Published by
NIGERIAN SOCIETY OF PHYSICAL SCIENCES
Available online @ https://journal.nsps.org.ng/index.php/jnsps

J. Nig. Soc. Phys. Sci. 8 (2026) 3609

Journal of the Nigerian Society of Physical Sciences

# Geometric admissibility of hypergeometric and integral operators for the synthesis of digital filters

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## Abstract

In this study, rigorous analytical criteria are developed to determine when the Gaussian hypergeometric function belongs to the Sakaguchi-type class $P(\delta, \sigma, t)$. A computable sufficient condition is established in Theorem 2.1, and the associated integral operator $G(\lambda, \mu; \nu; z)$ is shown to satisfy the admissibility conditions for membership in $P(\delta, \sigma, t)$ under the stated parameter assumptions. These theoretical results are interpreted in the context of digital filter synthesis through an admissibility-based design framework. The proposed approach provides a systematic procedure for selecting admissible parameter values that satisfy the required geometric conditions for filter construction. Numerical examples, including admissibility contour maps and frequency-response characteristics, illustrate the proposed framework and demonstrate its potential for digital filter design based on geometric function theory.


DOI: 10.46481/jnsps.2026.3609

*Keywords:* Harmonic mappings, Uniformly convex functions, Gaussian hypergeometric functions, Integral operators, Digital filter design

*Article History:*
Received: 15 June 2026
Received in revised form: 10 July 2026
Accepted for publication: 19 July 2026
Available online: 11 August 2026



Communicated by: B. J. Falaye

## 1. Introduction

Let $\mathcal{A}$ denote the class of analytic functions in $D = \{z : |z| < 1\}$ that are normalized and have the form

$$f(z) = z + \sum_{n=2}^{\infty} \lambda_n z^n. \tag{1}$$

The subclasses of univalent, starlike, and convex functions of order $\sigma$ are denoted by $\mathcal{S}$, $\mathcal{S}^*(\sigma)$, and $\mathcal{K}(\sigma)$, respectively, as described in Refs. [1, 2]. Goodman introduced the classes $\mathcal{UCV}$ and $\mathcal{UST}$ of uniformly convex and uniformly starlike functions. Rønning provided a one-variable analytic characterization of $\mathcal{UCV}$ in Ref. [3]; see also Ref. [4]. Thus, a function $f$ of the form in Eq. (1) belongs to $\mathcal{UCV}$ if and only if

$$\mathcal{R}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} \geq \left|\frac{zf''(z)}{f'(z)}\right|, \quad (z \in D).$$

Alexander's classical relation, $f(z) \in \mathcal{K} \Leftrightarrow zf'(z) \in \mathcal{S}^*$, does not extend directly to the classes $\mathcal{UCV}$ and $\mathcal{UST}$ (Ref. [5]). Rønning in Ref. [6] therefore introduced the class $\mathcal{S}_p$ such that $f(z) \in \mathcal{UCV} \Leftrightarrow zf'(z) \in \mathcal{S}_p$. The parameter $\sigma$ was subsequently incorporated into the classes $\mathcal{UCV}$ and $\mathcal{S}_p$ in Ref. [3]. A function $f$ of the form in Eq. (1) belongs to $\mathcal{S}_p(\sigma)$ when it satisfies the following analytic characterization:

$$\mathcal{R}\left\{\frac{zf'(z)}{f(z)} - \sigma\right\} \geq \left|\frac{zf'(z)}{f(z)} - 1\right|, \quad (\sigma \in \mathbb{R}, z \in D),$$

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According to Ref. [7], $f(z) \in \mathcal{UCV}(\sigma)$ if and only if $zf'(z) \in \mathcal{S}_p(\sigma)$. The following classes are based on the work of Srutha Keerthi *et al.* in Ref. [8].

**Definition 1.1.** *A function $f$ of the form in Eq. (1) belongs to $P(\delta,\sigma,t)$ if it satisfies*

$$\mathcal{R}\left[\frac{(1-t)\left[(2\delta^2-\delta)z^2f''(z)+zf'(z)\right]}{\begin{array}{c}4(\delta-\delta^2)(z-tz)\\ +(2\delta^2-\delta)z\left[f'(z)-t(f'(tz))\right]\\ +(2\delta^2-3\delta+1)\left[f(z)-f(tz)\right]\end{array}}-\sigma\right] \geq \left|\frac{(1-t)\left[(2\delta^2-\delta)z^2f''(z)+zf'(z)\right]}{\begin{array}{c}4(\delta-\delta^2)(z-tz)\\ +(2\delta^2-\delta)z\left[f'(z)-t(f'(tz))\right]\\ +(2\delta^2-3\delta+1)\left[f(z)-f(tz)\right]\end{array}}-1\right|. \tag{2}$$

*Here, $\sigma \in \mathbb{R}$, $0 \leq \delta \leq 1$, $z \in D$, $|t| \leq 1$, and $t \neq 1$.*

Note that $P(1,\sigma,0) = \mathcal{UCV}(\sigma)$ and $P(0,\sigma,0) = \mathcal{UST}(\sigma)$. Setting $\delta = 0$ yields the following class.

**Definition 1.2.** *Following Refs. [8, 9], a function $f$ of the form in Eq. (1) belongs to $Q(\sigma,t)$ if it satisfies*

$$\mathcal{R}\left[\frac{(1-t)[zf'(z)]}{[f(z)-f(tz)]}-\sigma\right] \geq \left|\frac{(1-t)[zf'(z)]}{[f(z)-f(tz)]}-1\right| \tag{3}$$

$(\sigma \in \mathbb{R}, z \in D, |t| \leq 1,\ t \neq 1)$.

We denote by $T$ the subclass of $\mathcal{S}$ consisting of functions of the form

$$f(z) = z - \sum_{n=2}^{\infty} \lambda_n z^n, \quad (\lambda_n \geq 0) \tag{4}$$

and let $P_T(\delta,\sigma,t) = P(\delta,\sigma,t) \cap T$.

The Gaussian hypergeometric function is defined by

$$F(\lambda,\mu;\nu;z) = \sum_{n=0}^{\infty} \frac{(\lambda)_n(\mu)_n}{(\nu)_n(1)_n} z^n,$$

For $\nu \neq 0,-1,-2,\ldots$, $(x)_n$ denotes the Pochhammer symbol, defined by

$$(x)_n = \begin{cases} 1 & \text{if } n = 0 \\ x(x+1)\ldots(x+n-1) & \text{if } n \in \mathbb{N} = \{1,2,\ldots\}. \end{cases}$$

According to Refs. [2, 7], $F(\lambda,\mu;\nu;1)$ converges when $\mathcal{R}(\nu-\lambda-\mu) > 0$ and is related to the Gamma functions by

$$F(\lambda,\mu;\nu;1) = \frac{\Gamma(\nu)\Gamma(\nu-\lambda-\mu)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}. \tag{5}$$

Several studies have advanced the classification of hypergeometric functions within geometric function theory. Merkes and Scott in Ref. [10] and Ruscheweyh and Singh in Ref. [11] used continued fractions to determine parameter ranges $(\lambda,\mu,\nu)$ for which functions belong to the class $\mathcal{S}^*(\sigma)$, where $\sigma < 1$. Carlson and Shaffer in Ref. [12] extended this approach by showing that linear operators can be used to apply convolution properties to hypergeometric functions in $\mathcal{S}^*(\sigma)$. Silverman in Ref. [13] subsequently established conditions under which $zF(\lambda,\mu;\nu;z)$ belongs to the subclasses $\mathcal{S}^*(\sigma)$ and $\mathcal{K}(\sigma)$.

This study examines the inclusion of the Gaussian hypergeometric function $zF(\lambda,\mu;\nu;z)$ in the Sakaguchi-type subclass $P(\delta,\sigma,t)$. The analytical framework yields sufficient conditions for this inclusion under appropriate restrictions on the parameters $\lambda$, $\mu$, and $\nu$. We further characterize the membership of $zF(\lambda,\mu;\nu;z)$ in the subclass $P_T(\delta,\sigma,t)$, which comprises functions with negative coefficients, by deriving necessary and sufficient conditions.

Furthermore, an integral operator $G(\lambda,\mu;\nu;z)$ associated with the hypergeometric function is analysed, and its role in preserving the geometric characteristics of these function classes is assessed using Refs. [13–21]. We demonstrate the application of the resulting analytical findings to signal processing via an admissibility design rule that mitigates Gibbs-type ringing artifacts by maintaining the transfer function inside a designated safe range. This geometric constraint produces smooth and monotonic filter responses with little overshoot, thereby providing a mathematically substantiated alternative to conventional digital-filter design principles discussed in Refs. [14, 16, 17].

Although our analysis builds upon the Sakaguchi-type function framework established in earlier studies, the present work makes several new contributions. In particular, we derive new sufficient conditions for the Gaussian hypergeometric function $zF(\lambda,\mu;\nu;z)$ to belong to the class $P(\delta,\sigma,t)$, establish corresponding admissibility conditions for the associated integral operator, and interpret these theoretical results within an admissibility-based framework for hypergeometric digital filter synthesis. Thus, while the underlying function class follows the existing literature, the hypergeometric formulations, integral-operator analysis, and digital filter interpretation presented here are original contributions of this work.

## 2. Coefficient inequality

The following lemmas are required to establish the main results.

