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Double Laplace–Sawi decomposition technique for solving nonlinear partial differential equations

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Abstract

This study develops a double Laplace–Sawi decomposition technique for nonlinear partial differential equations by coupling the double Laplace–Sawi transform with Adomian polynomials. In contrast with a purely formal transform calculation, explicit exponential-order hypotheses are imposed to guarantee existence of the single and double transforms and to justify interchange of integrations and differentiation. Separate recursive formulations are derived for first- and second-order time operators, thereby covering problems with one or two initial conditions without ambiguity. A sufficient contraction condition for convergence is established in a Banach space, together with an a posteriori truncation-error estimate. Three nonlinear problems are examined. The first corrects the transform and algebraic details of a nonlinear transport–reaction equation; the second demonstrates coefficient generation and finite-time blow-up for a cubic nonlinearity; and the third treats the inviscid Burgers equation, a genuinely nonlinear partial differential equation with coupled spatial dynamics and boundary data. For the Burgers problem, the decomposition coefficients are proved by induction, the exact characteristic solution is recovered, and explicit truncation errors are reported. The results show that the technique provides a systematic analytic construction on its stated convergence domain, while also identifying the limitations caused by local series convergence, singularities, and the increasing cost of high-order Adomian polynomials.

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1. Introduction

Nonlinear partial differential equations (PDEs) describe transport, wave propagation, diffusion, fluid motion, and many other processes in mathematical physics. Integral transforms reduce differential operators to algebraic expressions and can therefore simplify initial and boundary value problems. The Sawi transform has been applied to systems of ordinary differential equations and physical reaction models [1], whereas the double Sawi transform was developed for two-variable boundary value problems, including heat and wave equations [2]. Hybrid double transforms extend this idea by assigning a different transform to each independent variable. For example, the double Laplace–ARA transform was used to solve several linear PDEs [3].

The double Laplace–Sawi transform (DLW) combines the Laplace transform in the spatial variable with the Sawi transform in time. Ref. [4] introduced its existence, inversion, differentiation, and convolution properties. Closely related developments include the double ARA–Sawi transform [5], the double Sumudu–Sawi transform [6], and the conformable double Laplace–Sawi transform for fractional PDEs [7]. These recent

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studies demonstrate that Sawi-based hybrid transforms can organize multidimensional calculations, but they principally address linear equations. A nonlinear problem requires an additional device for representing the nonlinear operator.

1.1. Recent literature and research gap

The Adomian decomposition method (ADM) represents the unknown solution and its nonlinear term by compatible series [8]. Duan [9] developed recurrence algorithms that reduce the symbolic effort needed to generate Adomian polynomials. Within this journal, Ndam and Adedire [10] compared a modified ADM with truncated Taylor expansions for the van der Pol equation and showed that error assessment is essential when finite series are used. Akinola *et al.* [11] combined an integral transform with a weighted-residual construction for a semi-infinite boundary value problem. More recently, Yunus and Olayiwola [12] applied a Laplace–Adomian procedure to a nonlinear fractional epidemic model. Rawashdeh *et al.* [13] also emphasized convergence conditions in a fractional decomposition setting. Collectively, these studies motivate a hybrid transform–decomposition method, but they also show that simple examples alone do not establish convergence, accuracy, or applicability to genuine nonlinear PDEs.

The present work addresses this gap through four objectives: to state sufficient hypotheses for the existence and manipulation of the Laplace, Sawi, and double Laplace–Sawi transforms; to derive consistent DLW–ADM recursions for both first- and second-order time equations; to provide a sufficient convergence condition and a practical truncation-error criterion; and to test the construction on nonlinear problems of increasing complexity, including a Burgers equation with spatial coupling and boundary data.

The contribution is therefore not merely a new notation for ADM. It is a corrected transform-domain formulation with explicit analytical hypotheses, order-dependent recursions, convergence information, and a nontrivial PDE verification.

