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Neutrosophic parameterized time-effective soft sets: theory, properties, and a TOPSIS-based decision-making application

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Abstract

The parameterized time neutrosophic soft set supplies a time-indexed family of neutrosophic soft sets together with its basic operations, while the effective-parameter mechanism, which revises an assessment once external confirmatory evidence arrives, has so far been developed either for static neutrosophic soft sets or for time-indexed *fuzzy* soft sets, where each assessment is a single membership value. This paper places an effective-parameter operator, the Λ-adjustment, natively on neutrosophic truth–indeterminacy–falsity triples inside the parameterized time-indexed structure, giving the neutrosophic parameterized time-effective soft set (N-PTEFSS). The base structure and its operations are recalled from earlier work and are not claimed as new; what is established here is the adjustment itself and its algebraic behaviour, including a bound-preservation result showing that it respects the neutrosophic constraint $T + I + F \leq 3$, which has no counterpart in the single-valued fuzzy setting, together with monotonicity, Lipschitz continuity and order preservation. A neutrosophic TOPSIS decision procedure is developed downstream of the adjustment and applied to a medical-diagnosis ranking problem in which indeterminacy is a genuine and unavoidable part of clinical judgment. The model reduces, term for term at the definitional layer, to the time-effective fuzzy soft set when indeterminacy and falsity vanish, so it is a proper generalization of that fuzzy predecessor rather than a relabelling. In the worked application, time-weighting alone leaves the top of the ranking unchanged while the effective-parameter adjustment reverses the top two alternatives, showing that post-assessment confirmatory evidence is decision-relevant and that no purely pre-implementation analysis can capture it.

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1. Introduction

Most problems in engineering, medicine, economics, and the environment are shaped by uncertainty. Molodtsov [1] introduced soft sets as a mathematical tool for handling such uncertainty, and Maji *et al.* [2, 3] extended this to fuzzy soft sets, later applied to decision making by Roy and Maji [4]. In parallel, Smarandache [5] introduced neutrosophic sets, which represent each judgment by a triple of truth, indeterminacy, and falsity degrees rather than a single membership value, and Maji [6] combined these with soft sets to obtain neutrosophic soft sets. A recurring and practically important refinement is to make such models *time-aware*, so that a judgment recorded at one instant can be weighted against judgments recorded at others; time-fuzzy soft sets [7] and time-neutrosophic soft sets [8] are two representatives of this idea.

A second, independent refinement is the *effective-parameter* mechanism introduced by Alkhazaleh [9]: an adjustment that revises a membership value in the light of external evidence which only becomes available after an initial assess-

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ment has been made. This device was subsequently carried into the neutrosophic setting by Al-Hijjawi *et al.* [10], who let an effective set act on the truth, indeterminacy and falsity degrees of a neutrosophic soft set. Both of these lines are, however, static: the effective set adjusts a single snapshot, and neither model carries a time index.

Two of my own earlier papers are the immediate predecessors of the present work and fix precisely what is, and is not, new here. In [11] I introduced the time-effective fuzzy soft set (T-EFSS), which combines a time index with an effective mechanism in the *fuzzy* setting, where each assessment is a single membership value. In [12] my co-authors and I introduced the parameterized time neutrosophic soft set (P-TNSS), together with its complement, union, intersection, AND and OR operations and a decision-making application; that paper supplies the parameterized, time-indexed neutrosophic structure on which the present model is built.

The contribution of the present paper is therefore neither the effective set itself, which is due to [9, 10], nor the parameterized time-indexed neutrosophic structure and its basic operations, which are due to [12]. What is new is the Λ-*adjustment*: an effective-parameter operator defined natively on neutrosophic $\langle T, I, F\rangle$ triples and placed inside the P-TNSS pipeline, together with the algebraic guarantees it satisfies and the TOPSIS decision stage built downstream of it. Relative to [11], the step is from a single fuzzy membership to three interacting components, which raises a constraint with no fuzzy analogue: the joint bound $0 \le T + I + F \le 3$ must be shown to survive the adjustment (Proposition 4.12). Relative to [12], the step is the addition of post-assessment confirmatory evidence, which that model has no mechanism to represent.

1.1. Research gap and motivation

Five bodies of work sit closest to the structure proposed here.

Neutrosophic soft sets (Maji [6]; Cuong, Phong & Smarandache [13]) define union, intersection, complement, AND, and OR on neutrosophic soft sets, but carry no time dimension and no effective-parameter mechanism.

Time-indexed neutrosophic models, the dynamic interval-valued neutrosophic set (DIVNS) of Thong *et al.* [14] and the time-neutrosophic soft set (tNS-set) of Alkhazaleh [8], add a time dimension to neutrosophic structures and are the closest existing work. The tNS-set was later shown to be a special case of the more general neutrosophic-soft-set sequence of Bui *et al.* [15]. Neither construction, however, carries any effective-parameter mechanism: they define time-indexed neutrosophic operations and a decision procedure, but nothing that revises a $\langle T, I, F\rangle$ triple in the light of external, post-hoc confirmation.

Neutrosophic parameterized soft sets, the NP-soft set and NP-aggregation operator of Broumi, Deli & Smarandache [16], with Deli & Çağman's intuitionistic-fuzzy-parameterized soft set [17] as the closest structural precedent, attach weights to *parameters*, but not to alternatives, and carry no time dimension.

Neutrosophic TOPSIS (Biswas, Pramanik & Giri [18], and its many follow-ups) supplies the neutrosophic ranking machinery, neutrosophic positive/negative ideal solutions and a neutrosophic distance, used in Section 5, but operates on a single static neutrosophic decision matrix, with no time or effective-parameter mechanism upstream of it.

The two immediate predecessors. The time-effective fuzzy soft set of [11] already combines a time index with an effective mechanism, but does so on single fuzzy memberships: indeterminacy is not represented, and the triple-sum constraint that governs the neutrosophic case does not arise. The parameterized time neutrosophic soft set of [12] supplies exactly the parameterized, time-indexed neutrosophic structure used here, along with its complement, union, intersection, AND and OR operations, but contains no effective-parameter mechanism: it has no way to revise a $\langle T, I, F\rangle$ triple once external confirmatory evidence arrives after the initial assessment.

The gap, stated precisely. The parameterized time-indexed neutrosophic structure is available in [12], and the effective mechanism is available both in the static neutrosophic setting [9, 10] and in the time-indexed *fuzzy* setting [11]. What is not available is an effective-parameter adjustment that operates directly on $\langle T, I, F\rangle$ triples *inside* a parameterized time-indexed neutrosophic model, is proved to respect the neutrosophic bounds, and feeds a neutrosophic TOPSIS ranking stage. Supplying that operator, establishing its algebraic properties, and tracing its effect through to the ranking is the specific contribution of this paper. The base structure of Section 3 is recalled from [12] and is not claimed as new, it is restated here only so that the paper is self-contained and so that the Λ-adjustment can be defined on it precisely.

1.2. Novel contributions

The contributions of this paper are:

A native neutrosophic time-effective model (N-PTEFSS, Definition 3.1), obtained by equipping the parameterized time neutrosophic soft set of [12] with an effective-parameter mechanism, so that truth, indeterminacy, and falsity are each carried through both the time-weighting and the effective-parameter adjustment rather than collapsed to a scalar. The base time-indexed structure is due to [12], what is added here is the effective layer.

An effective-parameter (Λ-*adjustment*) operator on $\langle T, I, F\rangle$ triples (Definition 3.4), together with a proof that it preserves the neutrosophic bound $0 \le T + I + F \le 3$ (Proposition 4.12). Bound preservation by convexity is not by itself unique to the neutrosophic setting, what is specific here is that the joint triple-sum constraint $T + I + F \le 3$, which has no analogue for a single fuzzy membership, is maintained. The adjustment is further shown to be monotone in the confirmatory strength, Lipschitz, and order-preserving (Proposition 4.13), and a structural result (Proposition 5.1) characterizes why time-weighting alone cannot reverse a dominated alternative while the effective-parameter adjustment can.

Verification that the union, intersection, complement, AND and OR operations of [12] remain well behaved in the presence of the Λ-adjustment, with full proofs of complement involution, associativity, idempotency, distributivity, and De Mor-

gan's laws in the present setting (Section 4), together with the bound-preservation result that the adjustment makes necessary.

A neutrosophic TOPSIS decision procedure built on the N-PTEFSS pipeline (Section 5), and a worked medical-diagnosis application (Section 6).

A reduction result (Section 3.4) showing that the model specializes, term for term, to the time-effective fuzzy soft set (T-EFSS) of [11] when $I = F = 0$, establishing it as a proper generalization of that fuzzy predecessor rather than a relabelling.

1.3. Organization of the paper

Section 2 recalls the neutrosophic preliminaries. Section 3 defines the base structure, the neutrosophic time-weighting, and the Λ-adjustment, and states the reduction result. Section 4 defines the basic operations and proves their properties. Section 5 develops the neutrosophic TOPSIS algorithm. Section 6 works through the medical-diagnosis application. Section 8 concludes.

2. Preliminaries

Throughout, U is a nonempty universe of discourse and E a set of parameters, for $A \subseteq E$, $\mathcal{P}(U)$ denotes the power set of U and $\mathcal{F}(U)$ the set of all fuzzy subsets of U.

Definition 2.1 ([5]). A *single-valued neutrosophic set* (SVNS) A over U is

$$A = \{\langle u,\ T_A(u),\ I_A(u),\ F_A(u)\rangle : u \in U\},$$

where $T_A, I_A, F_A : U \to [0,1]$ are the truth-, indeterminacy-, and falsity-membership functions, subject to

$$0 \le T_A(u) + I_A(u) + F_A(u) \le 3, \qquad \forall u \in U.$$

We write $\mathrm{NS}(U)$ for the set of all SVNSs over U.

Definition 2.2 ([6]). A *neutrosophic soft set* over U with parameter set $A \subseteq E$ is a pair (F, A) with $F : A \to \mathrm{NS}(U)$. For $e \in A$ and $u \in U$, $F(e)(u) = \langle T_F(u),\ I_F(u),\ F_F(u)\rangle$.

Definition 2.3 ([7]). Let $T = \{t_1, \dots, t_m\}$ be a finite time index set. A family of fuzzy soft sets $(F, A)_t$ indexed by $t \in T$, with each $F_t : A \to \mathcal{F}(U)$, is a *time-fuzzy soft set* (T-FSS) over U.

Definition 2.4 ([8]). Let $T = \{t_1, \dots, t_m\}$ be a finite time index set. A family of neutrosophic soft sets $(F, A)_t$ indexed by $t \in T$, with each $F_t : A \to \mathrm{NS}(U)$, is a *time-neutrosophic soft set* (tNS-set) over U.