**Lemma 2.1.** *A sufficient condition for a function $f$ of the form in Eq. (1) to belong to $P(\delta,\sigma,t)$ is*

$$\begin{aligned}\sum_{n=2}^{\infty}\Big[&(4\delta^2-2\delta)n(n-1)+[2-(2\delta^2-\delta)(1+tu_n)(1+\sigma)]n\\ &-u_n[(2\delta^2-3\delta+1)(1+\sigma)]\Big]|\lambda_n|\\ &\leq (1-\sigma)-t(2\delta^2-\delta)(1+\sigma).\end{aligned} \tag{6}$$

*Proof.* Based on the definition of $P(\delta,\sigma,t)$ given in Ref. [22], it is sufficient to verify that

$$\left|\frac{(1-t)\left[(2\delta^2-\delta)z^2f''(z)+zf'(z)\right]}{\begin{array}{l}4(\delta-\delta^2)(z-tz)\\ +(2\delta^2-\delta)z\left[f'(z)-t(f'(tz))\right]\\ +(2\delta^2-3\delta+1)\left[f(z)-f(tz)\right]\end{array}}-1\right|$$

$$\leq \mathcal{R}\left[\frac{(1-t)\left[(2\delta^2-\delta)z^2f''(z)+zf'(z)\right]}{\begin{array}{l}4(\delta-\delta^2)(z-tz)\\ +(2\delta^2-\delta)z\left[f'(z)-t(f'(tz))\right]\\ +(2\delta^2-3\delta+1)\left[f(z)-f(tz)\right]\end{array}}-1\right]+1-\sigma.$$

We have

$$\left|\frac{(1-t)\left[(2\delta^2-\delta)z^2f''(z)+zf'(z)\right]}{\begin{array}{l}4(\delta-\delta^2)(z-tz)\\ +(2\delta^2-\delta)z\left[f'(z)-t(f'(tz))\right]\\ +(2\delta^2-3\delta+1)\left[f(z)-f(tz)\right]\end{array}}-1\right|$$

$$-\mathcal{R}\left[\frac{(1-t)\left[(2\delta^2-\delta)z^2f''(z)+zf'(z)\right]}{\begin{array}{l}4(\delta-\delta^2)(z-tz)\\ +(2\delta^2-\delta)z\left[f'(z)-t(f'(tz))\right]\\ +(2\delta^2-3\delta+1)\left[f(z)-f(tz)\right]\end{array}}-1\right]$$

$$\leq 2\left|\frac{(1-t)\left[(2\delta^2-\delta)z^2f''(z)+zf'(z)\right]}{\begin{array}{l}4(\delta-\delta^2)(z-tz)\\ +(2\delta^2-\delta)z\left[f'(z)-t(f'(tz))\right]\\ +(2\delta^2-3\delta+1)\left[f(z)-f(tz)\right]\end{array}}-1\right|$$

$$\leq \frac{\begin{array}{l}\sum_{n=2}^{\infty}\Big[2(2\delta^2-\delta)n(n-1)+2(1-(2\delta^2-\delta)(1+tu_n))n\\ \qquad -2u_n(2\delta^2-3\delta+1)\Big]|\lambda_n|-2t(2\delta^2-\delta)\end{array}}{\begin{array}{l}1+t(2\delta^2-\delta)\\ +\sum_{n=2}^{\infty}\Big[(2\delta^2-\delta)n(1+tu_n)+(2\delta^2-3\delta+1)u_n\Big]|\lambda_n|\end{array}},$$

where

$$u_n=\frac{1-t^n}{1-t},(n\in N).$$

The preceding expression is bounded above by $1-\sigma$ if Eq. (6) holds. This completes the proof.

For $\delta=0$, we obtain the following.

**Corollary 2.1.** *Following Ref. [8], a sufficient condition for a function $f$ of the form in Eq. (1) to belong to $Q(\sigma,t)$ is*

$$\sum_{n=2}^{\infty}[2n-u_n(1+\sigma)]|\lambda_n|\leq(1-\sigma), \tag{7}$$

**Lemma 2.2.** *A necessary and sufficient condition (see Ref. [1]) for $f(z)$ of the form in Eq. (4) to be in $P_T(\delta,\sigma,t)$ is that*

$$\begin{aligned}&\sum_{n=2}^{\infty}\Big[(2(2\delta^2-\delta)n(n-1)+[2-(2\delta^2-\delta)(1+tu_n)(1+\sigma)]n\\ &\qquad -u_n[(2\delta^2-3\delta+1)(1+\sigma)]\Big]\lambda_n\\ &\qquad \leq(1-\sigma)-t(2\delta^2-\delta)(1+\sigma).\end{aligned} \tag{8}$$

*Proof.* Let $f(z)$ be of the form in Eq. (4) and belong to the class $P_T(\delta,\sigma,t)$. Then,

$$(1-\sigma)+\mathcal{R}\left[\frac{(1-t)\left[(2\delta^2-\delta)z^2f''(z)+zf'(z)\right]}{\begin{array}{l}4(\delta-\delta^2)(z-tz)\\ +(2\delta^2-\delta)z\left[f'(z)-t(f'(tz))\right]\\ +(2\delta^2-3\delta+1)\left[f(z)-f(tz)\right]\end{array}}-\sigma\right]$$

$$\geq\left|\frac{(1-t)\left[(2\delta^2-\delta)z^2f''(z)+zf'(z)\right]}{\begin{array}{l}4(\delta-\delta^2)(z-tz)\\ +(2\delta^2-\delta)z\left[f'(z)-t(f'(tz))\right]\\ +(2\delta^2-3\delta+1)\left[f(z)-f(tz)\right]\end{array}}-1\right|.$$

Similarly, for $z\in D$,

$$(1-\sigma)-\mathcal{R}\left[\frac{\begin{array}{l}\sum_{n=2}^{\infty}\Big[(2\delta^2-\delta)n(n-1)+(1-(2\delta^2-\delta)(1+tu_n))n\\ \qquad -u_n(2\delta^2-3\delta+1)\Big]\lambda_nz^n-t(2\delta^2-\delta)\end{array}}{\begin{array}{l}1+t(2\delta^2-\delta)\\ -\sum_{n=2}^{\infty}\Big[(2\delta^2-\delta)n(1+tu_n)+(2\delta^2-3\delta+1)u_n\Big]\lambda_nz^n\end{array}}\right]$$

$$\geq\left|\frac{\begin{array}{l}\sum_{n=2}^{\infty}\Big[(2\delta^2-\delta)n(n-1)+(1-(2\delta^2-\delta)(1+tu_n))n\\ \qquad -u_n(2\delta^2-3\delta+1)\Big]\lambda_nz^n+t(2\delta^2-\delta)\end{array}}{\begin{array}{l}1+t(2\delta^2-\delta)\\ -\sum_{n=2}^{\infty}\Big[(2\delta^2-\delta)n(1+tu_n)+(2\delta^2-3\delta+1)u_n\Big]\lambda_nz^n\end{array}}\right|.$$

By selecting values of $z$ along the real axis so that the left-hand side of this inequality is real and then letting $z\to1$, as in Refs. [1, 23], we obtain

$$(1-\sigma)\Bigg([1+t(2\delta^2-\delta)]-\sum_{n=2}^{\infty}\Big[(2\delta^2-\delta)n(1+tu_n)$$

$$+ (2\delta^2 - 3\delta + 1)u_n\Big]\lambda_n\Big)$$

$$- \sum_{n=2}^{\infty}\Big[(2\delta^2-\delta)n(n-1) + (1-(2\delta^2-\delta)(1+tu_n))n - u_n(2\delta^2-3\delta+1)\Big]\lambda_n - t(2\delta^2-\delta)$$

$$\geq \sum_{n=2}^{\infty}\Big[(2\delta^2-\delta)n(n-1) + (1-(2\delta^2-\delta)(1+tu_n))n - u_n(2\delta^2-3\delta+1)\Big]\lambda_n + t(2\delta^2-\delta).$$

$$\Rightarrow \sum_{n=2}^{\infty}\Big[2(2\delta^2-\delta)n(n-1) + [2-(2\delta^2-\delta)(1+tu_n)(1+\sigma)]n - u_n[(2\delta^2-3\delta+1)(1+\sigma)]\Big]\lambda_n \leq (1-\sigma) - t(2\delta^2-\delta)(1+\sigma).$$

Conversely, assume that Eq. (8) holds, as in Ref. [1]. We then show that $f(z) \in P_T(\delta,\sigma,t)$.

$$\left|\frac{(1-t)\left[(2\delta^2-\delta)z^2f''(z)+zf'(z)\right]}{\begin{array}{l}4(\delta-\delta^2)(z-tz)\\ +(2\delta^2-\delta)z\left[f'(z)-t(f'(tz))\right]\\ +(2\delta^2-3\delta+1)\left[f(z)-f(tz)\right]\end{array}} - 1\right| - \mathcal{R}\left[\frac{(1-t)\left[(2\delta^2-\delta)z^2f''(z)+zf'(z)\right]}{\begin{array}{l}4(\delta-\delta^2)(z-tz)\\ +(2\delta^2-\delta)z\left[f'(z)-t(f'(tz))\right]\\ +(2\delta^2-3\delta+1)\left[f(z)-f(tz)\right]\end{array}} - 1\right] \leq (1-\sigma) - t(2\delta^2-\delta)(1+\sigma).$$