2. Laplace, Sawi, and double Laplace–Sawi transforms

Definition 2.1. Let $p : [0,\infty) \to \mathbb{C}$ be piecewise continuous on every bounded interval and satisfy

$$|p(z)| \le Me^{az}, \qquad z \ge 0, \tag{1}$$

for constants $M > 0$ and $a \in \mathbb{R}$. Its Laplace transform is

$$P(w) = \mathcal{L}_z\{p\}(w) = \int_0^\infty e^{-wz}p(z)\,dz, \qquad \operatorname{Re} w > a. \tag{2}$$

The exponential-order assumption, rather than continuity alone, guarantees absolute convergence in the stated half-plane. For functions and derivatives satisfying the same type of bound,

$$\mathcal{L}_z\{p_z\} = wP(w) - p(0), \tag{3}$$

$$\mathcal{L}_z\{p_{zz}\} = w^2P(w) - wp(0) - p_z(0). \tag{4}$$

Definition 2.2. Let $q : [0,\infty) \to \mathbb{C}$ be piecewise continuous on bounded intervals and satisfy $|q(t)| \le Me^{bt}$. Its Sawi transform is

$$\begin{aligned} Q(v) &= \mathcal{W}_t\{q\}(v) \\ &= \frac{1}{v^2}\int_0^\infty e^{-t/v}q(t)\,dt. \end{aligned} \tag{5}$$

For real parameters, the admissible domain in Equation (5) is $v > 0$ with $1/v > b$. If q_t and q_{tt} satisfy compatible exponential bounds, integration by parts gives

$$\mathcal{W}_t\{q_t\} = \frac{1}{v}Q(v) - \frac{1}{v^2}q(0), \tag{6}$$

$$\mathcal{W}_t\{q_{tt}\} = \frac{1}{v^2}Q(v) - \frac{1}{v^3}q(0) - \frac{1}{v^2}q_t(0). \tag{7}$$

Definition 2.3. Suppose that g is measurable on $[0,\infty)^2$, locally piecewise continuous, and

$$|g(z,t)| \le Me^{az+bt}. \tag{8}$$

The double Laplace–Sawi transform is

$$\begin{aligned} G(w,v) &= \mathcal{L}_z\mathcal{W}_t\{g\} \\ &= \frac{1}{v^2}\int_0^\infty\int_0^\infty e^{-wz-t/v}g(z,t)\,dt\,dz, \end{aligned} \tag{9}$$

where $\operatorname{Re} w > a$ and $\operatorname{Re}(1/v) > b$.

Under Equation (8), the integral in Equation (9) is absolutely convergent. Hence, Fubini's theorem justifies interchange of the two integrations and factorization for separable functions. Differentiation under the integral and the derivative identities used below are valid when the corresponding partial derivatives obey an integrable exponential majorant. Standard Bromwich inversion applies in w and, through the relation

$$\mathcal{W}_t\{q\}(v) = v^{-2}\mathcal{L}_t\{q\}(1/v), \tag{10}$$

inversion in v follows from Laplace inversion under the usual piecewise-smoothness assumptions.

For later use, define the r-fold Volterra operator, for $r = 1, 2$, by

$$\begin{aligned} (\mathcal{J}^r\phi)(z,t) = \frac{1}{(r-1)!}\int_0^t (t-\tau)^{r-1} \\ \times \phi(z,\tau)\,d\tau. \end{aligned} \tag{11}$$

Equations (6)–(7) imply

$$\mathcal{W}_t\{\mathcal{J}^r\phi\} = v^r\mathcal{W}_t\{\phi\}. \tag{12}$$

3. Double Laplace–Sawi decomposition technique

Consider the order-r nonlinear PDE

$$\partial_t^r g + Rg + N(g) = q, \qquad r \in \{1,2\}, \tag{13}$$

where R is a linear spatial or lower-order operator and N is a nonlinear operator. For $r = 1$, prescribe

$$g(z,0) = h_0(z), \tag{14}$$

whereas for $r = 2$, prescribe

$$g(z,0) = h_0(z), \qquad g_t(z,0) = h_1(z). \tag{15}$$

Application of DLW to Equation (13), followed by Equations (6)–(7), gives

$$G(w,v) = \mathcal{L}_z\mathcal{W}_t\{\Phi_r\} + v^r\mathcal{L}_z\mathcal{W}_t\{q - Rg - N(g)\}, \tag{16}$$

where

$$\Phi_1(z,t) = h_0(z), \qquad \Phi_2(z,t) = h_0(z) + th_1(z). \tag{17}$$

In particular, the correct second-order initial-data contribution in the transform domain is $v^{-1}\mathcal{L}_z\{h_0\} + \mathcal{L}_z\{h_1\}$, not terms multiplied by v^2 and v^3. Inverting Equation (16) and using Equation (12) yields the unambiguous integral equation

$$g = \Phi_r + \mathcal{J}^r[q - Rg - N(g)]. \tag{18}$$

Write

$$g = \sum_{m=0}^{\infty} g_m, \qquad N(g) = \sum_{m=0}^{\infty} A_m, \tag{19}$$

where

$$A_m = \frac{1}{m!}\frac{d^m}{d\lambda^m}N\left(\sum_{k=0}^{\infty}\lambda^k g_k\right)\Bigg|_{\lambda=0}, \qquad m \geq 0. \tag{20}$$

The corrected DLW decomposition recursion is

$$g_0 = \Phi_r + \mathcal{J}^r q, \tag{21}$$

$$g_{m+1} = -\mathcal{J}^r(Rg_m + A_m), \qquad m \geq 0. \tag{22}$$

Thus Equation (22) generates g_1 from $m = 0$ and applies separately and consistently to first- and second-order time operators.

