [13]. It extends the Laplace-transform technique and is more general than several modified or improved Laplace transforms; see Refs. [3, 10, 14, 15]. The SEE transform has been applied to several real-world problems; see Refs. [3–5, 9, 10, 13, 15].

The Adomian decomposition method (ADM) is a useful nu-

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merical method for obtaining approximate and, in some cases, exact solutions of nonlinear systems; see Ref. [16]. This study combines the ADM with the SEE transform to use the advantages of both methods in solving the tumor growth model. The combination simplifies linear and nonlinear differential equations, reduces the required computations, accelerates series convergence, and broadens the range of obtainable solutions.

2. Methodology

This section presents the definitions and properties required for the analysis.

Definition 2.1: SEE transform

The following definition is based on Ref. [13]. Assume that $f(\tau)$ is a function of the variable τ such that $\frac{1}{v^n}e^{-v\tau}f(\tau)$ is integrable on $0 \leqslant \tau < \infty$ for a complex parameter v, where $\mathrm{Re}(v) > 0$ and $n \in \mathbb{Z}$. The SEE transform is denoted by $S\{f(\tau)\}$ and is defined as

$$S\{f(\tau)\} = \frac{1}{v^n}\int_0^\infty e^{-v\tau} f(\tau)\,\mathrm{d}\tau = F(v). \tag{1}$$

Table 1 presents the SEE transforms of selected elementary functions; see Refs. [13, 17].

The following properties of the SEE transform are used to calculate the analytical solution of the tumor growth model.

Proposition 2.2: Linearity

Suppose that $f_1(\tau)$ and $f_2(\tau)$ are functions of τ, and let γ and σ be constants. According to Refs. [13, 17],

$$S\{\gamma f_1(\tau) \pm \sigma f_2(\tau)\} = \gamma S\{f_1(\tau)\} \pm \sigma S\{f_2(\tau)\}. \tag{2}$$

Proposition 2.3: Differentiation property of the SEE transform

Suppose that $f(\tau)$ is continuous for $0 \leqslant \tau < \infty$ and has the SEE transform $S\{f(\tau)\} = F(v)$. If $f'(\tau)$ possesses a SEE transform, then, according to Ref. [13],

$$\begin{aligned} S\{f^{(\mu)}(\tau)\} = & -\frac{1}{v^n}f^{(\mu-1)}(0) - \frac{1}{v^{n-1}}f^{(\mu-2)}(0) - \cdots \\ & - \frac{f(0)}{v^{n-\mu+1}} + v^\mu F(v), \end{aligned} \tag{3}$$

where $\mu = 1, 2, 3, \ldots$. For $\mu = 1$,

$$S\{f'(\tau)\} = -\frac{1}{v^n}f(0) + vF(v). \tag{4}$$

Definition 2.4: Inverse SEE transform

If $S\{f(\tau)\} = F(v)$, then $f(\tau)$ is the inverse SEE transform of $F(v)$, denoted by $S^{-1}\{\cdot\}$, and, according to Refs. [13, 17], it is represented as

$$S^{-1}\{F(v)\} = f(\tau) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} v^n e^{v\tau} F(v)\,\mathrm{d}v, \qquad a > 0. \tag{5}$$

Table 2 presents the inverse SEE transforms of selected functions.

3. Results and discussion

3.1. Application

This section combines the SEE transform and the ADM to obtain an analytical solution of the tumor growth problem described by Liu *et al.* in Ref. [11]. The model is

$$\frac{\mathrm{d}\eta(\tau)}{\mathrm{d}\tau} = R\eta(\tau)\left(1 - \frac{\eta(\tau)}{K}\right), \qquad K \neq 0. \tag{6}$$

This model describes changes in the number of cells, where τ is time, R is the growth rate of the tumor cells, $\eta(\tau)$ is the number of tumor cells, and K is the environmental carrying capacity. The factor $\left(1 - \frac{\eta(\tau)}{K}\right)$ represents the limiting effect of the environment on tumor-cell growth.

Equation (6) can be expressed as

$$\frac{\mathrm{d}\eta(\tau)}{\mathrm{d}\tau} - R\eta(\tau) + \frac{R}{K}\eta^2(\tau) = 0, \tag{7}$$

or

$$\eta'(\tau) - R\eta(\tau) + \frac{R}{K}\eta^2(\tau) = 0. \tag{8}$$

Applying the SEE transform to the equation gives

$$S\{\eta'(\tau)\} - RS\{\eta(\tau)\} + \frac{R}{K}S\{\eta^2(\tau)\} = 0, \tag{9}$$

and hence

$$-\frac{1}{v^n}\eta(0) + vF(v) - RF(v) + \frac{R}{K}S\{\eta^2(\tau)\} = 0. \tag{10}$$

Therefore,

$$(v - R)F(v) - \frac{1}{v^n}\eta(0) + \frac{R}{K}S\{\eta^2(\tau)\} = 0. \tag{11}$$

Using the initial condition $\eta(0) = \eta_0 = \alpha$, we obtain

$$F(v) = \frac{1}{v - R}\left[\frac{1}{v^n}\eta_0 - \frac{R}{K}S\{\eta^2(\tau)\}\right], \tag{12}$$

and

$$F(v) = \frac{\alpha}{v^n(v - R)} - \frac{R}{K(v - R)}S\{\eta^2(\tau)\}. \tag{13}$$