For $z \in D$,

$$\left|\frac{(1-t)\left[(2\delta^2-\delta)z^2f''(z)+zf'(z)\right]}{\begin{array}{l}4(\delta-\delta^2)(z-tz)\\ +(2\delta^2-\delta)z\left[f'(z)-t(f'(tz))\right]\\ +(2\delta^2-3\delta+1)\left[f(z)-f(tz)\right]\end{array}} - 1\right| - \mathcal{R}\left[\frac{(1-t)\left[(2\delta^2-\delta)z^2f''(z)+zf'(z)\right]}{\begin{array}{l}4(\delta-\delta^2)(z-tz)\\ +(2\delta^2-\delta)z\left[f'(z)-t(f'(tz))\right]\\ +(2\delta^2-3\delta+1)\left[f(z)-f(tz)\right]\end{array}} - 1\right].$$

$$\leq \left|\frac{\begin{array}{c}\sum_{n=2}^{\infty}\Big[(2\delta^2-\delta)n(n-1)+(1-(2\delta^2-\delta)(1+tu_n))n\\ -u_n(2\delta^2-3\delta+1)\Big]\lambda_n z^n + t(2\delta^2-\delta)\end{array}}{\begin{array}{l}1+t(2\delta^2-\delta)\\ -\sum_{n=2}^{\infty}\Big[(2\delta^2-\delta)n(1+tu_n)+(2\delta^2-3\delta+1)u_n\Big]\lambda_n z^n\end{array}}\right| + \mathcal{R}\left[\frac{\begin{array}{c}\sum_{n=2}^{\infty}\Big[(2\delta^2-\delta)n(n-1)+(1-(2\delta^2-\delta)(1+tu_n))n\\ -u_n(2\delta^2-3\delta+1)\Big]\lambda_n z^n + t(2\delta^2-\delta)\end{array}}{\begin{array}{l}1+t(2\delta^2-\delta)\\ -\sum_{n=2}^{\infty}\Big[(2\delta^2-\delta)n(1+tu_n)+(2\delta^2-3\delta+1)u_n\Big]\lambda_n z^n\end{array}}\right]$$

$$\leq \frac{\begin{array}{c}\sum_{n=2}^{\infty}\Big[2(2\delta^2-\delta)n(n-1)+2(1-(2\delta^2-\delta)(1+tu_n))n\\ -2u_n(2\delta^2-3\delta+1)\Big]\lambda_n + 2t(2\delta^2-\delta)\end{array}}{\begin{array}{l}1+t(2\delta^2-\delta)\\ -\sum_{n=2}^{\infty}\Big[(2\delta^2-\delta)n(1+tu_n)+(2\delta^2-3\delta+1)u_n\Big]\lambda_n\end{array}} \leq (1-\sigma) - t(2\delta^2-\delta)(1+\sigma).$$

Which is equivalent to Eq. (8). If $\delta = 0$, we get the following.

**Corollary 2.2.** *By Lemma 2.2 and Ref. [8], a function $f(z)$ belongs to $Q_T(\sigma,t)$ if it satisfies the following condition:*

$$\sum_{n=2}^{\infty}[2n - u_n(1+\sigma)]\lambda_n \leq (1-\sigma), \tag{9}$$

Using Lemma 2.1, we can find the following.

**Theorem 2.1.** *Following Ref. [7], if $\lambda,\mu > 0$ and $\nu > \lambda+\mu+2$, then a sufficient condition for $zF(\lambda,\mu;\nu;z)$ to belong to $P(\delta,\sigma,t)$ $(0 \leq \delta \leq 1,\ -1 \leq \sigma < 1, |t| \leq 1,\ t \neq 1)$ is*

$$\frac{\Gamma(\nu)\Gamma(\nu-\lambda-\mu)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}\left[\frac{(4\delta^2-2\delta)(\lambda)_2(\mu)_2}{(\nu-\lambda-\mu-2)_2\left[\begin{array}{c}2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\\ -\mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\end{array}\right]} + \frac{\left[(8\delta^2-4\delta+2)-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\right]\lambda\mu}{(\nu-\lambda-\mu-1)\left[\begin{array}{c}2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\\ -\mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\end{array}\right]} + 1\right]$$

$$\leq \frac{(1-\sigma)-t(2\delta^2-\delta)(1+\sigma)}{\left[\begin{array}{c}2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\\ -\mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\end{array}\right]} + 1. \tag{10}$$

*The condition in Eq. (10) is necessary and sufficient for the function $F_1$ in Eq. (8), defined by $F_1(\lambda,\mu;\nu;z) = z[2 - F(\lambda,\mu;\nu;z)]$, to belong to $P_T(\delta,\sigma,t)$.*

*Proof.* By Lemma 2.1 and Ref. [24], $zF(\lambda,\mu;\nu;z) = z + \sum_{n=2}^{\infty}\frac{(\lambda)_{n-1}(\mu)_{n-1}}{(\nu)_{n-1}(1)_{n-1}}z^n$.

$$\sum_{n=2}^{\infty}\Big[2(2\delta^2-\delta)n(n-1) + [2-(2\delta^2-\delta)(1+tu_n)(1+\sigma)]n$$

$$- u_n\big[(2\delta^2 - 3\delta + 1)(1+\sigma)\big]\Big]\frac{(\lambda)_{n-1}(\mu)_{n-1}}{(\nu)_{n-1}(1)_{n-1}}$$
$$\leq (1-\sigma) - t(2\delta^2-\delta)(1+\sigma).$$

Now

$$\sum_{n=2}^{\infty}\Big[2(2\delta^2-\delta)n(n-1) + [2-(2\delta^2-\delta)(1+tu_n)(1+\sigma)]n - u_n[(2\delta^2-3\delta+1)(1+\sigma)]\Big]\frac{(\lambda)_{n-1}(\mu)_{n-1}}{(\nu)_{n-1}(1)_{n-1}}$$
$$= \sum_{n=0}^{\infty}\Big[2(2\delta^2-\delta)(n+2)(n+1) + [2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)](n+2) - \mathcal{U}[(2\delta^2-3\delta+1)(1+\sigma)]\Big]\frac{(\lambda)_{n+1}(\mu)_{n+1}}{(\nu+1)_{n+1}(1)_{n+1}}. \quad (11)$$

Using $(x)_n = x(x+1)_{n-1}$ and Eq. (5), we apply a formal bounding argument. The term $u_{n+2}$ appearing in the summation is replaced by its uniform upper bound (supremum) over the index $n$, as follows:

$$\mathcal{U} = \sup_{n\geq 0}\left|\frac{1-t^{n+2}}{1-t}\right|.$$

Under this supremum-bound argument, the $n$-dependent quantity is bounded by its maximum value. Because the supremum $\mathcal{U}$ is independent of the summation index $n$, it may be treated as a constant and taken outside the summation.

$$\frac{(4\delta^2-2\delta)(\lambda)_2(\mu)_2}{(\nu)_2}\sum_{n=1}^{\infty}\frac{(\lambda+2)_{n-1}(\mu+2)_{n-1}}{(\nu+2)_{n-1}(1)_{n-1}} + \frac{\big[(8\delta^2-4\delta+2) - (2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\big]\lambda\mu}{\nu}\sum_{n=0}^{\infty}\frac{(\lambda+1)_n(\mu+1)_n}{(\nu+1)_n(1)_n}.$$

$$+\Big[2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma) - \mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\Big]\sum_{n=0}^{\infty}\frac{(\lambda)_{n+1}(\mu)_{n+1}}{(\nu)_{n+1}(1)_{n+1}}$$
$$= \frac{(4\delta^2-2\delta)(\lambda)_2(\mu)_2}{(\nu)_2}\frac{\Gamma(\nu+2)\Gamma(\nu-\lambda-\mu-2)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)} + \frac{\big[(8\delta^2-4\delta+2) - (2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\big]\lambda\mu}{\nu}\frac{\Gamma(\nu+1)\Gamma(\nu-\lambda-\mu-1)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}.$$

$$+\Big[2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma) - \mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\Big]\frac{\Gamma(\nu)\Gamma(\nu-\lambda-\mu)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}$$
$$-\Big[2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma) - \mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\Big].$$

$$= \frac{\Gamma(\nu)\Gamma(\nu-\lambda-\mu)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}\Bigg[\frac{(4\delta^2-2\delta)(\lambda)_2(\mu)_2}{(\nu-\lambda-\mu-2)_2} + \frac{\big[(8\delta^2-4\delta+2) - (2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\big]\lambda\mu}{\nu-\lambda-\mu-1} + \Big[2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma) - \mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\Big]\Bigg]$$
$$-\Big[2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma) - \mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\Big].$$

This expression is bounded above by $1-\sigma$ if and only if Eq. (10) holds, as in Ref. [1]. This completes the proof.

For $F_1(\lambda,\mu;\nu;z) = z - \sum_{n=2}^{\infty}\frac{(\lambda)_{n-1}(\mu)_{n-1}}{(\nu)_{n-1}(1)_{n-1}}z^n$, the requirement for $F_1$ to belong to $P_T(\delta,\sigma,t)$ follows from Lemma 2.2.

This is what we get when $\delta = 0$.