Definition 2.5 (Effective set, [9, 10]). An *effective set* Λ on a parameter universe A is a fuzzy set $\Lambda : A \to [0,1]$,

$$\Lambda = \{\langle a, \delta_\Lambda(a)\rangle : a \in A\},$$

where $\delta_\Lambda(a) \in [0,1]$ is the degree to which parameter a enhances or maintains the assessment of the elements under consideration.

3. The neutrosophic parameterized time-effective soft set

3.1. Base structure

Definition 3.1 (N-PTEFSS base structure). Fix a universe U, a parameter set $A \subseteq E$, and a finite, ordered time set $T = \{t_1, \dots, t_m\}$. A *neutrosophic parameterized time-effective soft set* (N-PTEFSS) is a family

$$\{(F_{t_i}, A) : t_i \in T\}, \qquad F_{t_i} : A \longrightarrow \mathrm{NS}(U),$$

so that for each parameter $e \in A$, each alternative $u \in U$, and each time instant $t_i \in T$,

$$F_{t_i}(e)(u) = \langle T_i(u),\ I_i(u),\ F_i(u)\rangle \in [0,1]^3,$$
$$0 \le T_i(u) + I_i(u) + F_i(u) \le 3.$$

The parameter e is held fixed and suppressed below, since every definition in this section is applied parameter-by-parameter. This object differs from the T-EFSS mapping $F : A \to \mathcal{F}(U)$ of [11]: three functions are carried through every instant instead of one.

3.2. Neutrosophic time-weighting

Definition 3.2 (Time-weighting). Fix nonnegative time weights $w_1, \dots, w_m$ with $\sum_i w_i = 1$ (reflecting, e.g., the recency or reliability of each instant). The *time-aggregated triple* at u is defined componentwise by the weighted average

$$T(u) = \sum_{i=1}^{m} w_i\, T_i(u), \qquad I(u) = \sum_{i=1}^{m} w_i\, I_i(u),$$
$$F(u) = \sum_{i=1}^{m} w_i\, F_i(u).$$

Remark 3.3 (Choice of the aggregation rule). Three aggregation rules were considered: (i) the componentwise weighted average above, (ii) the single-valued neutrosophic weighted average (SVNSWA) operator, which aggregates T multiplicatively as $1 - \prod_i (1 - T_i)^{w_i}$ and I, F as $\prod_i I_i^{w_i}$, $\prod_i F_i^{w_i}$, and (iii) the recency-ratio rule $\sum_j j\, F_j / (m \sum_j F_j)$ familiar from time-indexed fuzzy soft models [7], applied componentwise. This paper adopts (i), for two model-specific reasons. First, the weighted average is a convex combination, so it maps $[0,1]$ into $[0,1]$ and satisfies $\sum_i w_i(T_i + I_i + F_i) \le 3$, hence it preserves the neutrosophic bound automatically (Proposition 4.12).

Second, at $I = F = 0$ it collapses term-for-term to a fuzzy weighted average on T, giving the clean reduction of Section 3.4. The recency-ratio rule (iii) is not a convex combination: a short calculation shows that with time indices $1, \dots, m$ and normalizer $m \sum_j F_j$, any nonzero component is mapped to at least $1/m$ of full membership regardless of how small the raw readings are, an artifact with no clear meaning for an indeterminacy or falsity degree. The weighted average avoids this. The SVNSWA operator (ii) is the natural multiplicative alternative when the reduction property is not required.

3.3. The effective parameter set and the Λ-adjustment

Definition 3.4 (Neutrosophic Λ-adjustment)**.** Let $\widetilde{E} \subseteq A$ be a designated set of *effective parameters*, parameters for which independent, external confirmatory evidence exists (a reference-lab result, a specialist re-read, a second data source). For each alternative $u \in U$ and each $a \in \widetilde{E}$, let $\delta_{\Lambda_u}(a) \in [0,1]$ be the confirmatory support a contributes for u, and set the *average effective contribution*

$$\delta_\Lambda(u) = \frac{1}{|\widetilde{E}|} \sum_{a \in \widetilde{E}} \delta_{\Lambda_u}(a) \ \in \ [0,1].$$

The neutrosophic effective set Λ adjusts the time-aggregated triple $\langle T(u),\ I(u),\ F(u)\rangle$ componentwise:

$$\langle T_\Lambda(u),\ I_\Lambda(u),\ F_\Lambda(u)\rangle =$$

$$\begin{cases} \langle\, T(u) + (1 - T(u))\,\delta_\Lambda(u), \\ \quad I(u)(1 - \delta_\Lambda(u)), \\ \quad F(u)(1 - \delta_\Lambda(u))\,\rangle, \\ \text{if } T(u) \in (0,1), \\ \langle T(u),\ I(u),\ F(u)\rangle\,, \\ \text{otherwise.} \end{cases}$$

Strong external support pushes truth up toward 1 and shrinks both indeterminacy and falsity toward 0 by the same proportional factor, but, matching the fuzzy source, only when the truth-degree is still interior, an alternative already fully true or fully false under this parameter is left unchanged, since there is no partial state left for confirmatory evidence to move.

Remark 3.5 (Boundary behavior)**.** Within the interior branch, $\delta_\Lambda(u) = 0$ (no confirmatory support) gives the identity map $\langle T,\ I,\ F\rangle$, and $\delta_\Lambda(u) = 1$ (full confirmation) gives the fully-true state $\langle 1,\ 0,\ 0\rangle$, as required.

Remark 3.6 (Symmetric treatment of I and F)**.** Definition 3.4 shrinks I and F by the same factor $(1 - \delta_\Lambda)$. This symmetric rule is the conservative default. An asymmetric variant, for instance, shrinking F faster than I on the view that confirmatory evidence should first resolve an "unknown" reading into an affirmative rather than directly overturn a negative reading, is defensible in a diagnostic setting, but would require its own justification and its own reduction check, we therefore record it as an open extension (Section 8) rather than building it into the base definition.

Remark 3.7 (Scope of the adjustment: alternative-level versus parameter-level)**.** Definition 3.4 applies a single alternative-level $\delta_\Lambda(u)$, itself an average of confirmatory contributions over the effective set $\widetilde{E}$, to *every* parameter $e \in A$, not only to $e \in \widetilde{E}$. This is a deliberate default, not an oversight: it encodes the view that partial confirmatory evidence raises overall confidence in an alternative's whole assessment (an analyst who sees one claim corroborated reasonably becomes more confident in related, unconfirmed claims about the same alternative too), and it is the direct neutrosophic analogue of the scope already used in the T-EFSS formula of [11] (Proposition 3.9), which likewise applies $\delta_\Lambda(u_j)$ uniformly.

It is, however, a genuine design choice with a real alternative: restricting the adjustment to $e \in \widetilde{E}$ only, or replacing $\delta_\Lambda(u)$ with a parameter-specific $\delta_\Lambda(u,e)$ that uses each effective parameter's own confirmatory value rather than their average. Section 6.6 reports an ablation across all three variants on the running example, the ranking reversal that motivates this paper (Section 6) turns out to hold under all of them, though the closeness coefficients themselves differ, so the choice of scope is not immaterial and should be stated explicitly whenever the model is applied.

Remark 3.8 (The boundary case, and a finer alternative)**.** Definition 3.4 leaves the triple unchanged unless $T(u) \in (0,1)$. Stated in terms of T alone, this rule is inherited directly from the fuzzy source formula and is what makes the reduction of Proposition 3.9 exact, but it is worth being explicit about its neutrosophic reading, because T and F are independent here in a way they are not in the fuzzy setting. Skipping the adjustment at $T(u) = 1$ is unambiguous: the alternative is already fully consistent under this parameter, and there is no room left for confirmatory evidence to raise T. Skipping at $T(u) = 0$, however, conflates two genuinely different situations: the extreme point $\langle 0,0,1\rangle$, which is fully decided (definitely inconsistent) and correctly left alone, and a state such as $\langle 0, 0.7, 0.3\rangle$, which has zero truth but high indeterminacy and is *not* decided, here confirmatory evidence could reasonably raise T from 0 and lower I, yet the T-only rule freezes it.

A finer boundary rule ties the skip to the joint triple rather than to T alone: apply the adjustment unless (T,I,F) is an extreme, fully-decided point, $\langle 1,0,0\rangle$ or $\langle 0,0,1\rangle$, and otherwise adjust, so that a zero-truth-but-indeterminate state is still updated. For the truth-update at an interior or partially-indeterminate point the same convex-combination form is used, and the bound-preservation proof of Proposition 4.12 carries over verbatim, since it only uses $\delta_\Lambda \in [0,1]$ and convexity. This finer rule does not change any number in the worked example, every Stage B truth degree there is strictly interior ($0.3 \le T \le 0.76$), so no boundary branch is ever taken, but it removes the semantic awkwardness the T-only statement carries, and we recommend it whenever an application can produce genuine $T = 0$, $I > 0$ readings. The base definition is stated in the simpler T-only form so that the reduction to the T-EFSS of [11] is a clean identity, the two rules agree everywhere except on the measure-zero, high-indeterminacy boundary just described.

To make the practical difference concrete, consider a single cell $\langle 0, 0.7, 0.3\rangle$, zero truth, high indeterminacy, low falsity, the "unknown-negative" state, receiving confirmatory strength $\delta_\Lambda = 0.5$. The T-only rule freezes it at $\langle 0, 0.7, 0.3\rangle$, the decided-only rule updates it to $\langle 0.5, 0.35, 0.15\rangle$, raising truth from 0 and halving both indeterminacy and falsity (the triple sum stays admissible at 1.0). In a two-alternative ranking where one patient carries such a cell, this single change moves that patient's closeness coefficient from 0.128 to 0.354 under otherwise identical settings, a substantial shift, even though it does not flip the or-

der here. The lesson is that the two rules diverge exactly when confirmatory evidence meets genuine indeterminacy rather than a decided negative, which is precisely the situation the neutrosophic model exists to represent, an application in which zero-truth-with-indeterminacy readings are common should prefer the decided-only rule.

3.4. Reduction to the time-effective fuzzy soft set

The fuzzy predecessor of the present model is the time-effective fuzzy soft set (T-EFSS) of [11], which combines a time index with an effective mechanism on single fuzzy memberships. The reduction to that model holds at three levels, and it is worth being precise about which are exact numerical identities and which is structural.