3.1. Convergence and truncation

Let X be a Banach space of spatial functions on which R and N map the closed ball under consideration into X, and let $C([0,T];X)$ carry the supremum norm. For derivative nonlinearities, the spatial regularity and operator domains must be chosen so that this mapping property holds.

Theorem 3.1. Assume that $R : X \to X$ is bounded with norm β, that N is analytic and Lipschitz with constant L_N on a closed ball containing the iterates, and that

$$\kappa = \frac{T^r}{r!}(\beta + L_N) < 1. \tag{23}$$

Then the mapping defined by the right-hand side of Equation (18) is a contraction. Consequently, Equation (13) has a unique solution in that ball, and the compatible absolutely convergent Adomian series generated by Equations (21)–(22) converges to this solution.

Proof. For u, v in the ball,

$$\|\mathcal{J}^r[R(u-v) + N(u) - N(v)]\|_\infty \leq \frac{T^r}{r!}(\beta + L_N)\|u - v\|_\infty \tag{24}$$

$$= \kappa\|u - v\|_\infty. \tag{25}$$

The contraction principle gives existence and uniqueness. Analyticity of N makes its Adomian expansion compatible with the fixed-point expansion; absolute convergence permits termwise application of the bounded Volterra operator. □

For computation, define

$$S_M(z,t) = \sum_{m=0}^{M} g_m(z,t). \tag{26}$$

If the computed components satisfy $\|g_{m+1}\|_\infty \leq \rho\|g_m\|_\infty$ for all $m \geq M+1$ with $0 < \rho < 1$, then the a posteriori tail estimate is

$$\|g - S_M\|_\infty \leq \frac{\|g_{M+1}\|_\infty}{1-\rho}. \tag{27}$$

Equations (23) and (27) make explicit that convergence is local in general. Near singularities or outside a contractive interval, many terms may be required and the series may fail to converge.

4. Applications and discussion

Example 1 (Nonlinear transport–reaction equation)**.** Consider

$$g_t + g_z + 2g^2 = 0, \qquad g(z,0) = \frac{1}{z+1}, \qquad z,t \geq 0, \tag{28}$$

with the boundary condition $g(0,t) = \frac{1}{1+t}$. The spatial Laplace transform of the initial datum is

$$H(w) = \mathcal{L}_z\left\{\frac{1}{z+1}\right\} = e^w E_1(w), \qquad \operatorname{Re} w > 0, \tag{29}$$

where E_1 is the exponential integral. Thus the transform-domain initial term must be retained as $H(w)/v$; it is not e^{-zw}/v. Using the first-order form of Equation (22),

$$g_0 = \frac{1}{z+1}, \tag{30}$$

$$g_{m+1} = -\int_0^t [(g_m)_z + 2A_m](z,\tau)\,d\tau, \tag{31}$$

where $A_m = \sum_{k=0}^{m} g_k g_{m-k}$. The first components are

$$g_1 = -\frac{t}{(z+1)^2}, \qquad g_2 = \frac{t^2}{(z+1)^3}, \qquad g_3 = -\frac{t^3}{(z+1)^4}. \tag{32}$$

In particular, the time integration in Equation (31) gives the denominator $(z+1)^3$ in both contributions to g_2 before simplification. Therefore,

$$g(z,t) = \frac{1}{z+1}\sum_{m=0}^{\infty}\left(-\frac{t}{z+1}\right)^m = \frac{1}{z+t+1}. \tag{33}$$

The decomposition series converges for $|t/(z+1)| < 1$. The closed form in Equation (33) extends analytically beyond that disk wherever $z + t + 1 \neq 0$; this continuation is distinct from convergence of the original series.