**Remark 2.1.** *The assumption $\nu > \lambda+\mu+2$ guarantees that $\nu-\lambda-\mu-2 > 0$, ensuring that the Gamma function $\Gamma(\nu-\lambda-\mu-2)$ appearing in the closed-form expressions is finite and well defined.*

**Corollary 2.3.** *If $\lambda,\mu > 0$ and $\nu > \lambda+\mu+2$, then a sufficient condition for $zF(\lambda,\mu;\nu;z)$ to belong to $Q(\sigma,t)$ $(-1 \leq \sigma < 1, |t| \leq 1,\ t \neq 1)$ is given in Ref. [8] by*

$$\frac{\Gamma(\nu)\Gamma(\nu-\lambda-\mu)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}\left[\frac{2\lambda\mu}{(2-\mathcal{U}(1+\sigma))(\nu-\lambda-\mu-1)}+1\right] \leq \frac{1-\sigma}{2-\mathcal{U}(1+\sigma)}+1. \quad (12)$$

**Theorem 2.2.** *Following Ref. [25], if $\lambda,\mu > -1$, $\lambda\mu < 0$, and $\nu > \lambda+\mu+2$, then $zF(\lambda,\mu;\nu;z) \in P_T(\delta,\sigma,t)$ if and only if*

$$\frac{\Gamma(\nu+1)\Gamma(\nu-\lambda-\mu-2)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}\Bigg[(4\delta^2-2\delta)(\lambda+1)(\mu+1) + \Big\{(8\delta^2-4\delta+2) - (2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\Big\}(\nu-\lambda-\mu-2) + \frac{\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\Big\}(\nu-\lambda-\mu-2)_2}{\lambda\mu} - \frac{\mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)(\nu-\lambda-\mu-2)_2}{\lambda\mu}\Bigg]$$
$$\leq \Bigg[\Big\{(1-\sigma) - t(2\delta^2-\delta)(1+\sigma)\Big\}\frac{\nu}{|\lambda\mu|} + \Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma) - \mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\Big\}\frac{\nu}{\lambda\mu}\Bigg]. \quad (13)$$

*Proof.* Since

$$zF(\lambda,\mu;\nu;z) = z + \frac{\lambda\mu}{\nu}\sum_{n=2}^{\infty}\frac{(\lambda+1)_{n-2}(\mu+1)_{n-2}}{(\nu+1)_{n-2}(1)_{n-1}}z^n$$
$$= z - \left|\frac{\lambda\mu}{\nu}\right|\sum_{n=2}^{\infty}\frac{(\lambda+1)_{n-2}(\mu+1)_{n-2}}{(\nu+1)_{n-2}(1)_{n-1}}z^n. \quad (14)$$

By Lemma 2.2 and Ref. [25], it is sufficient to show that

$$\sum_{n=2}^{\infty}\Big[2(2\delta^2-\delta)n(n-1) + \{2-(2\delta^2-\delta)(1+tu_n)(1+\sigma)\}n - u_n\{(2\delta^2-3\delta+1)(1+\sigma)\}\Big]\frac{(\lambda+1)_{n-2}(\mu+1)_{n-2}}{(\nu+1)_{n-2}(1)_{n-1}}$$

$$\leq \left|\frac{\nu}{\lambda\mu}\right|[(1-\sigma)-t(2\delta^2-\delta)(1+\sigma)]. \tag{15}$$

Now

$$\sum_{n=0}^{\infty}\Big[2(2\delta^2-\delta)(n+2)(n+1)+\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\}$$
$$(n+2)-\mathcal{U}\{(2\delta^2-3\delta+1)(1+\sigma)\}\Big]\frac{(\lambda+1)_n(\mu+1)_n}{(\nu+1)_n(1)_{n+1}}$$
$$=(4\delta^2-2\delta)\sum_{n=1}^{\infty}\frac{(\lambda+1)_n(\mu+1)_n}{(\nu+1)_n(1)_{n-1}}$$
$$+[(8\delta^2-4\delta+2)-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)]$$
$$\times\sum_{n=0}^{\infty}\frac{(\lambda+1)_n(\mu+1)_n}{(\nu+1)_n(1)_n}$$
$$+[2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)-\mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)]$$
$$\times\sum_{n=0}^{\infty}\frac{(\lambda+1)_n(\mu+1)_n}{(\nu+1)_n(1)_{n+1}}.$$

$$=\frac{\Gamma(\nu+1)\Gamma(\nu-\lambda-\mu-2)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}\Big[(4\delta^2-2\delta)(\lambda+1)(\mu+1)$$
$$+\Big\{(8\delta^2-4\delta+2)-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\Big\}(\nu-\lambda-\mu-2)$$
$$+\frac{\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)-\mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\Big\}}{\lambda\mu}$$
$$\times(\nu-\lambda-\mu-2)_2\Big]$$
$$-\frac{\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)-\mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\Big\}}{\lambda\mu}.$$

$$=\frac{\Gamma(\nu+1)\Gamma(\nu-\lambda-\mu-2)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}\Big[(4\delta^2-2\delta)(\lambda+1)(\mu+1)$$
$$+\Big\{(8\delta^2-4\delta+2)-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\Big\}(\nu-\lambda-\mu-2)$$
$$+\frac{\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)-\mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\Big\}}{\lambda\mu}$$
$$\times(\nu-\lambda-\mu-2)_2\Big]$$
$$\leq\Big[\Big\{(1-\sigma)-t(2\delta^2-\delta)(1+\sigma)\Big\}\frac{\nu}{|\lambda\mu|}$$
$$+\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)-\mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\Big\}\frac{\nu}{\lambda\mu}\Big].$$

For $\delta=0$, the following result is obtained.

**Corollary 2.4.** *Following Ref. [8], if $\lambda,\mu>-1$, $\lambda\mu<0$, and $\nu>\lambda+\mu+2$, then a necessary and sufficient condition for $zF(\lambda,\mu;\nu;z)$ to belong to $Q_T(\sigma,t)$ $(-1\leq\sigma<1,|t|\leq1,\ t\neq1)$ is that*

$$\frac{\Gamma(\nu+1)\Gamma(\nu-\lambda-\mu-2)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}\Big[2(\nu-\lambda-\mu-2)$$
$$+\frac{[2-\mathcal{U}(1+\sigma)](\nu-\lambda-\mu-2)_2}{\lambda\mu}\Big]$$
$$\leq[(1-\sigma)]\frac{\nu}{|\lambda\mu|}+[2-\mathcal{U}(1+\sigma)]\frac{\nu}{\lambda\mu}. \tag{16}$$

## 3. An integral operator

Following Ref. [2], we now obtain analogous results for the integral operator $G(\lambda,\mu;\nu;z)$ associated with $F(\lambda,\mu;\nu;z)$:

$$G(\lambda,\mu;\nu;z)=\int_0^z F(\lambda,\mu;\nu;s)\,ds. \tag{17}$$

**Theorem 3.1.**

(i) *For $\lambda,\mu>1$ and $\nu>\lambda+\mu+1$, a sufficient condition for $G(\lambda,\mu;\nu;z)$ defined by Eq. (17) to belong to the class $P(\delta,\sigma,t)$, as in Ref. [26], is*

$$\frac{\Gamma(\nu)\Gamma(\nu-\lambda-\mu)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}\Big[\frac{(4\delta^2-2\delta)\lambda\mu}{(\nu-\lambda-\mu-1)}$$
$$+\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\Big\}$$
$$-\frac{\mathcal{U}\Big\{(2\delta^2-3\delta+1)(1+\sigma)(\nu-\lambda-\mu)\Big\}}{(\lambda-1)(\mu-1)}\Big]$$
$$+\frac{\mathcal{U}\Big\{(2\delta^2-3\delta+1)(1+\sigma)\Big\}}{(\lambda-1)(\mu-1)}$$
$$\leq\Big[\Big\{(1-\sigma)-t(2\delta^2-\delta)(1+\sigma)\Big\}$$
$$+\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)$$
$$-\mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\Big\}\Big]. \tag{18}$$

(ii) *If $\lambda,\mu>-1$, $\lambda\mu<0$ and $\nu>\max\{0,\lambda+\mu+1\}$, then $G(\lambda,\mu;\nu;z)$ given by Eq. (17) is in $P_T(\delta,\sigma,t)$ if and only if*

$$\frac{\Gamma(\nu+1)\Gamma(\nu-\lambda-\mu-1)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}\Big[(4\delta^2-2\delta)$$
$$+\frac{\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\Big\}(\nu-\lambda-\mu-1)}{\lambda\mu}$$
$$-\frac{\mathcal{U}\Big\{(2\delta^2-3\delta+1)(1+\sigma)\Big\}(\nu-\lambda-\mu-1)_2}{(\lambda-1)_2(\mu-1)_2}\Big]$$
$$+\frac{\mathcal{U}\Big\{(2\delta^2-3\delta+1)(1+\sigma)\Big\}(\nu-1)_2}{(\lambda-1)_2(\mu-1)_2}$$
$$\leq\Big[\Big\{(1-\sigma)-t(2\delta^2-\delta)(1+\sigma)\Big\}\frac{\nu}{|\lambda\mu|}$$
$$+\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)$$
$$-\mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\Big\}\frac{\nu}{\lambda\mu}\Big]. \tag{19}$$

*Proof.* Since

$$G(\lambda,\mu;\nu;z)=z+\sum_{n=2}^{\infty}\frac{(\lambda)_{n-1}(\mu)_{n-1}}{(\nu)_{n-1}(1)_n}z^n,$$

we note that

$$\sum_{n=2}^{\infty}\Big[2(2\delta^2-\delta)n(n-1)+[2-(2\delta^2-\delta)(1+tu_n)(1+\sigma)]n$$
$$-u_n[(2\delta^2-3\delta+1)(1+\sigma)]\Big]\frac{(\lambda)_{n-1}(\mu)_{n-1}}{(\nu)_{n-1}(1)_n}$$

$$= \sum_{n=1}^{\infty}\Big[2(2\delta^2-\delta)n(n+1)+[2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)]$$
$$(n+1)-\mathcal{U}[(2\delta^2-3\delta+1)(1+\sigma)]\Big]\frac{(\lambda)_n(\mu)_n}{(\nu)_n(1)_{n+1}}$$
$$= (4\delta^2-2\delta)\sum_{n=1}^{\infty}\frac{(\lambda)_n(\mu)_n}{(\nu)_n(1)_{n-1}}+\sum_{n=1}^{\infty}\frac{(\lambda)_n(\mu)_n}{(\nu)_n(1)_n}$$
$$[2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)]$$
$$-\sum_{n=1}^{\infty}\frac{(\lambda)_n(\mu)_n}{(\nu)_n(1)_{n+1}}\times\mathcal{U}[(2\delta^2-3\delta+1)(1+\sigma)]$$

$$= \frac{\Gamma(\nu)\Gamma(\nu-\lambda-\mu)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}\Bigg[\frac{(4\delta^2-2\delta)\lambda\mu}{(\nu-\lambda-\mu-1)}$$
$$+\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\Big\}$$
$$-\frac{\mathcal{U}\Big\{(2\delta^2-3\delta+1)(1+\sigma)(\nu-\lambda-\mu)\Big\}}{(\lambda-1)(\mu-1)}\Bigg]$$
$$+\frac{\mathcal{U}\Big\{(2\delta^2-3\delta+1)(1+\sigma)\Big\}}{(\lambda-1)(\mu-1)}$$
$$-\Big\{(2\delta^2-3\delta+1)(1+\sigma)$$
$$-\mathcal{U}\Big[(2\delta^2-3\delta+1)(1+\sigma)\Big]\Big\}.$$

.