Proposition 3.9 (Reduction, definitional layer)**.** *Setting* $I \equiv F \equiv 0$ *throughout:*

(a) the time-weighting of Definition 3.2 collapses to the fuzzy weighted average $T(u) = \sum_i w_i T_i(u)$*, with* $I(u) = F(u) = 0$*,*

(b) the Λ*-adjustment of Definition 3.4 collapses to*

$$T_\Lambda(u) = \begin{cases} T(u) + (1 - T(u))\delta_\Lambda(u), & T(u) \in (0,1), \\ T(u), & \text{otherwise,} \end{cases}$$

$$I_\Lambda(u) = F_\Lambda(u) = 0,$$

which is term-for-term the effective adjustment of the time-effective fuzzy soft set of [11], with T *in the role of the fuzzy membership* μ*, the same formula is the effective fuzzy soft set adjustment of Alkhazaleh [9], and equivalently the single-component restriction of the effective neutrosophic adjustment of Al-Hijjawi et al. [10].*

Proof. (a) With $I_i = F_i = 0$ for all i, the sums $\sum_i w_i I_i$ and $\sum_i w_i F_i$ are 0, and $\sum_i w_i T_i$ is exactly the fuzzy weighted average. (b) The I- and F-updates in Definition 3.4 are the pure scalings $I(1-\delta_\Lambda)$ and $F(1-\delta_\Lambda)$, which vanish identically when $I = F = 0$, the boundary case is stated purely in terms of T, so it is inherited unchanged, and the T-update is literally the fuzzy formula. □

At the *ranking* layer the reduction is structural rather than numerical. When $I = F = 0$, every decision-matrix entry becomes a triple $\langle T, 0, 0\rangle$ carrying exactly the fuzzy membership in its first component, and the neutrosophic ranking procedure of Section 5 returns the same order as fuzzy Feng's TOPSIS on the corresponding fuzzy data. The closeness-coefficient *values* generally differ, because Feng's fuzzy TOPSIS normalizes each column ($r_j = d_j / \sqrt{\sum_i d_{ij}^2}$) before measuring distance, whereas the neutrosophic TOPSIS of Section 5 operates on the triples through a normalized $\langle T, I, F\rangle$ distance without column pre-normalization. These are different distance geometries, so the coefficients need not coincide even when the indeterminacy and falsity channels vanish. The reduction is therefore *exact at the definitional layer* (time-weighting and Λ) and *structural at the ranking layer* (same matrix structure, same order).

4. Basic operations and their properties

4.1. Operations

Remark 4.1 (Standing assumption)**.** Unless stated otherwise, union and intersection on triples use the operators

$$(\max, \min, \min) \quad \text{and} \quad (\min, \max, \max),$$

respectively, and the complement uses $c(x) = 1 - x$ on the middle component, the convention of Maji [6], Cuong–Phong–Smarandache [13], and Smarandache's own worked examples. All propositions below are stated for this fixed pair of operators, the associativity, idempotency, and distributivity results do not automatically transfer to an arbitrary t-norm/t-conorm pair (e.g. the algebraic product $t(x,y) = xy$ is not idempotent), which would require separate proofs.

Remark 4.2 (On the indeterminacy component)**.** A competing convention (e.g. Al-Quran & Hassan [19]) aggregates indeterminacy by averaging, $I_K = \frac{1}{2}(I_F + I_G)$, rather than by min or max. We deliberately do not use the averaging convention, for a load-bearing rather than stylistic reason: although $\frac{1}{2}(I+I) = I$ gives idempotency for a single operation, averaging fails associativity across three operands, so the idempotency of the *compound* operations and the De Morgan duality proved below would both break. The (min, max, max)/(max, min, min) convention keeps each component inside a single distributive lattice, which is exactly what makes Propositions 4.8–4.10 go through.

Definition 4.3 (Union and intersection)**.** For neutrosophic soft sets (F, A) and (G, A) over U, $e \in A$, $u \in U$:

$$(F \mathbin{\widetilde{\cup}} G)(e)(u) = \langle \max(T_F, T_G),\ \min(I_F, I_G),\ \min(F_F, F_G)\rangle,$$
$$(F \mathbin{\widetilde{\cap}} G)(e)(u) = \langle \min(T_F, T_G),\ \max(I_F, I_G),\ \max(F_F, F_G)\rangle.$$

For two N-PTEFSSs, these are applied pointwise at every time instant $t \in T$, on the union of differing parameter sets $C = A \cup B$, a parameter present in only one operand is carried over unchanged, exactly as in the T-EFSS union of [11].

Definition 4.4 (Complement)**.** $F^c(e)(u) = \langle F_F(u),\ 1 - I_F(u),\ T_F(u)\rangle$, the standard neutrosophic complement.

Definition 4.5 (AND, OR)**.** For (F, A) and (G, B), $(F \wedge G) = (H, A \times B)$ with $H(a,b) = F(a) \mathbin{\widetilde{\cap}} G(b)$ componentwise as in Definition 4.3, $(F \vee G) = (H, A \times B)$ is defined identically with $\widetilde{\cup}$ in place of $\widetilde{\cap}$.

Definition 4.6 (Null and absolute N-PTEFSS)**.** $\widetilde{\emptyset}(e)(u) = \langle 0, 1, 1\rangle$ and $\widetilde{U}(e)(u) = \langle 1, 0, 0\rangle$ for all e, u. Note $\widetilde{\emptyset}^c = \widetilde{U}$, consistent with Definition 4.4.

4.2. Properties

Proposition 4.7 (Complement involution and null/absolute identities)**.** *For any N-PTEFSS* F*:* (a) $(F^c)^c = F$, (b) $F \mathbin{\widetilde{\cup}} \widetilde{\emptyset} = F$, $F \mathbin{\widetilde{\cap}} \widetilde{U} = F$, $F \mathbin{\widetilde{\cup}} \widetilde{U} = \widetilde{U}$, $F \mathbin{\widetilde{\cap}} \widetilde{\emptyset} = \widetilde{\emptyset}$.

Proof. (a) $F^c = \langle F_F, 1 - I_F, T_F\rangle$, applying the complement again swaps the outer components and re-complements the middle one: $(F^c)^c = \langle T_F, 1-(1-I_F), F_F\rangle = \langle T_F, I_F, F_F\rangle = F$. (b) Each identity is a direct substitution using $\max(x,0) = x$, $\min(x,1) = x$, $\min(x,0) = 0$, $\max(x,1) = 1$ for $x \in [0,1]$, for example $F \mathbin{\widetilde{\cup}} \widetilde{\emptyset} = \langle \max(T,0), \min(I,1), \min(F,1)\rangle = \langle T, I, F\rangle = F$, and $F \mathbin{\widetilde{\cup}} \widetilde{U} = \langle \max(T,1), \min(I,0), \min(F,0)\rangle = \langle 1, 0, 0\rangle = \widetilde{U}$. The other two are analogous. □

Proposition 4.8 (Associativity and idempotency)**.** *$\widetilde{\cup}$ and $\widetilde{\cap}$ are each associative, commutative, and idempotent.*

Proof. Each operation acts componentwise through max or min, and $(\mathbb{R}, \max)$ and $(\mathbb{R}, \min)$ are each associative, commutative, and idempotent. Since $\widetilde{\cup}$ applies $(\max, \min, \min)$ to the three components independently, it inherits all three properties pointwise, likewise $\widetilde{\cap}$ with $(\min, \max, \max)$. □

Proposition 4.9 (Distributivity)**.** $F \mathbin{\widetilde{\cup}} (G \mathbin{\widetilde{\cap}} H) = (F \mathbin{\widetilde{\cup}} G) \mathbin{\widetilde{\cap}} (F \mathbin{\widetilde{\cup}} H)$*, and dually* $F \mathbin{\widetilde{\cap}} (G \mathbin{\widetilde{\cup}} H) = (F \mathbin{\widetilde{\cap}} G) \mathbin{\widetilde{\cup}} (F \mathbin{\widetilde{\cap}} H)$.

Proof. Fix e, u and write $a = T_F(u)$, $b = T_G(u)$, $c = T_H(u)$ for the truth components (the I- and F-components follow the same argument with the roles of max and min interchanged). The claim reduces to

$$\max(a, \min(b,c)) = \min(\max(a,b), \max(a,c)),$$

the distributive law of the totally ordered set $([0,1], \le)$ under $(\max, \min)$. Since $\le$ is a total order, we may assume without loss of generality that $b \le c$, the case $c \le b$ being identical after interchanging b and c (both sides of the identity are symmetric in b and c). With $b \le c$ we have $\min(b,c) = b$ and $\max(a,b) \le \max(a,c)$, so the identity to be proved becomes

$$\max(a,b) = \min\big(\max(a,b), \max(a,c)\big) = \max(a,b),$$

which holds identically. The dual identity $\min(a, \max(b,c)) = \max(\min(a,b), \min(a,c))$ follows by the same argument: assuming $b \le c$ gives $\max(b,c) = c$ and $\min(a,b) \le \min(a,c)$, so both sides equal $\min(a,c)$. Applying this to each of the three components gives the result. □

Proposition 4.10 (De Morgan's laws)**.** $(F \mathbin{\widetilde{\cup}} G)^c = F^c \mathbin{\widetilde{\cap}} G^c$ *and* $(F \mathbin{\widetilde{\cap}} G)^c = F^c \mathbin{\widetilde{\cup}} G^c$.

Proof. For the first identity, $(F \mathbin{\widetilde{\cup}} G) = \langle \max(T_F, T_G), \min(I_F, I_G), \min(F_F, F_G)\rangle$, so by Definition 4.4,

$$(F \mathbin{\widetilde{\cup}} G)^c = \langle \min(F_F, F_G), \\ 1 - \min(I_F, I_G), \max(T_F, T_G)\rangle.$$

On the other side, $F^c \mathbin{\widetilde{\cap}} G^c = \langle \min(F_F, F_G), \max(1-I_F, 1-I_G), \max(T_F, T_G)\rangle$. Since $1 - x$ is order-reversing, $\max(1-I_F, 1-I_G) = 1 - \min(I_F, I_G)$, so the two expressions agree in all three components. The second identity follows identically with $\widetilde{\cup}/\widetilde{\cap}$ and max / min interchanged. □

Example 4.11 (Numerical check of De Morgan)**.** Let $U = \{u_1\}$, one parameter e, with $F(e)(u_1) = \langle 0.7, 0.2, 0.1\rangle$ and $G(e)(u_1) = \langle 0.4, 0.1, 0.6\rangle$. Then $F \mathbin{\widetilde{\cup}} G = \langle 0.7, 0.1, 0.1\rangle$, so $(F \mathbin{\widetilde{\cup}} G)^c = \langle 0.1, 0.9, 0.7\rangle$. Also $F^c = \langle 0.1, 0.8, 0.7\rangle$ and $G^c = \langle 0.6, 0.9, 0.4\rangle$, so $F^c \mathbin{\widetilde{\cap}} G^c = \langle \min(0.1, 0.6), \max(0.8, 0.9), \max(0.7, 0.4)\rangle = \langle 0.1, 0.9, 0.7\rangle$, matching $(F \mathbin{\widetilde{\cup}} G)^c$ exactly.