The ADM is now introduced into the tumor model as described in Ref. [16]. The nonlinear term η^2 is represented by an infinite series of Adomian polynomials B_m:

$$\eta(\tau) = \sum_{m=0}^\infty \eta_m(\tau), \tag{14}$$

and

$$\eta^2(\tau) = \sum_{m=0}^\infty B_m. \tag{15}$$

For the nonlinear term η^2, the Adomian polynomials are evaluated as follows:

$$B_0 = \eta_0^2, \qquad \eta_0(\tau) = \alpha e^{R\tau}, \tag{16}$$

Table 1. SEE transforms of selected elementary functions

$f(\tau)$	$S\{f(\tau)\} = \dfrac{1}{v^n}\displaystyle\int_0^\infty e^{-v\tau} f(\tau)\,\mathrm{d}\tau = F(v)$
β	$\dfrac{\beta}{v^{n+1}}$, where β is a constant and $v > 0$.
τ^m	$\dfrac{m!}{v^{n+m+1}}$ for $m = 1, 2, 3, \ldots$; $\dfrac{\Gamma(m+1)}{v^{n+m+1}}$ for $m > -1$, where $\Gamma(\cdot)$ is the gamma function and $v > 0$.
$e^{\beta\tau}$	$\dfrac{1}{v^n(v-\beta)}$, where $v > \beta$.
$\sin(\beta\tau)$	$\dfrac{\beta}{v^n(v^2+\beta^2)}$.
$\cos(\beta\tau)$	$\dfrac{v}{v^n(v^2+\beta^2)}$.
$\cosh(\beta\tau)$	$\dfrac{v}{v^n(v^2-\beta^2)}$.
$\sinh(\beta\tau)$	$\dfrac{\beta}{v^n(v^2-\beta^2)}$.

Table 2. Inverse SEE transforms of selected functions

$F(v)$	$S^{-1}\{F(v)\} = f(\tau)$
$\dfrac{\beta}{v^{n+1}}$	β, where β is a constant.
$\dfrac{m!}{v^{n+m+1}}$	τ^m, for all $m \in \mathbb{N}$.
$\dfrac{\Gamma(m+1)}{v^{n+m+1}}$	τ^m, where $m > -1$.
$\dfrac{1}{v^n(v+\beta)}$	$e^{-\beta\tau}$.
$\dfrac{\beta}{v^n(v^2+\beta^2)}$	$\sin(\beta\tau)$.
$\dfrac{1}{v^{n-1}(v^2+\beta^2)}$	$\cos(\beta\tau)$.
$\dfrac{\beta}{v^n(v^2-\beta^2)}$	$\sinh(\beta\tau)$.
$\dfrac{1}{v^{n-1}(v^2-\beta^2)}$	$\cosh(\beta\tau)$.

$$B_1 = 2\eta_0\eta_1, \qquad \eta_1(\tau) = -\frac{\alpha^2}{K}\left(e^{2R\tau} - e^{R\tau}\right), \tag{17}$$

and

$$B_2 = \eta_1^2 + 2\eta_0\eta_2, \quad \eta_2(\tau) = -\frac{R}{K}S^{-1}\left\{\frac{1}{v-R}S\{B_1\}\right\}, \ldots \tag{18}$$

Thus,

$$\eta(\tau) = \eta_0(\tau) + \eta_1(\tau) + \eta_2(\tau) + \cdots . \tag{19}$$

The infinite series leads to the closed-form analytical solution of the tumor growth model. Substitution of the series into the SEE transform gives

$$S\left\{\sum_{m=0}^{\infty}\eta_m(\tau)\right\} = \frac{\alpha}{v^n(v-R)} - \frac{R}{K(v-R)}S\left\{\sum_{m=0}^{\infty}B_m\right\}. \tag{20}$$

For $m \geqslant 0$, the recursive relation is

$$S\{\eta_{m+1}(\tau)\} = -\frac{R}{K(v-R)}S\{B_m\}. \tag{21}$$

Applying the inverse SEE transform gives

$$\eta_0 = S^{-1}\left\{\frac{\alpha}{v^n(v-R)}\right\}, \tag{22}$$

so that

$$\eta_0(\tau) = \alpha e^{R\tau}. \tag{23}$$

Next, $B_0 = \eta_0^2 = \alpha^2 e^{2R\tau}$ is used to evaluate $\eta_1(\tau)$:

$$S\{B_0\} = S\{\alpha^2 e^{2R\tau}\} = \frac{\alpha^2}{v^n(v-2R)}. \tag{24}$$

Then,

$$\begin{aligned} S\{\eta_1(\tau)\} &= -\frac{R}{K(v-R)}\left(\frac{\alpha^2}{v^n(v-2R)}\right) \\ &= -\frac{R}{K(v-R)}\left(\frac{\alpha^2}{v^n(v-2R)}\right). \end{aligned} \tag{25}$$

Using the partial-fraction decomposition described in Ref. [16] and applying S^{-1}, the series is

$$\eta(\tau) = \eta_0(\tau) + \eta_1(\tau) + \eta_2(\tau) + \cdots , \tag{26}$$

and the closed-form exact solution is

$$\eta(\tau) = \frac{K\alpha}{\alpha + (K-\alpha)e^{-R\tau}}. \tag{27}$$

The combination of the SEE integral transform and the ADM facilitates the solution of the tumor growth equation by converting the nonlinear differential model into an algebraic formulation. This reduces the difficulty of solving the biological system step by step. The solution shows how tumor-cell growth is bounded by environmental factors and depends on the growth rate R. Partial fractions and the inverse SEE transform yield an explicit expression for the number of tumor cells at time τ.

4. Conclusion

This study shows that the SEE transform, combined with the ADM, can be used to obtain the analytical solution of the tumor growth differential model.

The number of tumor cells, $\eta(\tau)$, at any time τ can be evaluated by substituting the initial tumor-cell population α, growth rate R, and environmental carrying capacity K into Eq. (27). The model can be used to predict the number of tumor cells over time, which is relevant to diagnosis and treatment planning. The approach may also provide a basis for future studies combining the SEE transform with fractional methods for more realistic tumor modeling.

Data availability

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this manuscript.

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