This expression is bounded above by $1-\sigma$ if and only if Eq. (18) holds, as in Ref. [2]. This proves part (i).

To prove (ii) we apply Lemma 2.2 to

$$G(\lambda,\mu;\nu;z) = z-\frac{|\lambda\mu|}{\nu}\sum_{n=2}^{\infty}\frac{(\lambda+1)_{n-2}(\mu+1)_{n-2}}{(\nu+1)_{n-2}(1)_n}z^n.$$

It is sufficient to show that

$$\sum_{n=2}^{\infty}\Big[2(2\delta^2-\delta)n(n-1)+[2-(2\delta^2-\delta)(1+tu_n)(1+\sigma)]n$$
$$-u_n[(2\delta^2-3\delta+1)(1+\sigma)]\Big]\frac{(\lambda+1)_{n-2}(\mu+1)_{n-2}}{(\nu+1)_{n-2}(1)_n}$$
$$\le\frac{[(1-\sigma)-t(2\delta^2-\delta)(1+\sigma)]\nu}{|\lambda\mu|}.$$

Now

$$\sum_{n=0}^{\infty}\Big[2(2\delta^2-\delta)(n+2)(n+1)+[2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)](n+2)$$
$$-\mathcal{U}[(2\delta^2-3\delta+1)(1+\sigma)]\Big]\frac{(\lambda+1)_n(\mu+1)_n}{(\nu+1)_n(1)_{n+2}}$$

$$= (4\delta^2-2\delta)\sum_{n=0}^{\infty}\frac{(\lambda+1)_n(\mu+1)_n}{(\nu+1)_n(1)_n}$$
$$+\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\Big\}\sum_{n=0}^{\infty}\frac{(\lambda+1)_n(\mu+1)_n}{(\nu+1)_n(1)_{n+1}}$$
$$-\mathcal{U}\Big\{(2\delta^2-3\delta+1)(1+\sigma)\Big\}\sum_{n=0}^{\infty}\frac{(\lambda+1)_{n+1}(\mu+1)_{n+1}}{(\nu+1)_{n+1}(1)_{n+2}}$$
$$= \frac{\Gamma(\nu+1)\Gamma(\nu-\lambda-\mu-1)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}\Bigg[(4\delta^2-2\delta)$$
$$+\frac{\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\Big\}(\nu-\lambda-\mu-1)}{\lambda\mu}$$
$$-\frac{\mathcal{U}\Big\{(2\delta^2-3\delta+1)(1+\sigma)\Big\}(\nu-\lambda-\mu-1)_2}{(\lambda-1)_2(\mu-1)_2}\Bigg]$$
$$+\frac{\mathcal{U}\Big\{(2\delta^2-3\delta+1)(1+\sigma)\Big\}(\nu-1)_2}{(\lambda-1)_2(\mu-1)_2}$$
$$-\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)$$
$$-\mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\Big\}$$

$$= \frac{\Gamma(\nu+1)\Gamma(\nu-\lambda-\mu-1)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}\Bigg[(4\delta^2-2\delta)$$
$$+\frac{\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)\Big\}(\nu-\lambda-\mu-1)}{\lambda\mu}$$
$$-\frac{\mathcal{U}\Big\{(2\delta^2-3\delta+1)(1+\sigma)\Big\}(\nu-\lambda-\mu-1)_2}{(\lambda-1)_2(\mu-1)_2}\Bigg]$$
$$+\frac{\mathcal{U}\Big\{(2\delta^2-3\delta+1)(1+\sigma)\Big\}(\nu-1)_2}{(\lambda-1)_2(\mu-1)_2}$$
$$\le\Bigg[\Big\{(1-\sigma)-t(2\delta^2-\delta)(1+\sigma)\Big\}\frac{\nu}{|\lambda\mu|}$$
$$+\Big\{2-(2\delta^2-\delta)(1+t\mathcal{U})(1+\sigma)$$
$$-\mathcal{U}(2\delta^2-3\delta+1)(1+\sigma)\Big\}\frac{\nu}{\lambda\mu}\Bigg].$$

This completes the proof.

For $\delta=0$, we obtain the following.

**Corollary 3.1.**

(i) *If $\lambda,\mu>1$ and $\nu>\lambda+\mu$, a sufficient condition for $G(\lambda,\mu;\nu;z)$ defined by Eq. (17) to belong to the class $Q(\sigma,t)$ is*

$$\frac{\Gamma(\nu)\Gamma(\nu-\lambda-\mu)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}\Bigg[2-\frac{\mathcal{U}(1+\sigma)(\nu-\lambda-\mu)}{(\lambda-1)(\mu-1)}\Bigg]$$
$$+\frac{\mathcal{U}(1+\sigma)}{(\lambda-1)(\mu-1)}$$
$$\le(1-\sigma)+\Big[2-\mathcal{U}(1+\sigma)\Big]. \tag{20}$$

(ii) *Following Ref. [8], if $\lambda,\mu>-1$, $\lambda\mu<0$, and $\nu>\max\{0,\lambda+\mu\}$, then $G(\lambda,\mu;\nu;z)$ given by Eq. (17) is in $Q_T(\sigma,t)$ if*

$$\frac{\Gamma(\nu+1)\Gamma(\nu-\lambda-\mu-1)}{\Gamma(\nu-\lambda)\Gamma(\nu-\mu)}\Bigg[\frac{2(\nu-\lambda-\mu-1)}{\lambda\mu}$$
$$-\frac{\mathcal{U}(1+\sigma)(\nu-\lambda-\mu-1)_2}{(\lambda-1)_2(\mu-1)_2}\Bigg]$$
$$+\frac{\mathcal{U}(1+\sigma)(\nu-1)_2}{(\lambda-1)_2(\mu-1)_2}$$
$$\le(1-\sigma)\frac{\nu}{|\lambda\mu|}+\Big[2-\mathcal{U}(1+\sigma)\Big]\frac{\nu}{\lambda\mu}. \tag{21}$$

## 4. Numerical implementation

All numerical illustrations presented in Figures 1–5 were generated using the fixed parameter values $\delta=0.5$, $\sigma=0$, $t=0$, together with $\nu=\lambda+\mu+2.5$, so that the condition $\nu>\lambda+\mu+2$ required in Theorem 2.1 is satisfied. For every parameter pair $(\lambda,\mu)$, Eq. (10) was evaluated numerically using these fixed values. The admissible region (safe zone) is defined as the set of all parameter pairs for which the left-hand side (LHS) of Eq. (10) does not exceed the right-hand side (RHS), that is, LHS $\leq$ RHS. Parameter pairs for which LHS $>$ RHS are classified as non-admissible. The boundary of the safe zone corresponds to the equality case.

Figure 1 was obtained by evaluating Eq. (10) on a uniform grid in the $(\lambda,\mu)$-plane and classifying each grid point according to whether the admissibility condition is satisfied. For the illustrative admissible parameter set $(\lambda,\mu,\nu) = (3,4,9.5)$, Eq. (10) yields LHS $=$ 1.260, RHS $=$ 1.500, and hence $1.260 \leq 1.500$, confirming that this parameter set belongs to the admissible region. Similarly, the comparison parameter set $(\lambda,\mu,\nu) = (7,8,17.5)$ was evaluated using the same numerical setup. Since the inequality in Eq. (10) is not satisfied for this parameter choice, it is classified as non-admissible and is used throughout the numerical illustrations as the comparison case. The frequency responses shown in Figure 2 were computed from the truncated Gaussian hypergeometric series $H(z) = zF(\lambda,\mu;\nu;z)$, evaluated on the unit circle $z = e^{i\omega}$, $0 \leq \omega \leq \pi$, using the first 30 terms of the power-series expansion. The magnitude responses were normalized by their respective maximum values before plotting. Figure 3 presents illustrative time-domain responses obtained from finite-order realizations corresponding to the selected admissible and comparison parameter sets and is intended to demonstrate the qualitative behaviour associated with the admissibility condition rather than to provide additional theoretical results. Figure 4 illustrates the geometric action of the hypergeometric function and the associated integral operator by numerically mapping uniformly distributed boundary points of the unit disk. Finally, Figure 5 summarizes the complete admissibility-based filter-design workflow, combining parameter verification through Eq. (10), frequency-response evaluation, and realization via the integral operator.