Proposition 4.12 (Bound preservation)**.** (a) *If F, G have each component in $[0,1]$, so do $F \mathbin{\widetilde{\cup}} G$ and $F \mathbin{\widetilde{\cap}} G$, hence $0 \le T + I + F \le 3$ is preserved.* (b) *The Λ-adjustment of Definition 3.4 preserves each component's $[0,1]$ range, hence preserves $0 \le T + I + F \le 3$.*

Proof. (a) max and min are closed on $[0,1]$, so each of the three components stays in $[0,1]$, and a sum of three numbers each in $[0,1]$ lies in $[0,3]$. (b) If $T(u) \in \{0,1\}$ the triple is unchanged and trivially admissible. Otherwise $T(u) \in (0,1)$, and each update is a convex combination:

$$T_\Lambda = (1-\delta_\Lambda)T + \delta_\Lambda \cdot 1,$$
$$I_\Lambda = (1-\delta_\Lambda)I + \delta_\Lambda \cdot 0,$$
$$F_\Lambda = (1-\delta_\Lambda)F + \delta_\Lambda \cdot 0.$$

Since $\delta_\Lambda \in [0,1]$ and $[0,1]$ is convex, each of $T_\Lambda, I_\Lambda, F_\Lambda$ stays in $[0,1]$, summing gives $0 \le T_\Lambda + I_\Lambda + F_\Lambda \le 3$. □

Beyond bound preservation, the Λ-adjustment has several structural properties that are worth recording, both because they characterize its behaviour and because they answer natural questions about stability and consistency of the induced ranking. Throughout, the interior branch $T \in (0,1)$ is assumed (the extreme points are fixed by the boundary rule).

Proposition 4.13 (Structural properties of the Λ-adjustment)**.** *Write $\Lambda_\delta\langle T, I, F\rangle = \langle T + (1-T)\delta, I(1-\delta), F(1-\delta)\rangle$ for $\delta \in [0,1]$ and $T \in (0,1)$. Then:*

(a) (Monotonicity in the evidence δ.) *T_Λ is nondecreasing in δ, and I_Λ, F_Λ are nonincreasing in δ, explicitly $\partial T_\Lambda/\partial\delta = 1 - T \ge 0$, $\partial I_\Lambda/\partial\delta = -I \le 0$, $\partial F_\Lambda/\partial\delta = -F \le 0$. Stronger confirmatory evidence never lowers truth and never raises indeterminacy or falsity.*

(b) (Lipschitz continuity.) *For fixed input, $\delta \mapsto \Lambda_\delta$ is Lipschitz: $|T_\Lambda(\delta_1) - T_\Lambda(\delta_2)| = (1-T)|\delta_1 - \delta_2|$, and likewise with constants I and F for the other two components, so the map is Lipschitz in δ with constant $\sqrt{(1-T)^2 + I^2 + F^2} \le \sqrt{3}$ in the Euclidean norm on triples. A small error in eliciting δ_Λ produces a proportionally small change in the adjusted triple, no amplification.*

(c) (Order preservation / isotonicity.) *Equip triples with the componentwise "quality" order $\langle T_1, I_1, F_1\rangle \le \langle T_2, I_2, F_2\rangle$ iff $T_1 \le T_2$, $I_1 \ge I_2$, and $F_1 \ge F_2$ (more truth, less indeterminacy, less falsity). For a fixed δ, Λ_δ is isotone: $a \preceq b \implies \Lambda_\delta(a) \preceq \Lambda_\delta(b)$. Applying the same confirmatory strength to two alternatives cannot invert their quality order under this partial order.*

Proof. (a) Differentiate each component of Λ_δ with respect to δ: $\partial_\delta[T + (1-T)\delta] = 1 - T$, $\partial_\delta[I(1-\delta)] = -I$, $\partial_\delta[F(1-\delta)] = -F$. Since $T \le 1$ and $I, F \ge 0$, the three signs are $\ge 0, \le 0, \le 0$ respectively. (b) $T_\Lambda(\delta_1) - T_\Lambda(\delta_2) = (1-T)(\delta_1 - \delta_2)$, and the I, F differences are $-I(\delta_1 - \delta_2)$, $-F(\delta_1 - \delta_2)$, taking absolute values gives the stated component Lipschitz constants, and the Euclidean joint constant is $\sqrt{(1-T)^2 + I^2 + F^2}$, which is $\le \sqrt{3}$ since each of $1-T, I, F \in [0, 1]$. (c) Suppose $a \preceq b$, i.e. $T_a \le T_b$, $I_a \ge I_b$, $F_a \ge F_b$. Then $T_{\Lambda,b} - T_{\Lambda,a} = (T_b - T_a)(1-\delta) \ge 0$, $I_{\Lambda,a} - I_{\Lambda,b} = (I_a - I_b)(1-\delta) \ge 0$, and $F_{\Lambda,a} - F_{\Lambda,b} = (F_a - F_b)(1-\delta) \ge 0$. These are exactly the three inequalities defining $\Lambda_\delta(a) \preceq \Lambda_\delta(b)$. □

Property (c) is the one with a direct bearing on the decision stage: it says the adjustment is coherent with the underlying notion of a "better" assessment, so the reversals the model produces (as in Section 6) arise from *differing* evidence strengths $\delta_\Lambda(u)$ across alternatives, never from the adjustment scrambling an order that already held under equal evidence. In particular, if all alternatives receive an identical δ_Λ, part (c) applied in both directions shows the pre-adjustment and post-adjustment quality orders coincide on every pair that was comparable, so a uniform adjustment cannot invert the quality order of any pair that was comparable. Whether the induced TOPSIS ranking is likewise unchanged is a separate question, not settled by part (c), Section 6.10 reports a numerical check of it on the running example.

5. Neutrosophic TOPSIS on the N-PTEFSS pipeline

5.1. Distance and ideal solutions

Adopted from Biswas–Pramanik–Giri [18]. For two SVN triples $a = \langle T_a, I_a, F_a\rangle$ and $b = \langle T_b, I_b, F_b\rangle$, the normalized Euclidean distance is

$$d(a,b) = \sqrt{\tfrac{1}{3}\left[(T_a - T_b)^2 + (I_a - I_b)^2 + (F_a - F_b)^2\right]}.$$

Given a Λ-adjusted neutrosophic decision matrix with alternatives u_k (rows) and parameters e_j (columns), the positive and negative ideal solutions are taken column-wise from the data:

$$\mathrm{PIS}_j = \left\langle \max_k T_{jk},\ \min_k I_{jk},\ \min_k F_{jk} \right\rangle,$$
$$\mathrm{NIS}_j = \left\langle \min_k T_{jk},\ \max_k I_{jk},\ \max_k F_{jk} \right\rangle.$$

5.2. Algorithm

1. Decision matrices. Build a neutrosophic decision matrix $D_i = [F_{t_i}(e_j)(u_k)]$ for each $t_i \in T$.

2. Time aggregation. Aggregate across time by Definition 3.2 to obtain $\widetilde{D} = [\langle T_{jk}, I_{jk}, F_{jk}\rangle]$.

3. Λ-adjustment. Apply Definition 3.4 using $\widetilde{E}$ and each alternative's $\delta_\Lambda(u_k)$ to obtain $\widetilde{D}_\Lambda$.

4. Weighted distances. Fix parameter weights $\omega_j \ge 0$, $\sum_j \omega_j = 1$ (equal if unspecified), and compute

$$d_\omega(u_k, \mathrm{PIS}) = \sqrt{\sum_j \omega_j\, d\Big(\langle T_{jk}, I_{jk}, F_{jk}\rangle, \mathrm{PIS}_j\Big)^2},$$

and symmetrically $d_\omega(u_k, \mathrm{NIS})$.

5. Closeness coefficient. $CC(u_k) = \dfrac{d_\omega(u_k, \mathrm{NIS})}{d_\omega(u_k, \mathrm{PIS}) + d_\omega(u_k, \mathrm{NIS})}$.

6. Ranking. Rank alternatives by descending CC.

5.3. Computational complexity

With m time instants, n alternatives, and p parameters, Steps 1–2 cost $O(m\,n\,p)$ componentwise operations, and Steps 3–6 add $O(n\,p)$. The asymptotic structure matches classical Feng's TOPSIS on fuzzy soft sets, each cell now carries a $\langle T, I, F\rangle$ triple instead of a single number, so every per-cell arithmetic operation costs a constant factor of 3. The overall complexity class is unchanged.

5.4. When time-weighting cannot change the winner but Λ can

The application in Section 6 shows a ranking that time-weighting leaves fixed but the Λ-adjustment reverses. This is not a coincidence of the chosen numbers, there is a structural reason, which the following makes precise. Recall the componentwise quality order $\preceq$ from Proposition 4.13(c): $a \preceq b$ iff b has truth at least as high and indeterminacy and falsity no higher than a.

Proposition 5.1 (Dominance is time-stable but not Λ-stable, fixed-corner variant)**.** Throughout this proposition the ranking is computed by the fixed-corner TOPSIS variant, in which the ideal solutions are the constant corners $\mathrm{PIS} = \langle 1, 0, 0\rangle$ and $\mathrm{NIS} = \langle 0, 1, 1\rangle$ rather than the data-dependent, column-wise ideals of Section 5. *Fix a parameter and suppose alternative b* dominates *alternative a at every time instant, i.e. $a \preceq b$ in each snapshot t_i.*

(a) *Then b dominates a after time-weighting as well: the componentwise weighted average of Definition 3.2 satisfies $\bar{a} \preceq \bar{b}$. Consequently, if b dominates a on* every *parameter at every instant, no choice of time weights can make a outrank b: measured against the fixed ideal corners $\langle 1, 0, 0\rangle$ (PIS) and $\langle 0, 1, 1\rangle$ (NIS), dominance gives $d(\bar{b}, \mathrm{PIS}) \le d(\bar{a}, \mathrm{PIS})$ and $d(\bar{b}, \mathrm{NIS}) \ge d(\bar{a}, \mathrm{NIS})$, hence $CC(b) \ge CC(a)$.*

(b) *The Λ-adjustment can nonetheless reverse this, but only through* differential *evidence. If a and b receive equal confirmatory strength $\delta_\Lambda(a) = \delta_\Lambda(b)$, isotonicity (Proposition 4.13(c)) preserves $\bar{a} \preceq \bar{b}$ and the winner is unchanged. If instead $\delta_\Lambda(a) > \delta_\Lambda(b)$, the adjustment can produce $\Lambda\bar{a} \not\preceq \Lambda\bar{b}$, allowing a to overtake b.*