Example 2 (Cubic nonlinearity and finite-time blow-up)**.** Consider

$$g_t - z^2 g^3 = 0, \qquad g(z,0) = 1. \tag{34}$$

This equation is an instructive coefficient test, although it is an ordinary differential equation in t for every fixed z. With $x = z^2 t$ and $g = \sum_{m\geq 0} c_m x^m$, the decomposition recursion gives

$$c_0 = 1, \qquad (m+1)c_{m+1} = \sum_{i+j+k=m} c_i c_j c_k. \tag{35}$$

The unique coefficients satisfying Equation (35) are

$$c_m = \frac{(2m)!}{2^m (m!)^2}. \tag{36}$$

Indeed, the generating function associated with Equation (36) is $C(x) = (1-2x)^{-1/2}$, for which $C'(x) = C^3(x)$ and $C(0) = 1$. Equality of coefficients proves Equation (35), and uniqueness completes the induction. Hence,

$$g(z,t) = \sum_{m=0}^{\infty} \frac{(2m)!}{2^m (m!)^2} (z^2 t)^m = \frac{1}{\sqrt{1-2z^2 t}}. \tag{37}$$

The series converges only for $|2z^2 t| < 1$. For real $z \neq 0$ and $t \geq 0$, the maximal classical interval is $0 \leq t < 1/(2z^2)$, and the solution blows up when $1 - 2z^2 t = 0$.

Example 3 (Inviscid Burgers equation)**.** To test coupled spatial dynamics, consider the genuinely nonlinear PDE

$$\begin{aligned} g_t + g\, g_z &= 0, \qquad & g(z,0) &= z, \\ g(0,t) &= 0, \qquad & 0 \leq z &\leq Z. \end{aligned} \tag{38}$$

For $N(g) = g g_z$, the Adomian polynomials are

$$A_m = \sum_{k=0}^{m} g_k (g_{m-k})_z. \tag{39}$$

Equations (21)–(22) give

$$g_0 = z, \qquad g_{m+1} = -\int_0^t A_m(z,\tau)\, d\tau. \tag{40}$$

We prove by induction that

$$g_m(z,t) = (-1)^m z t^m. \tag{41}$$

The result is immediate for $m = 0$. If it holds through index m, then Equation (39) gives

$$A_m = (m+1)(-1)^m z t^m, \tag{42}$$

and Equation (40) yields $g_{m+1} = (-1)^{m+1} z t^{m+1}$. Therefore,

$$g(z,t) = z \sum_{m=0}^{\infty} (-t)^m = \frac{z}{1+t}, \qquad |t| < 1. \tag{43}$$

The method of characteristics independently gives the same closed-form solution, and the boundary condition in Equation (38) is satisfied. The series is local to $|t| < 1$, while the real closed-form solution exists for $t > -1$.

Table 1: Absolute truncation error for the Burgers solution at $z = 1$.

t	$M = 2$	$M = 4$	$M = 6$
0.2	6.667×10^{-3}	2.667×10^{-4}	1.067×10^{-5}
0.5	8.333×10^{-2}	2.083×10^{-2}	5.208×10^{-3}
0.8	2.844×10^{-1}	1.820×10^{-1}	1.165×10^{-1}

For the partial sum $S_M = z \sum_{m=0}^{M} (-t)^m$, the exact error is

$$|g - S_M| = \frac{|z|\,|t|^{M+1}}{|1+t|}. \tag{44}$$

Table 1 reports this error at $z = 1$. It shows rapid decay for $t = 0.2$ but slower convergence as t approaches the series boundary $t = 1$, illustrating rather than concealing the truncation limitation.

5. Conclusion

The novel contribution of this work is a mathematically consistent DLW decomposition framework for first- and second-order nonlinear time evolution equations. The revision supplies transform-existence hypotheses, corrects the second-order initial-data factors, removes ambiguous notation, and derives the recursion from an explicitly defined Volterra operator. A contraction condition and a posteriori tail estimate clarify when the decomposition is justified.

The theoretical implication is that the hybrid transform and ADM can be combined without treating the resulting series as automatically global. The practical implication is a reproducible symbolic procedure for nonlinear transport and evolution models. The Burgers example, its characteristic verification, and its error table show how spatial coupling, boundary data, and truncation accuracy can be assessed.

The method nevertheless has limitations. The contraction condition is sufficient rather than necessary; convergence can be local; Adomian polynomials become costly for complicated nonlinearities; and singularities or discontinuities require separate analysis. Future work should investigate adaptive restarting or Padé continuation, fractional and coupled systems, discontinuous solutions and shock formation, automated polynomial generation, and quantitative comparisons with spectral and finite-difference solvers.

Data availability

No external dataset was used or generated. All analytical expressions and the numerical values used to produce Table 1 are given explicitly in Equations (43) and (44); therefore, a separate data repository is not applicable.

Declaration of competing interest

The authors declare no conflict of interest.

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