## 5. Methodology

The proposed methodology uses the admissibility design rule, a three-phase analytical and numerical workflow for synthesizing digital filters that satisfy the admissibility criterion. In phase 1 (pre-certification), potential filter parameters $(\lambda,\mu,\nu)$ for the transfer function $H(z) = zF(\lambda,\mu;\nu;z)$ are evaluated against the Gamma-ratio inequality established in Theorem 2.1; this analytical gate determines whether the function satisfies the sufficient condition for membership in $P(\delta,\sigma,t)$. In phase 2 (admissibility verification), the selected parameters are plotted on a two-dimensional contour map to confirm that they lie within the safe zone. The point $(\lambda,\mu) = (3,4)$ with $\nu = \lambda+\mu+2.5$ is used only as a representative numerical example. Finally, phase 3 (structural realisation) uses the integral operator $G(\lambda,\mu;\nu;z)$, whose admissibility is established in Theorem 3.1 under the stated parameter conditions. The corresponding numerical response is then evaluated for the chosen finite truncation.

## 6. Proposed algorithm: $\mathcal{UCV}$ hypergeometric filter design

This method for generating artifact-resistant digital filters is termed the *admissible design rule*. It incorporates geometric function theory into a three-step engineering process whose objective is to verify that the filter transfer function $H(z)$ falls within the uniformly convex ($\mathcal{UCV}$) subclass $P(\delta,\sigma,t)$.

*Inputs.*

- Filter parameters $\lambda,\mu > 0$ and $\nu > \lambda+\mu+2$.
- Subclass parameters $(\delta,\sigma,t)$, where the parameter $\delta$ should be selected according to the desired subclass; the numerical illustrations in the present work use $\delta = 0.5$ to demonstrate the admissibility conditions for the general class $P(\delta,\sigma,t)$.
- Normalization frequency $\omega_c$.

In the numerical illustrations, the representative parameter combination $(\lambda,\mu) = (3,4)$ together with $\nu = \lambda+\mu+2.5$ was selected according to the admissibility condition established in Theorem 2.1. Since Theorem 2.1 requires only that $\nu > \lambda+\mu+2$, the choice $\nu = \lambda+\mu+2.5$ provides a comfortable margin above the theoretical lower bound, placing the selected parameters well within the admissible (safe zone) determined by the Gamma-function inequality. For the finite truncation considered, this representative choice yields a smooth numerical frequency response with negligible ringing, while satisfying the admissibility conditions used in Theorems 2.1 and 3.1. It should be emphasized that $\nu = \lambda+\mu+2.5$ is not a theoretical requirement; any admissible parameter combination satisfying the same inequality may also be employed.

*Phase 1: pre-certification (the mathematical gate).*

1. Write the transfer function in terms of the Gaussian hypergeometric series in Ref. [12]:

$$H(z) = z\,F(\lambda,\mu;\nu;z) = z + \sum_{n=2}^{\infty} \frac{(\lambda)_{n-1}(\mu)_{n-1}}{(\nu)_{n-1}(1)_{n-1}} z^n.$$

2. Evaluate the Gamma-function admissibility criterion in Eq. (10) of Theorem 2.1.
3. If the inequality holds, the filter satisfies the stated sufficient admissibility criterion; otherwise, adjust the parameters and repeat the evaluation.

*Phase 2: admissibility verification (parametric mapping).*

1. Plot the selected parameter pair $(\lambda,\mu)$ on the admissibility contour map.
2. Confirm that the selected point lies within the *safe zone*. The point $(\lambda,\mu) = (3,4)$ with $\nu = \lambda+\mu+2.5$ is the representative configuration used in the numerical illustrations.
3. Compute the magnitude response $|H(\omega)|$ using a truncated power-series expansion of approximately 25–30 terms and inspect the response for sharp changes or spikes.

*Phase 3: final realisation (structural preservation).*

1. The filter is implemented by the integral operator

$$G(\lambda,\mu;\nu;z) = \int_0^z F(\lambda,\mu;\nu;s)\,ds.$$

2. Apply Theorem 3.1 to verify that the integral operator $G(\lambda,\mu;\nu;z)$ satisfies the admissibility conditions for the chosen parameters.
3. Evaluate the time-domain step response and report the observed numerical behaviour.

*Output.* The output is a realised digital filter whose transfer function satisfies the stated geometric admissibility criterion and remains within the subclass $P(\delta,\sigma,t)$ under the selected parameter conditions.

## 7. Significance

The significance of the present work from an engineering perspective lies in the development of an admissibility-based mathematical framework for the design of digital filters using geometric function theory. The sufficient conditions established for the subclass $P(\delta, \sigma, t)$, particularly the Gamma-function criterion given in Theorem 2.1, provide a systematic procedure for selecting parameter values for which the associated Gaussian hypergeometric function satisfies the required admissibility conditions. These theoretical results offer analytical guidance for filter parameter selection prior to numerical implementation.

Furthermore, Theorem 3.1 establishes sufficient conditions under which the particular integral operator

$$G(\lambda, \mu; \nu; z) = \int_0^z F(\lambda, \mu; \nu; s)\, ds$$

belongs to the class $P(\delta, \sigma, t)$ for the specified parameter values. Consequently, the theorem provides a mathematical basis for employing this operator within the proposed admissibility-based filter design framework.

The uniformly convex ($\mathcal{UCV}$) family was selected because it satisfies stronger geometric conditions than the classical convex and starlike families. While convex and starlike functions ensure analyticity and univalence, uniformly convex functions possess additional geometric properties that make them an attractive class for constructing admissible transfer functions. For this reason, the present study focuses on the $\mathcal{UCV}$ family when deriving admissibility conditions for the associated Gaussian hypergeometric functions.

The numerical examples presented in this paper illustrate that the finite-order filter realizations obtained from the admissible parameter sets exhibit smooth frequency responses and favourable time-domain behaviour for the representative parameter values considered. These observations are intended to demonstrate the practical applicability of the proposed mathematical framework and should be interpreted as numerical illustrations rather than as general theoretical consequences of membership in the class $P(\delta, \sigma, t)$.

## 8. $\mathcal{UCV}$ hypergeometric functions in digital filter design

The geometric properties established in Sections 2 and 3 have a direct application to digital signal processing. For filters based on hypergeometric functions whose parameters satisfy the $P(\delta, \sigma, t)$ admissibility condition, the numerical examples in this section illustrate smooth frequency responses for the representative admissible parameter values considered. This section formalises the connection between the analytical theory of Sections 2–3 and the engineering design of such filters.

### 8.1. Artifacts in standard filters

Classical finite impulse response (FIR) and infinite impulse response (IIR) digital filters often exhibit Gibbs-type oscillations (ringing artifacts) near sharp spectral transitions. These arise from the use of filter coefficients that are derived from non-geometric criteria, leading to transfer functions whose phase and amplitude responses are not jointly controlled. In the complex-analytic setting, such filters correspond to functions $f(z)$ lying outside the class $P(\delta, \sigma, t)$, and in particular outside the uniformly convex class $\mathcal{UCV}$.

The admissibility rule in Eq. (10), derived in Theorem 2.1, provides an explicit sufficient condition on the parameters $(\lambda, \mu, \nu)$ of the hypergeometric function to guarantee membership in $P(\delta, \sigma, t)$.

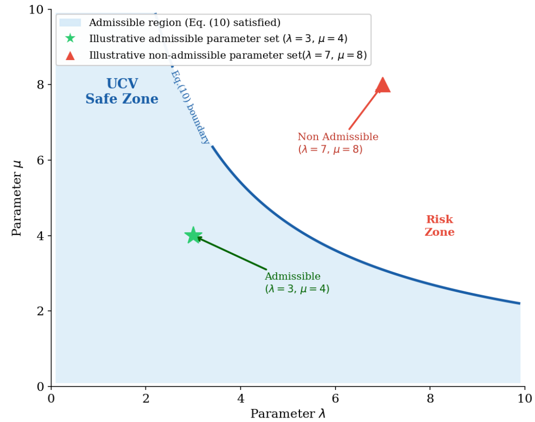


Figure 1. Admissibility contour plot in the $(\lambda, \mu)$ parameter space. The shaded area represents the safe zone in which Eq. (10) from Theorem 2.1 is satisfied; this condition ensures that $zF(\lambda, \mu; \nu; z) \in P(\delta, \sigma, t)$. The star indicates the representative admissible parameter set $(\lambda, \mu) = (3, 4)$.

Parameters that satisfy Eq. (10) are within the safe zone of the admissibility contour map, while the parameter sets outside this zone in the illustrated numerical cases exhibit artifact-prone responses.

### 8.2. Filter construction using the admissibility criterion

Following Ref. [27], the transfer function of a $\mathcal{UCV}$ hypergeometric filter is written as $H(z) = zF(\lambda, \mu; \nu; z) = z + \sum_{n=2}^{\infty} \frac{(\lambda)_{n-1}(\mu)_{n-1}}{(\nu)_{n-1}(1)_{n-1}} z^n$. The design procedure is as follows.

*Phase 1: pre-certification.* Select parameters $\lambda, \mu > 0$ and $\nu > \lambda + \mu + 2$.

The Gamma-ratio inequality in Eq. (10) of Theorem 2.1 is then evaluated. If the inequality holds, the filter satisfies the stated sufficient condition for membership in $P(\delta, \sigma, t)$.

*Phase 2: admissibility verification.* Construct the contour of admissibility as given in Figure 1, so that the selected pair of parameters $(\lambda, \mu)$ falls within the safe zone. The numerical example uses $(\lambda, \mu) = (3, 4)$ and $\nu = \lambda + \mu + 2.5$ as a representative admissible configuration. Although the numerical illustrations employ the representative parameter set $(\lambda, \mu) = (3, 4)$ with $\nu = \lambda + \mu + 2.5$, the proposed theoretical results are not restricted to this particular choice. The admissibility criterion established in Theorem 2.1 and the admissibility result established in Theorem 3.1 remain valid for all parameter combinations satisfying the required conditions. Therefore, the numerical example presented in this work serves only to illustrate the proposed design methodology, while the conclusions apply to the entire admissible parameter region.