Proof. (a) For a single instant, $\preceq$ is preserved by convex combination: if $a_i \preceq b_i$ for all i, then in the truth component $\sum_i w_i T_{a_i} \le \sum_i w_i T_{b_i}$, and in the indeterminacy and falsity components $\sum_i w_i I_{a_i} \ge \sum_i w_i I_{b_i}$ and $\sum_i w_i F_{a_i} \ge \sum_i w_i F_{b_i}$, each is a nonnegative-weighted sum of the per-instant inequalities, so $\bar{a} \preceq \bar{b}$. Distances to the fixed corners are monotone under $\preceq$: with PIS $= \langle 1, 0, 0\rangle$, the squared distance $d(\cdot, \mathrm{PIS})^2 = \frac{1}{3}[(1-T)^2 + I^2 + F^2]$ is nonincreasing in T and nondecreasing in I, F, so $\bar{a} \preceq \bar{b}$ gives $d(\bar{b}, \mathrm{PIS}) \le d(\bar{a}, \mathrm{PIS})$, the symmetric argument at NIS $= \langle 0, 1, 1\rangle$ gives $d(\bar{b}, \mathrm{NIS}) \ge d(\bar{a}, \mathrm{NIS})$. Hence $CC(b) = \frac{d(\bar{b},\mathrm{NIS})}{d(\bar{b},\mathrm{PIS})+d(\bar{b},\mathrm{NIS})} \ge CC(a)$. (b) The equal-evidence case is Proposition 4.13(c). For the reversal, one parameter suffices: let $\bar{b} = \langle 0.6, 0.2, 0.2\rangle$ dominate $\bar{a} = \langle 0.5, 0.3, 0.3\rangle$. With $\delta_\Lambda(a) = 0.9$ and $\delta_\Lambda(b) = 0$, the adjusted triples are $\Lambda\bar{a} = \langle 0.95, 0.03, 0.03\rangle$ and $\Lambda\bar{b} = \langle 0.6, 0.2, 0.2\rangle$, now a dominates b, so the order has reversed. □

Remark 5.2 (Why the fixed-corner restriction is needed)**.** The restriction to fixed corners is not cosmetic. The algorithm of Section 5.2 builds PIS_j and NIS_j column-wise from the decision matrix itself, so the ideals move when the matrix moves. Under those data-dependent ideals the argument above breaks at its final step: dominance still gives $\bar{a} \preceq \bar{b}$, but the two alternatives are then measured against ideals that they themselves helped to define, and the monotonicity of $d(\cdot, \mathrm{PIS})$ in the components no longer transfers to a comparison of closeness coefficients. Proposition 5.1 is therefore stated and proved only for the fixed-corner variant, where the ideals are constants independent of the data. The asymmetry it identifies between the two mechanisms is nonetheless the operative one in the application: the numerical behaviour reported in Section 6 is computed with the column-wise ideals of Section 5.2, and is presented there as an observed instance, not as a consequence of this proposition.

The content of the proposition is the asymmetry between the two mechanisms. Time-weighting only ever forms convex combinations *within* an alternative's own history, so it can never rescue an alternative that is dominated at every instant, the winner among dominant alternatives is fixed before any time weights are chosen. The Λ-adjustment injects information from *outside* that history, the confirmatory evidence δ_Λ, and it is exactly the *difference* in that evidence across alternatives, not its presence, that can move the decision. This is the precise sense in which the effect demonstrated numerically in Section 6 is structural rather than incidental.

6. Application: A medical-diagnosis ranking problem

We illustrate the model on a diagnostic ranking problem, a setting in which indeterminacy is not a modeling convenience but a genuine and unavoidable feature: a clinician assessing whether a patient's presentation is consistent with a condition D must routinely record "inconclusive" as a real answer, distinct from both "consistent" and "inconsistent." The scenario below is deliberately built with four candidate patients and four diagnostic parameters, so that the ranking, the effective-parameter mechanism, and the sensitivity of the conclusion can all be examined on a case rich enough to be realistic. All numerical values reported below were computed independently and verified before being written in.

6.1. Problem description

Four patients $U = \{u_1, u_2, u_3, u_4\}$ (the alternatives) are assessed against four diagnostically relevant parameters,

e_1 = biomarker elevation,
e_2 = imaging finding,
e_3 = clinical symptom score,
e_4 = response to a therapeutic trial,

at two successive visits t_1, t_2, with time weights $w_1 = 0.4$ and $w_2 = 0.6$ reflecting greater confidence in the more recent visit. For each (patient, parameter, visit) the clinician records a triple $\langle T, I, F\rangle$, where T is the degree to which the finding is consistent with D, I the degree to which it cannot be determined from that reading alone, and F the degree to which it is inconsistent with D. The raw assessments are given in Table 1. Patient u_4 is included specifically to exercise indeterminacy in a second way: the symptom score e_3 is genuinely inconclusive for this patient at both visits ($I = 0.6$), representing a real and common clinical situation, a presentation that stays ambiguous even on repeated review, rather than a value chosen only to be low.

The parameters carry unequal weight in the ranking, reflecting their differing diagnostic value: $\omega_{e_1} = 0.35$, $\omega_{e_2} = 0.25$, $\omega_{e_3} = 0.25$, $\omega_{e_4} = 0.15$ (the biomarker is treated as the single most informative parameter, and treatment response as the least, consistent with standard diagnostic-workup weighting). The effective parameter set is $\widetilde{E} = \{e_1, e_3\}$: the biomarker carries an external reference-lab confirmation, and the symptom score a specialist re-read, both of which arrive only after the initial assessment. The resulting average effective contributions are

$$\delta_\Lambda(u_1) = 0.30, \quad \delta_\Lambda(u_2) = 0.10,$$
$$\delta_\Lambda(u_3) = 0.50, \quad \delta_\Lambda(u_4) = 0.35.$$

6.2. Three-stage analysis

To isolate the contribution of each mechanism on the same data, the analysis proceeds in three progressively richer stages: Stage A (static baseline: visit t_2 only, no time-weighting, no Λ-adjustment), Stage B (time-weighted, Definition 3.2), Stage C (Stage B further adjusted by the Λ-adjustment, Definition 3.4). Table 2 gives Stages B and C.

How Table 2 was computed (sample: u_1, e_1). Stage B applies Definition 3.2 to the raw pair from Table 1, $t_1 = \langle 0.6, 0.3, 0.1\rangle$, $t_2 = \langle 0.7, 0.2, 0.1\rangle$, componentwise:

$$T = 0.4(0.6) + 0.6(0.7) = 0.660,$$
$$I = 0.4(0.3) + 0.6(0.2) = 0.240,$$
$$F = 0.4(0.1) + 0.6(0.1) = 0.100,$$

Table 1: Raw neutrosophic assessments $\langle T,\ I,\ F\rangle$ at visits t_1 and t_2.

Patient	Parameter	t_1	t_2
u_1	e_1	$\langle 0.6, 0.3, 0.1\rangle$	$\langle 0.7, 0.2, 0.1\rangle$
u_1	e_2	$\langle 0.5, 0.4, 0.1\rangle$	$\langle 0.6, 0.3, 0.1\rangle$
u_1	e_3	$\langle 0.7, 0.2, 0.1\rangle$	$\langle 0.8, 0.1, 0.1\rangle$
u_1	e_4	$\langle 0.5, 0.3, 0.2\rangle$	$\langle 0.6, 0.2, 0.2\rangle$
u_2	e_1	$\langle 0.5, 0.4, 0.1\rangle$	$\langle 0.4, 0.4, 0.2\rangle$
u_2	e_2	$\langle 0.7, 0.2, 0.1\rangle$	$\langle 0.8, 0.1, 0.1\rangle$
u_2	e_3	$\langle 0.4, 0.4, 0.2\rangle$	$\langle 0.4, 0.4, 0.2\rangle$
u_2	e_4	$\langle 0.4, 0.4, 0.2\rangle$	$\langle 0.4, 0.4, 0.2\rangle$
u_3	e_1	$\langle 0.7, 0.2, 0.1\rangle$	$\langle 0.8, 0.1, 0.1\rangle$
u_3	e_2	$\langle 0.6, 0.3, 0.1\rangle$	$\langle 0.7, 0.2, 0.1\rangle$
u_3	e_3	$\langle 0.5, 0.3, 0.2\rangle$	$\langle 0.6, 0.3, 0.1\rangle$
u_3	e_4	$\langle 0.5, 0.3, 0.2\rangle$	$\langle 0.6, 0.2, 0.2\rangle$
u_4	e_1	$\langle 0.5, 0.3, 0.2\rangle$	$\langle 0.6, 0.3, 0.1\rangle$
u_4	e_2	$\langle 0.5, 0.3, 0.2\rangle$	$\langle 0.5, 0.3, 0.2\rangle$
u_4	e_3	$\langle 0.3, 0.6, 0.1\rangle$	$\langle 0.3, 0.6, 0.1\rangle$
u_4	e_4	$\langle 0.7, 0.2, 0.1\rangle$	$\langle 0.8, 0.1, 0.1\rangle$

Table 2: Stage B (time-weighted) and Stage C (time-weighted + Λ) assessments.

Patient (δ_Λ)	Parameter	Stage B $\langle T,\ I,\ F\rangle$	Stage C $\langle T,\ I,\ F\rangle$
u_1 (0.30)	e_1	$\langle 0.660, 0.240, 0.100\rangle$	$\langle 0.762, 0.168, 0.070\rangle$
u_1 (0.30)	e_2	$\langle 0.560, 0.340, 0.100\rangle$	$\langle 0.692, 0.238, 0.070\rangle$
u_1 (0.30)	e_3	$\langle 0.760, 0.140, 0.100\rangle$	$\langle 0.832, 0.098, 0.070\rangle$
u_1 (0.30)	e_4	$\langle 0.560, 0.240, 0.200\rangle$	$\langle 0.692, 0.168, 0.140\rangle$
u_2 (0.10)	e_1	$\langle 0.440, 0.400, 0.160\rangle$	$\langle 0.496, 0.360, 0.144\rangle$
u_2 (0.10)	e_2	$\langle 0.760, 0.140, 0.100\rangle$	$\langle 0.784, 0.126, 0.090\rangle$
u_2 (0.10)	e_3	$\langle 0.400, 0.400, 0.200\rangle$	$\langle 0.460, 0.360, 0.180\rangle$
u_2 (0.10)	e_4	$\langle 0.400, 0.400, 0.200\rangle$	$\langle 0.460, 0.360, 0.180\rangle$
u_3 (0.50)	e_1	$\langle 0.760, 0.140, 0.100\rangle$	$\langle 0.880, 0.070, 0.050\rangle$
u_3 (0.50)	e_2	$\langle 0.660, 0.240, 0.100\rangle$	$\langle 0.830, 0.120, 0.050\rangle$
u_3 (0.50)	e_3	$\langle 0.560, 0.300, 0.140\rangle$	$\langle 0.780, 0.150, 0.070\rangle$
u_3 (0.50)	e_4	$\langle 0.560, 0.240, 0.200\rangle$	$\langle 0.780, 0.120, 0.100\rangle$
u_4 (0.35)	e_1	$\langle 0.560, 0.300, 0.140\rangle$	$\langle 0.714, 0.195, 0.091\rangle$
u_4 (0.35)	e_2	$\langle 0.500, 0.300, 0.200\rangle$	$\langle 0.675, 0.195, 0.130\rangle$
u_4 (0.35)	e_3	$\langle 0.300, 0.600, 0.100\rangle$	$\langle 0.545, 0.390, 0.065\rangle$
u_4 (0.35)	e_4	$\langle 0.760, 0.140, 0.100\rangle$	$\langle 0.844, 0.091, 0.065\rangle$