*Phase 3: final realisation.* Theorem 3.1 establishes that the function $G(\lambda, \mu; \nu; z)$ belongs to $P(\delta, \sigma, t)$ under the stated parameter conditions.

For the numerical frequency-response analysis, the admissible parameter set $(\lambda, \mu, \nu) = (3, 4, 9.5)$ was selected, where $\nu = \lambda + \mu + 2.5$ satisfies the admissibility condition $\nu > \lambda + \mu + 2$. For comparison, the parameter set $(\lambda, \mu, \nu) = (7, 8, 17.5)$ was also considered.

The frequency response was computed on the unit circle $z = e^{i\omega}$, $0 \leq \omega \leq \pi$, using a truncated hypergeometric series consisting of the first 30 terms. The magnitude response was normalized with respect to its maximum value before plotting.

Figure 2 illustrates a smoother frequency response for the representative parameter set obtained from the 30-term truncation.

### 8.3. Digital filter model

The proposed digital filter is represented by the normalized transfer function

$$H(z) = zF(\lambda,\mu;\nu;z) = z + \sum_{n=2}^{\infty} \frac{(\lambda)_{n-1}(\mu)_{n-1}}{(\nu)_{n-1}(1)_{n-1}} z^n,$$

where $F(\lambda,\mu;\nu;z)$ denotes the Gaussian hypergeometric function. The corresponding coefficient sequence is

$$h_1 = 1, \qquad h_n = \frac{(\lambda)_{n-1}(\mu)_{n-1}}{(\nu)_{n-1}(1)_{n-1}}, \qquad n \geq 2.$$

The frequency response of the filter is obtained by evaluating the transfer function on the unit circle,

$$H(e^{i\omega}), \qquad 0 \leq \omega \leq \pi,$$

which is used to generate the numerical frequency-response characteristics presented in this work.

The transfer function is expressed in positive powers of $z$, following the analytic-function convention adopted throughout this work. Therefore, $H(z)$ is interpreted as a formal transfer function rather than the standard causal digital signal processing (DSP) transfer function written in negative powers of $z$. The analyticity of $H(z)$ in the open unit disk is a property of the underlying analytic function and is not interpreted as digital-filter stability. For numerical implementation, the infinite series is truncated to the first 30 terms, yielding a finite polynomial FIR transfer function with no finite poles. Hence, the implemented filter has a finite impulse response, an entire region of convergence, and satisfies the absolute-summability condition, ensuring bounded-input bounded-output (BIBO) stability. Consequently, the proposed transfer function provides a mathematically stable framework for filter synthesis within the adopted geometric function theory setting. The present work is concerned with the theoretical transfer function and its numerical realization; a hardware realization and practical DSP implementation are beyond the scope of this study.

The transfer function is normalized by dividing its magnitude response by its maximum value before comparison with other filters. This normalization facilitates a consistent comparison of the frequency responses without altering the relative spectral characteristics.

For numerical implementation, the infinite hypergeometric series is approximated by its first 30 terms. This finite-order approximation provides an accurate representation of the ideal transfer function for the parameter values considered in the numerical experiments. The implementation consists of computing the hypergeometric coefficients, constructing the truncated transfer function, evaluating $H(e^{i\omega})$, and plotting the normalized magnitude response for admissible parameter values.

### 8.4. Time-domain artifact elimination

Uniform convexity also has a time-domain interpretation in the present numerical setting. In the examples considered here, the finite-order approximation of a filter with transfer function $H(z) \in P(\delta,\sigma,t)$ exhibits negligible overshoot. This numerical behaviour is consistent with the joint restriction imposed by $P(\delta,\sigma,t)$ on the real part of the logarithmic derivative of $f(z)$ and the associated phase–amplitude coupling. The computed step response therefore suggests reduced ringing for the tested parameter values.

In addition, according to Theorem 3.1, the integral operator $G(\lambda,\mu;\nu;z)$ ensures that the resulting filter stage belongs to the class $P(\delta,\sigma,t)$. Therefore, under the conditions of Theorem 3.1, the function generated by the integral operator belongs to the admissible class $P(\delta,\sigma,t)$.

*Phase response and group delay.* Membership in the uniformly convex class $P(\delta,\sigma,t)$ may also influence the phase behaviour of the designed filter. Because the admissibility conditions constrain the logarithmic derivative of the transfer function, the illustrated numerical example exhibits a smoother phase response. However, this paper establishes no explicit theoretical relation between membership in $P(\delta,\sigma,t)$ and group delay. Any reduction in phase distortion or improvement in waveform restoration should therefore be interpreted as a numerical observation for the example considered. Explicit formulas for phase response and group delay are beyond the scope of this work; nevertheless, the geometric conditions provide a basis for future investigation.

### 8.5. Geometric interpretation of the integral operator

The main theorem in Section 3 establishes sufficient conditions under which the integral operator $G(\lambda,\mu;\nu;z)$ belongs to the class $P(\delta,\sigma,t)$. This result may be interpreted within the proposed admissibility-based filter design framework. In particular, Theorem 3.1 provides mathematical conditions under which the function generated by the integral operator satisfies the admissibility requirements considered in this work.

From an engineering perspective, the operator $G(\lambda,\mu;\nu;z)$ may be viewed as an accumulation stage in the cascade structure of a digital filter. Similar to an integrator in a digital filtering framework, the operator smooths sharp variations in the transfer function, while Theorem 3.1 establishes that the resulting function satisfies the admissibility conditions for the stated parameter values. Consequently, the integral operator provides a mathematically justified framework for constructing the particular admissible filter considered in this study.

Namely, it is known that the image of $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ under the mapping $H(z) = zF(\lambda,\mu;\nu;z)$ is a uniformly convex domain provided that the conditions of Theorem 2.1 are satisfied. The corresponding integral operator

$$G(\lambda,\mu;\nu;z) = \int_0^z F(\lambda,\mu;\nu;s)\,ds$$

generates another domain corresponding to the function $G(\lambda,\mu;\nu;z)$, which belongs to the class $P(\delta,\sigma,t)$ under the parameter conditions established in Theorem 3.1.

From a signal processing viewpoint, the admissibility result established in Theorem 3.1 provides a mathematical justification for employing the integral operator within the proposed filter-design framework. Accordingly, the numerical examples presented in this paper illustrate the geometric behaviour of the admissible function generated by the operator and demonstrate the applicability of the proposed admissibility-based approach to digital filter design.

### 8.6. Comparison with classical filters

Classical filter structures, including Butterworth, Chebyshev Type I, and elliptic filters, are widely used in signal-processing applications because of their frequency-selective properties (Refs. [27–30]). Such filters are usually designed in terms of magnitude-response specifications and do not necessarily impose additional restrictions on the geometry of the admissible function space. Consequently, certain

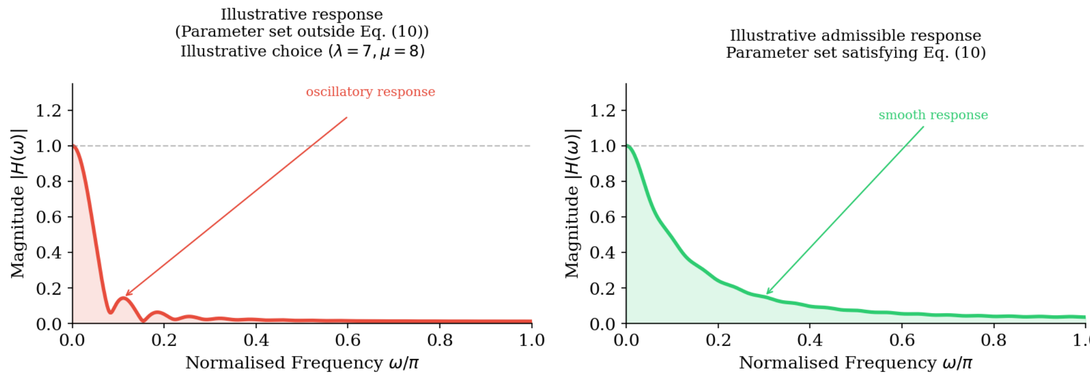


Figure 2. Normalized frequency responses of the hypergeometric transfer function $H(z) = zF(\lambda, \mu; \nu; z)$ computed on the unit circle $z = e^{i\omega}$. The admissible parameter set $(\lambda, \mu, \nu) = (3, 4, 9.5)$ satisfies the condition $\nu > \lambda + \mu + 2$, whereas $(7, 8, 17.5)$ is used as a comparison parameter set. The responses are obtained using a truncated 30-term hypergeometric series and normalized by their maximum magnitude.

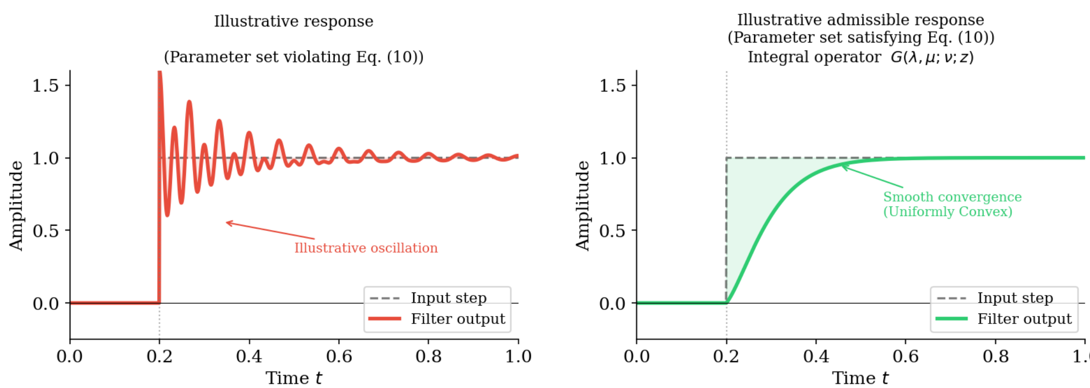


Figure 3. Step response for a typical filter (left) and step response for the $\mathcal{UCV}$ hypergeometric filter produced using the integral operator $G(\lambda, \mu; \nu; z)$ (right). The step response exhibits negligible overshoot. This is a numerical observation and is consistent with the admissibility conditions established in Theorem 3.1.