giving Stage B = $\langle 0.660, 0.240, 0.100\rangle$, as in Table 2. Stage C then applies the Λ-adjustment (Definition 3.4) with $\delta_\Lambda(u_1) = 0.30$, since $T = 0.660 \in (0, 1)$, the interior branch applies:

$$\begin{aligned} T_\Lambda &= 0.660 + (1 - 0.660)(0.30) = 0.762, \\ I_\Lambda &= 0.240(1 - 0.30) = 0.168, \\ F_\Lambda &= 0.100(1 - 0.30) = 0.070 \end{aligned}$$

giving Stage C = $\langle 0.762, 0.168, 0.070\rangle$, matching Table 2. Every other row follows the identical two-step procedure with that patient's own raw data and δ_Λ, the full derivation for all sixteen rows is reproduced by the script in the Appendix.

6.3. Neutrosophic TOPSIS ranking

Applying the algorithm of Section 5.2 with the parameter weights above yields the distances to the positive and negative ideal solutions, written $D^+(u_k) := d_\omega(u_k, \text{PIS})$ and $D^-(u_k) := d_\omega(u_k, \text{NIS})$ for brevity, and the closeness coefficients in Table 3, visualized in Figure 1.

How Table 3 was computed (sample: u_3, Stage C). First, the positive/negative ideal solutions are read off each column of the Stage C matrix (Table 2). For e_1: the four patients' Stage C values are u_1=$\langle 0.762, 0.168, 0.070\rangle$, u_2=$\langle 0.496, 0.360, 0.144\rangle$, u_3=$\langle 0.880, 0.070, 0.050\rangle$, u_4=$\langle 0.714, 0.195, 0.091\rangle$, so PIS_{e_1} = $\langle 0.880, 0.070, 0.050\rangle$ (in fact u_3's own value) and NIS_{e_1} = $\langle 0.496, 0.360, 0.144\rangle$ (u_2's value). The distance from u_3 to each,

Table 3: Distances and closeness coefficients CC at each stage. D^+ is the distance to PIS, D^- to NIS.

	Stage A			Stage B			Stage C		
Patient	D^+	D^-	CC	D^+	D^-	CC	D^+	D^-	CC
u_1	0.1095	0.2540	0.6987	0.1095	0.2210	0.6686	0.0855	0.1944	0.6945
u_2	0.2550	0.1258	0.3305	0.2188	0.1182	0.3507	0.2416	0.0474	0.1639
u_3	0.1065	0.2349	0.6881	0.1013	0.2001	0.6639	0.0276	0.2332	0.8942
u_4	0.2503	0.1443	0.3657	0.2274	0.1192	0.3440	0.1490	0.1502	0.5020

by the formula in Section 5, is

$$\begin{aligned} &d(u_{3,e_1},\mathrm{PIS}_{e_1}) = 0,\\ &d(u_{3,e_1},\mathrm{NIS}_{e_1})\\ &\quad= \sqrt{\tfrac{1}{3}[(0.384)^2 + (-0.290)^2 + (-0.094)^2]}\\ &\quad\approx 0.2831. \end{aligned}$$

Repeating this for e_2, e_3, e_4 gives the four per-parameter distance pairs, which are combined with the weights ω_{e_1}=0.35, ω_{e_2}=0.25, ω_{e_3}=0.25, ω_{e_4}=0.15:

$$D^+(u_3) = \sqrt{\textstyle\sum_j \omega_{e_j}\, d(u_{3,e_j},\mathrm{PIS}_{e_j})^2} = 0.0276,$$
$$D^-(u_3) = \sqrt{\textstyle\sum_j \omega_{e_j}\, d(u_{3,e_j},\mathrm{NIS}_{e_j})^2} = 0.2332,$$

so $CC(u_3) = D^-(u_3)/(D^+(u_3) + D^-(u_3)) = 0.2332/0.2608 = 0.8942$, matching Table 3. The other three patients and both other stages follow the same procedure, the complete computation is in the Appendix.

6.4. Discussion: the Λ-adjustment changes the decision

The rankings by stage are

$$\begin{aligned} &\text{Stage A: } u_1 \succ u_3 \succ u_4 \succ u_2,\\ &\text{Stage B: } u_1 \succ u_3 \succ u_2 \succ u_4,\\ &\text{Stage C: } u_3 \succ u_1 \succ u_4 \succ u_2. \end{aligned}$$

Time-weighting (Stage A to Stage B) leaves the top of the ranking unchanged, u_1 and u_3 remain first and second, separated by a narrow and stable margin (0.6987 vs. 0.6881 in Stage A, 0.6686 vs. 0.6639 in Stage B), and only reorders the two weaker patients. The effective-parameter adjustment (Stage B to Stage C), by contrast, *reverses the top two*: u_3 overtakes u_1, and the separation widens to 0.8942 vs. 0.6945. The mechanism responsible is exactly the one the model was built to capture: u_3 carries the strongest external confirmation ($\delta_\Lambda = 0.50$), so once that post-assessment evidence is incorporated, u_3's truth degrees rise and its indeterminacy and falsity fall more sharply than u_1's ($\delta_\Lambda = 0.30$), moving u_3 decisively ahead.

Patient u_4 illustrates a second role for indeterminacy: its symptom score e_3 never resolves ($I = 0.6$ throughout, even after Λ-adjustment I only falls to 0.39), yet u_4 still rises from last to third once its strong treatment-response parameter e_4 is confirmed, showing that persistent indeterminacy in one parameter does not by itself prevent a patient from being correctly ranked once the other evidence is weighed. No purely pre-implementation assessment, static or time-weighted, has any mechanism to represent this information, so neither Stage A nor Stage B can recover the Stage C decision. The lowest-ranked patient u_2 is never fully dominated, it is strongest on the imaging parameter e_2, so its closeness coefficient stays strictly positive throughout, confirming that the ranking reflects a genuine trade-off across parameters rather than a degenerate ordering.

6.5. Sensitivity analysis

A ranking reversal is only informative if it is not a fragile artifact of one arbitrarily chosen number. Two sensitivity analyses were run against the Stage C conclusion: one against the effective-contribution parameter that drives the Λ-adjustment, and one against the parameter weights that drive the TOPSIS ranking itself.

6.5.1. Sensitivity to $\delta_\Lambda(u_3)$

Holding $\delta_\Lambda(u_1) = 0.30$, $\delta_\Lambda(u_2) = 0.10$, and $\delta_\Lambda(u_4) = 0.35$ fixed, $\delta_\Lambda(u_3)$ was varied from 0.10 to 0.60 and the closeness coefficients of u_1 and u_3 recomputed at each value (Figure 2). A bisection search on $CC(u_1) - CC(u_3) = 0$ locates the crossover precisely at

$$\delta_\Lambda(u_3^*) \approx 0.3075.$$

For every $\delta_\Lambda(u_3) < 0.3075$, u_1 remains the top-ranked patient, for every value above it, u_3 takes over. The scenario's value of 0.50 lies well clear of this threshold on the u_3 side, so the reversal reported above is not a knife-edge result: it would survive any confirmatory-evidence reading down to roughly 0.31, a wide margin given the reference-lab-plus-specialist-reread evidence that motivates $\delta_\Lambda(u_3) = 0.50$ in the first place.

6.5.2. Sensitivity to the parameter weight ω_{e_1}

The second analysis asks whether the reversal depends on the specific parameter weights chosen. Holding the Stage C matrix fixed, the biomarker weight ω_{e_1} was varied from 0.05 to 0.65, with the remaining weight redistributed across e_2, e_3, e_4 in their original 0.25 : 0.25 : 0.15 proportions at every step (Table 4). Across this entire range, more than a thirteen-fold change in ω_{e_1}, the ranking stays exactly $u_3 \succ u_1 \succ u_4 \succ u_2$, and u_3's margin over u_1 never falls below 0.17. The Stage C conclusion is therefore not an artifact of the particular weights in Section 6.1, it is driven by the effective-parameter evidence itself, which is the property the sensitivity analysis was designed to check.

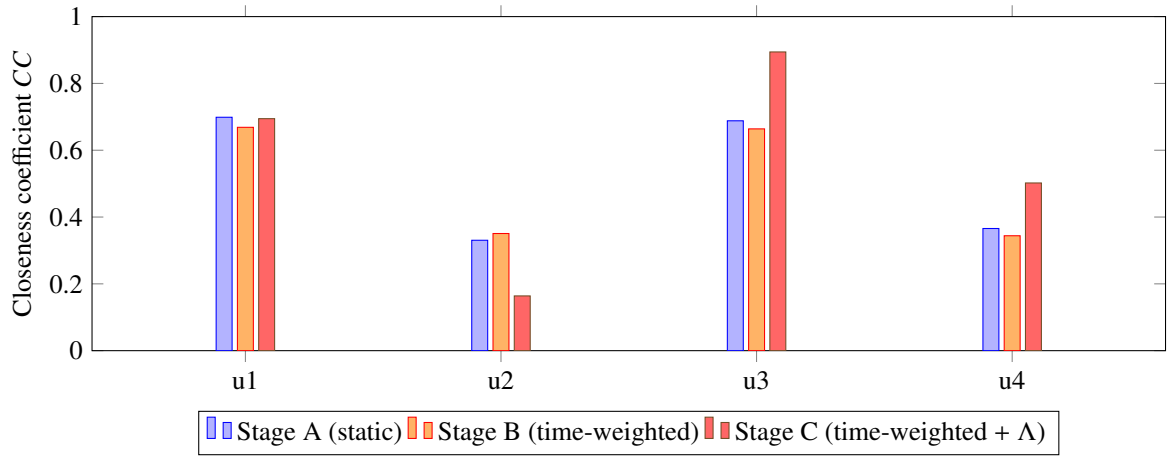


Figure 1: Closeness coefficients by patient across the three stages. u_3 overtakes u_1 only at Stage C.

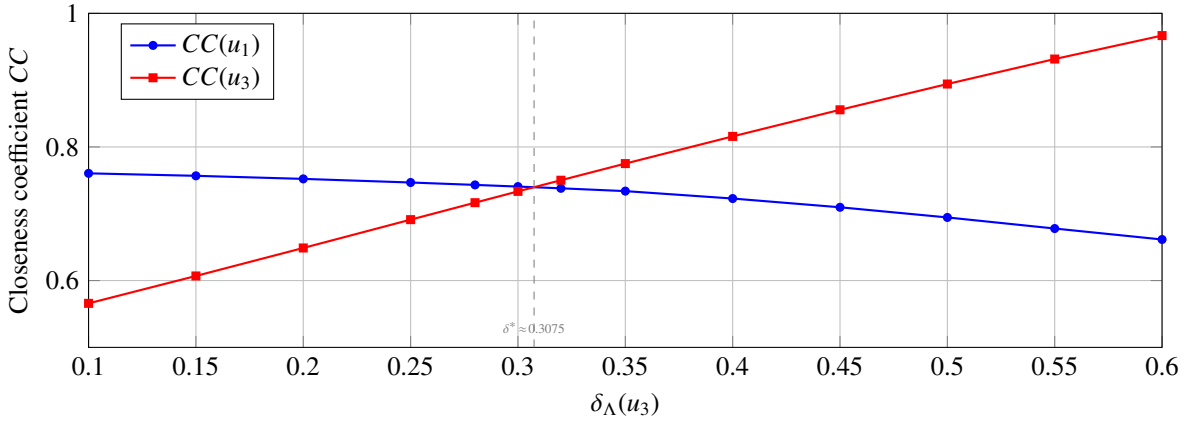


Figure 2: Closeness coefficients of u_1 and u_3 as $\delta_\Lambda(u_3)$ varies, with the other patients' effective contributions fixed. The lines cross at $\delta_\Lambda(u_3^*) \approx 0.3075$; the scenario value $\delta_\Lambda(u_3) = 0.50$ sits well inside the $u_3 \succ u_1$ region.

Table 4: Closeness coefficients as ω_{e_1} varies (Stage C matrix, other weights rescaled proportionally). The ranking $u_3 \succ u_1 \succ u_4 \succ u_2$ holds throughout.

ω_{e_1}	$CC(u_1)$	$CC(u_2)$	$CC(u_3)$	$CC(u_4)$
0.05	0.6992	0.2067	0.8608	0.4757
0.15	0.6976	0.1924	0.8724	0.4843
0.25	0.6961	0.1783	0.8835	0.4930
0.35	0.6945	0.1639	0.8942	0.5020
0.45	0.6930	0.1492	0.9050	0.5112
0.55	0.6915	0.1338	0.9158	0.5207
0.65	0.6900	0.1173	0.9271	0.5305

How Table 4 was computed (sample: $\omega_{e_1} = 0.55$). At each step, ω_{e_1} is fixed to the target value and the remaining weight $1-\omega_{e_1}$ is split across e_2, e_3, e_4 in their original $0.25 : 0.25 : 0.15$ ratio. For $\omega_{e_1} = 0.55$, the remaining weight is $1 - 0.55 = 0.45$, giving

$$\omega_{e_2} = 0.45 \cdot \tfrac{0.25}{0.65} = 0.1731,$$
$$\omega_{e_3} = 0.45 \cdot \tfrac{0.25}{0.65} = 0.1731,$$
$$\omega_{e_4} = 0.45 \cdot \tfrac{0.15}{0.65} = 0.1038$$

which sum to 1 as required. The Stage C matrix (Table 2) is then re-ranked with the algorithm of Section 5.2 using this weight vector in place of the baseline weights, exactly as in the worked $CC(u_3)$ example above, giving the row $CC(u_1)$=0.6915, $CC(u_2)$=0.1338, $CC(u_3)$=0.9158, $CC(u_4)$=0.5207 reported in Table 4. All seven rows were produced this way, see the Appendix.

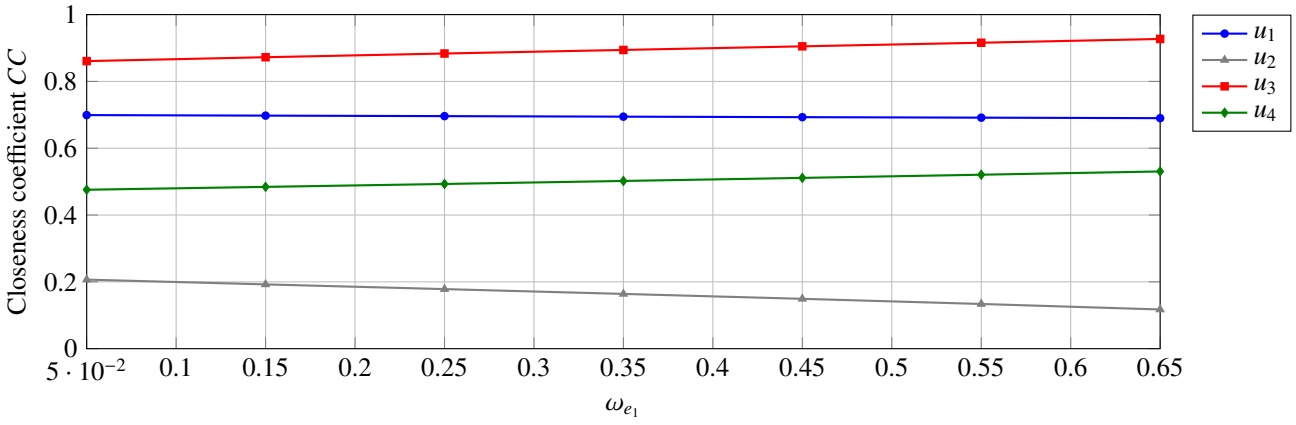


Figure 3: Closeness coefficients of all four patients as the biomarker weight ω_{e_1} varies (Stage C). No curve crosses another across the tested range: the ranking $u_3 \succ u_1 \succ u_4 \succ u_2$ is robust to the choice of parameter weights.

6.6. Ablation: Scope of the Λ-adjustment

Remark 3.7 noted that Definition 3.4 applies $\delta_\Lambda(u)$ to every parameter of an alternative, not only to $e \in \widetilde{E}$, and that this is a genuine design choice rather than the only reasonable one. Three variants were compared on the Stage B matrix of this application, holding the underlying confirmatory evidence fixed:

(A) *Current definition*: $\delta_\Lambda(u)$, averaged over $\widetilde{E} = \{e_1, e_3\}$, is applied to all four parameters.

(B) *$\widetilde{E}$-restricted*: the same averaged $\delta_\Lambda(u)$ is applied only to e_1, e_3, e_2, e_4 are left at their Stage B values.

(C) *Per-parameter*: e_1 and e_3 are each adjusted by their own confirmatory value $\delta_\Lambda(u, e)$ rather than the averaged one, with e_2, e_4 again left unadjusted. The per-parameter values used were chosen so that their average over $\widetilde{E}$ recovers exactly the scenario's original $\delta_\Lambda(u)$ from Section 6.1, for instance u_3's lab confirmation $\delta_\Lambda(u_3, e_1) = 0.60$ and specialist re-read $\delta_\Lambda(u_3, e_3) = 0.40$ average to 0.50, so all three variants are grounded in the same underlying evidence and differ only in how it is aggregated and scoped.

The ranking that motivates this paper is unchanged under all three variants, which is reassuring: the reversal reported in Section 6.4 is not an artifact of applying the adjustment too broadly. The coefficients themselves are not identical, however, Variant A produces the largest separation between u_3 and u_2, since it lets confirmed evidence on e_1, e_3 also lift the unconfirmed e_2, e_4 readings, while Variants B and C are more conservative and keep u_2 noticeably closer to the pack ($CC(u_2)$ rises from 0.164 under A to 0.287–0.294 under B/C). This is exactly the trade-off Remark 3.7 anticipates: scope affects magnitude, and in a case with sparser or more contested confirmatory coverage than this one it could plausibly affect the ranking too. Variant A is retained as the default definition for the reasons given in Remark 3.7, but Variants B and C are fully specified alternatives that a practitioner can substitute directly, and reporting all three, as here, is recommended whenever the choice is not obvious from the application domain.

6.7. Cost-type criteria

The parameters in the running example are all of benefit type: a higher truth degree is more consistent with the target condition and therefore more favourable, so the positive ideal takes the column-wise maximum of T. Many realistic diagnostic panels also contain *cost-type* parameters, where a higher truth degree is unfavourable, for example a "treatment-associated risk" score, for which strong evidence of risk should count against an alternative. The N-PTEFSS pipeline accommodates cost criteria in the standard neutrosophic-TOPSIS way, by replacing each cost column with its neutrosophic complement (Definition 4.4) before constructing the ideal solutions, the complement $\langle F, 1 - I, T\rangle$ turns "strong truth of a bad thing" into "strong falsity of a good thing," after which the ordinary benefit-type PIS/NIS construction of Section 5 applies unchanged. This is the route used by Biswas–Pramanik–Giri [18] and avoids introducing a separate ideal-solution rule for cost columns.

To illustrate, reinterpret e_4 (previously "response to a therapeutic trial," a benefit) as a cost criterion, say a treatment-associated risk score, where a high T is unfavourable, and leave e_1, e_2, e_3 as benefits. Complementing the e_4 column of the Stage C matrix and re-running the algorithm gives

$$CC(u_1) = 0.6492, \quad CC(u_2) = 0.3529,$$
$$CC(u_3) = 0.6969, \quad CC(u_4) = 0.3621,$$

with ranking $u_3 \succ u_1 \succ u_4 \succ u_2$, the same order as the all-benefit case, though the coefficients change (the spread compresses, since the strong e_4 readings that previously helped u_3 and u_4 now count against them). The point is not that the ranking must stay fixed, with a different cost structure it need not, but that cost criteria are handled by a single, citable preprocessing step rather than a special case in the ranking rule.

6.8. Robustness to the distance and aggregation choices

Two components of the pipeline were fixed by convention in Sections 3.2 and 5: the componentwise weighted average for time aggregation, and the normalized Euclidean distance of

Table 5: Closeness coefficients under the three Λ-adjustment scope variants. The ranking $u_3 \succ u_1 \succ u_4 \succ u_2$ is unchanged across all three.