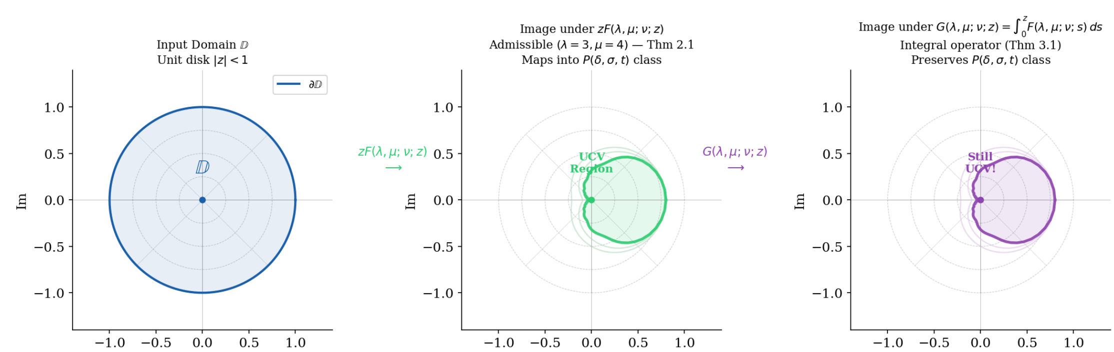


Figure 4. Geometric interpretation of the integral operator $G(\lambda, \mu; \nu; z)$. Left: input unit disc $\mathbb{D}$. Centre: the image of $zF(\lambda, \mu; \nu; z)$ (green), satisfying the admissibility conditions of Theorem 2.1. Right: the image corresponding to the integral operator $G(\lambda, \mu; \nu; z)$ (purple), illustrating the membership result established in Theorem 3.1 for the function $G(\lambda, \mu; \nu; z)$. The concentric circles for $r = 0.5$, $0.75$, and $1$ visualize the admissible image associated with the stated parameter conditions.

transients, such as Gibbs-type ringing, may occur under specific operating conditions.

Unlike these classical filters, the hypergeometric filter considered here is constructed in the uniformly convex setting using the criterion in Theorem 2.1. The geometric admissibility properties of the associated integral operator are established analytically in Theorem 3.1. Thus, the requirements imposed on the transfer function by $P(\delta, \sigma, t)$ provide an analytical framework for controlling smoothness. The proposed filtering algorithm may therefore serve as a mathematically grounded alternative for the applications considered.

In contrast to Butterworth and Chebyshev filters, whose design is primarily governed by frequency-response specifications, the proposed

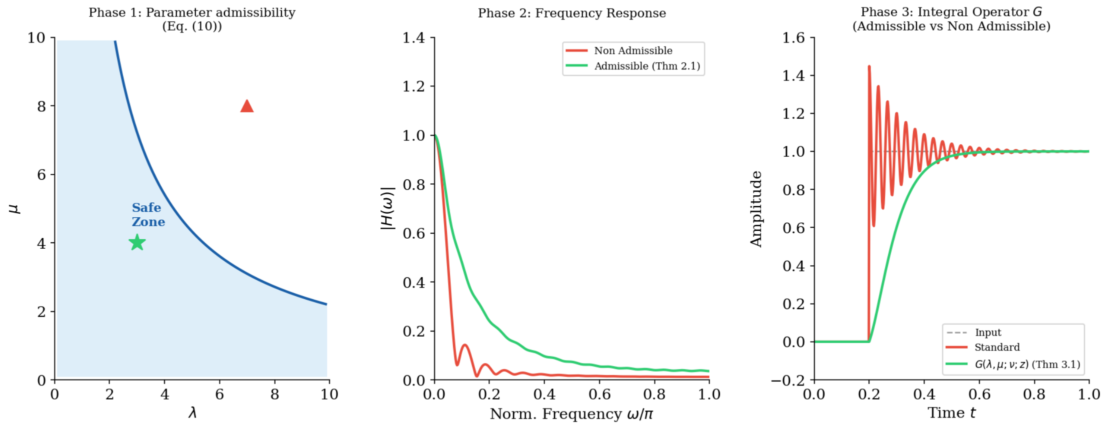


Figure 5. Summary of the admissibility design rule in three phases. Phase 1 (left column): admissibility contour map, where the safe zone indicates $\mathcal{UCV}$-admissible parameters. Phase 2 (middle column): frequency response comparison demonstrating how $\mathcal{UCV}$ parameters (green) avoid overshooting found in non-$\mathcal{UCV}$ parameters (red). Phase 3 (right column): time domain response using $G(\lambda,\mu;\nu;z)$, exhibiting an illustrative time-domain response obtained using the admissible parameter set.

hypergeometric filter is constructed from geometric admissibility conditions. Consequently, the framework supports smooth transfer-function behaviour analytically rather than relying solely on empirical design, while preserving admissibility under the stated conditions for cascaded implementations.

### 8.7. Summary of the three-phase design rule

Figure 5 gives an overview of all three phases involved in $\mathcal{UCV}$ hypergeometric filter design in a single diagram. In phase 1, we define the parameter space using Eq. (10). In phase 2, we discuss the advantages in the frequency domain. In phase 3, we show that $G(\lambda,\mu;\nu;z)$ is a smoothness-preserving cascaded stage.

The engineering significance of the present findings is summarized as follows. Theorem 2.1 furnishes a computable criterion in the form of a theorem, stated solely in terms of Gamma functions, for distinguishing between artifact-producing parameter sets and geometrically admissible parameter sets. Theorem 3.1 establishes that, under its stated parameter assumptions, the integral operator $G(\lambda,\mu;\nu;z)$ belongs to the class $P(\delta,\sigma,t)$.

Collectively, these results provide signal-processing engineers with a well-defined mathematical foundation, based on a Gamma-function criterion, for designing digital filters whose geometric properties are established within the theory of uniformly convex hypergeometric functions.

**Remark 8.1.** *The results established in this work apply to the ideal continuous transfer functions generated by the proposed hypergeometric and integral operators. The geometric admissibility conditions guarantee the desired analytic and uniform convexity properties of these ideal models. However, practical digital filter implementations involve finite-series truncation, coefficient quantization, and finite-precision arithmetic, which are beyond the scope of the present theoretical analysis. Therefore, the proposed framework provides a rigorous mathematical foundation for filter synthesis, while experimental validation and implementation-specific performance remain topics for future investigation.*

## 9. Conclusion and future scope

In this study, we investigated conditions under which the Gaussian hypergeometric function $zF(\lambda,\mu;\nu;z)$ and an associated integral operator belong to the Sakaguchi-type class $P(\delta,\sigma,t)$. The sufficient membership conditions were obtained as explicit inequalities involving Gamma functions. These inequalities can therefore be used to identify parameter values for which the hypergeometric function belongs to the class under consideration. We also analysed the membership conditions for the integral operator $G(\lambda,\mu;\nu;z)$; under the stated assumptions, Theorem 3.1 gives sufficient conditions for the operator to belong to $P(\delta,\sigma,t)$. The operator-theoretic approach adopted in the present work is consistent with related studies on integral operators in geometric function theory in Ref. [31].

In addition to these theoretical results, we considered applications of the geometric properties to digital filter construction. The proposed admissibility criterion provides a mathematical prescription for distinguishing regions of parameter space in which the designed filters exhibit improved smoothness from regions in which oscillatory artifacts may occur. For the representative parameters tested, the numerical experiments produced smooth frequency- and time-domain responses with minimal ringing. The theoretical results suggest that the integral operator may be useful for multistage filter construction when the assumptions of Theorem 3.1 are satisfied.

These findings suggest several directions for future research. One direction is to investigate whether the membership conditions derived in this paper can be tightened to obtain necessary and sufficient conditions. More advanced hypergeometric systems, such as those involving Appell and Lauricella functions, could also be explored to incorporate multidimensional geometry into image and signal processing.

Another key area is the analysis of fractional-order integrators. Because fractional calculus is increasingly relevant to modern filters and control systems, examining whether the geometric properties are preserved under fractional integration could lead to more flexible filter structures. Another avenue is to optimise admissible parameter combinations for objectives such as passband-ripple reduction or attenuation.

Practical studies should consider hypergeometric filter construction using experimental data, including biomedical signals, speech, radar signals, and images. Potential biomedical applications include electrocardiogram (ECG) and electroencephalogram (EEG) signal enhancement, magnetic resonance imaging (MRI) and computed tomography (CT) image reconstruction, ultrasound imaging, and microscopic biomedical image analysis, where suppression of ringing artifacts and preservation of smooth structural features are highly desirable. Such examples could clarify the applicability of geometrically justified filters and identify settings in which ringing reduction is especially important. Relatively little work has addressed the intersection of geometric function theory and engineering applications, and the present approach suggests several opportunities for future development.

## Data availability

No external datasets were used or analyzed during the current study. All data generated and presented in this manuscript are included within the article.

## Declaration of competing interest

The authors declare that they have no known competing financial or personal interests that could have appeared to influence the work reported in this article.

## Funding

The authors received no specific funding from any public, commercial, or not-for-profit organisation for this research.

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