Variant	$CC(u_1)$	$CC(u_2)$	$CC(u_3)$	$CC(u_4)$	Ranking
(A) current, all parameters	0.6945	0.1639	0.8942	0.5020	$u_3 \succ u_1 \succ u_4 \succ u_2$
(B) $\widetilde{E}$-restricted	0.6313	0.2943	0.7552	0.4664	$u_3 \succ u_1 \succ u_4 \succ u_2$
(C) per-parameter	0.6300	0.2866	0.7476	0.4719	$u_3 \succ u_1 \succ u_4 \succ u_2$

Biswas–Pramanik–Giri for the TOPSIS stage. Because both are choices rather than necessities, we re-ran the Stage C ranking under one standard alternative for each, to check that the reported ranking does not depend on them (Table 6).

For the distance, the normalized Hamming distance $d_H(a,b) = \frac{1}{3}(|T_a - T_b| + |I_a - I_b| + |F_a - F_b|)$ was substituted for the Euclidean one. For the aggregation, the single-valued neutrosophic weighted average (SVNSWA) operator, which combines T multiplicatively as $1 - \prod_i (1 - T_i)^{w_i}$ and I, F as $\prod_i I_i^{w_i}$, $\prod_i F_i^{w_i}$, was substituted for the weighted average at the time-aggregation step, with the Λ-adjustment and ranking otherwise unchanged. In both cases the ranking is exactly $u_3 \succ u_1 \succ u_4 \succ u_2$, identical to the baseline, and the closeness coefficients move only slightly.

That the two distances agree here is an empirical observation about the present data, established by the check just described rather than by appeal to any general principle: the choice of distance is known to matter in other settings, and the comparison of distance measures for decision-making is itself an active question, as the study of modified intuitionistic-fuzzy distances by Ejegwa *et al.* [20] illustrates. The paper's default choices are motivated in Remark 3.3 and Section 5 (the weighted average for its clean reduction property, the Euclidean distance for its standard status), this check confirms that they are defaults of convenience, not load-bearing assumptions on which the conclusion depends.

6.9. Weighting order: Ideal-then-weight versus weight-then-ideal

The algorithm of Section 5.2 derives the ideal solutions from the (unweighted) Λ-adjusted matrix and introduces the criterion weights ω_j only in the distance aggregation, an "ideal-then-weight" order. A common TOPSIS variant instead multiplies each column by ω_j first, builds a weighted decision matrix, reads the ideal solutions off *that* matrix, and then measures unweighted distances, "weight-then-ideal." The two are not identical in general, because scaling a column changes where its per-column ideal sits. Re-running Stage C under the weight-then-ideal order gives $CC = (0.6978, 0.1568, 0.9081, 0.4848)$ for (u_1, u_2, u_3, u_4), versus the baseline $(0.6945, 0.1639, 0.8942, 0.5020)$: the coefficients shift by at most about 0.017, and the ranking is unchanged at $u_3 \succ u_1 \succ u_4 \succ u_2$.

The choice of weighting order is therefore immaterial to the conclusion on this instance, we report the ideal-then-weight form in the main algorithm because it keeps the ideal solutions interpretable as assessments that some alternative actually attains. Under the weight-then-ideal order the entries of the weighted matrix still lie in $[0,1]^3$, since $0 \le \omega_j \le 1$ and componentwise multiplication by ω_j maps $[0,1]$ into itself, what is lost is not membership in the cube but interpretability, because a scaled triple $\langle \omega_j T, \omega_j I, \omega_j F \rangle$ is no longer the assessment of any alternative under parameter e_j, and the ideals read off the scaled matrix are correspondingly scaled rather than attainable profiles. A practitioner preferring the weighted-matrix convention obtains the same ranking here.

6.10. Degenerate-case sanity check

Proposition 4.13(c) states that for a fixed δ the adjustment is isotone with respect to the componentwise quality order $\preceq$: if $a \preceq b$, then $\Lambda_\delta(a) \preceq \Lambda_\delta(b)$. It is worth being explicit about what this does and does not give. It guarantees that a uniform adjustment cannot invert the quality order of a *comparable* pair. It does not by itself guarantee that the TOPSIS ranking is unchanged, for two reasons: alternatives that are incomparable under $\preceq$ are outside its scope entirely, and the ranking is produced by closeness coefficients computed against data-dependent column-wise ideals, which themselves move when the matrix is adjusted.

Establishing ranking invariance in general would require a separate TOPSIS-invariance theorem, which is not proved here. The following is therefore an instance-specific numerical check on the running example, not a general consequence of Proposition 4.13(c). Starting from the Stage B matrix (ranking $u_1 \succ u_3 \succ u_2 \succ u_4$) and applying a single uniform δ_Λ to all four patients, the post-adjustment ranking is $u_1 \succ u_3 \succ u_2 \succ u_4$ for every tested $\delta_\Lambda \in \{0, 0.2, 0.35, 0.5, 0.8\}$, identical to Stage B in all cases. On this instance, therefore, the reversal reported in Section 6 requires *differential* evidence across alternatives (there δ_Λ ranges from 0.10 to 0.50), and is not an artifact the adjustment produces on its own.

This is the behaviour one wants from an evidence-incorporation operator: with no differential evidence, it leaves the decision where the time-weighted assessment put it. The check is consistent with the isotonicity of Proposition 4.13(c), but is not implied by it, and we make no claim that uniform adjustment leaves the TOPSIS ranking unchanged on arbitrary data.

6.11. Comparison with a time-neutrosophic baseline

The closest existing model without an effective-parameter mechanism is the time-neutrosophic soft set (tNS-set) of Alkhazaleh [8], whose decision method aggregates the time-indexed triples and ranks alternatives by a neutrosophic score

Table 6: Stage C closeness coefficients under alternative distance and aggregation choices. All three rows give the ranking $u_3 \succ u_1 \succ u_4 \succ u_2$.

Configuration	$CC(u_1)$	$CC(u_2)$	$CC(u_3)$	$CC(u_4)$
Baseline (weighted avg. + Euclidean)	0.6945	0.1639	0.8942	0.5020
Hamming distance	0.6990	0.1657	0.8970	0.5153
SVNSWA aggregation	0.6957	0.1681	0.8894	0.4997

function, with no stage that incorporates post-assessment confirmation. Applying it to the same data (Table 1) isolates precisely what the Λ-adjustment contributes. Because the tNS-set method leaves the choice of score function open, we report the two standard forms: the basic score $s_1 = T - F$, which ignores indeterminacy, and the acceptability score $s_2 = \frac{1}{3}(T+(1-I)+(1-F))$, which incorporates it. Both are aggregated across parameters with the same weights ω_j used throughout, for a fair comparison. The results are in Table 7.

Two things stand out. First, no version of the tNS-set baseline has any way to represent the confirmatory evidence that distinguishes the patients: its ranking is determined entirely by the time-aggregated readings, so it cannot reproduce the Stage-C decision *for the reason* Stage C reaches it. Where the baseline does place u_3 first (score s_2), it does so only because u_3 happens to carry marginally lower average indeterminacy in the raw data, a fragile, incidental margin of 0.770 vs. 0.763, not because u_3's diagnosis was externally confirmed. Under the other standard score s_1 the baseline cannot separate u_1 and u_3 at all (both 0.530).

The N-PTEFSS pipeline, by contrast, places u_3 first at Stage C by a decisive margin (0.894 vs. 0.695) and does so through an explicit, auditable mechanism, u_3's stronger δ_Λ, that a clinician can trace to the reference-lab confirmation and specialist re-read. Second, our own model *without* the Λ stage (Stage B) ranks u_1 first, matching the un-confirmed baseline, it is only the Λ-adjustment that moves u_3 ahead. This is the concrete, same-data evidence that the effective-parameter mechanism, not the time-indexing, which the tNS-set already provides, is what changes the decision, and changes it for an interpretable reason rather than an accident of the score function.

7. Discussion and related work

The N-PTEFSS model sits at the intersection of three established lines, and it is worth stating precisely how it relates to each, including alternatives it does not adopt.

Operator conventions and matrix formulations. This paper fixes the (max, min, min)/(min, max, max) algebra of Maji [6] and Cuong–Phong–Smarandache [13], chosen (Remark 4.1) to keep every component in a single distributive lattice so that the properties of Section 4 hold. A parallel body of work develops neutrosophic soft sets through generic t-norm/t-conorm operators and matrix representations, Deli & Broumi's neutrosophic soft matrices [21] and Karaaslan's revision of Maji's operations [22], which trade the lattice guarantees for a broader operator family and a computational (matrix) form convenient for implementation. The Λ-adjustment is defined independently of this choice: it acts on the (T, I, F) triples themselves, so it could be carried over to a matrix-based, generic-operator formulation, at the cost of re-deriving the idempotency and De Morgan properties for whichever t/s pair is adopted. We keep the max/min algebra here because the reduction to the T-EFSS of [11] and the lattice proofs are cleaner, but nothing in the Λ-mechanism depends on it.

Reliability-adjusted decision making. The effective-parameter idea has a conceptual cousin in reliability- or confidence-weighted MADM, where each assessment carries a separate reliability index that discounts or reinforces it. The Λ-adjustment differs in two ways that are worth making explicit rather than eliding. First, it is directional and post-hoc: it models evidence that arrives *after* an initial time-indexed assessment and specifically raises truth while lowering indeterminacy and falsity, rather than a symmetric confidence weight applied uniformly at elicitation. Second, it is bound-preserving by construction (Proposition 4.12), whereas a raw reliability multiplier need not keep $T + I + F$ admissible. A fuller comparison, mapping δ_Λ onto a reliability index and contrasting their behaviour on the same data, is a natural next study, we flag it here rather than claiming the mechanisms are equivalent, because they are not.

Richer uncertainty representations. The model here uses single-valued neutrosophic triples. Interval-valued neutrosophic soft sets [23], complex triple-valued neutrosophic soft sets [24], and possibility neutrosophic soft sets represent uncertainty more richly, replacing each of T, I, F by an interval, by a complex-valued amplitude–phase pair, or attaching a possibility degree. The complex-valued line of work is instructive here for a specific reason: Hatamleh *et al.* [24] rank alternatives by a cotangent similarity measure rather than by ideal-solution distance, which shows that the ranking stage of a neutrosophic soft pipeline is genuinely modular, and hence that the Λ-adjustment, which acts strictly upstream of ranking, is not tied to the TOPSIS stage adopted here. The Λ-adjustment extends to the interval-valued setting in principle, applying the same convex-combination update to each interval endpoint, but the bound-preservation proof and the reduction argument would both need to be redone for intervals, and the ordering step would require an interval-neutrosophic distance in place of the single-valued one. We regard this as a worthwhile extension rather than a routine corollary, and leave it open.

8. Conclusion

This paper introduced the neutrosophic parameterized time-effective soft set (N-PTEFSS), a model in which parameteri-

Table 7: The tNS-set decision method (no Λ-adjustment) on the data of Table 1, versus the N-PTEFSS pipeline. Scores s_1, s_2 are aggregated with the criterion weights ω_j, CC rows are neutrosophic-TOPSIS closeness coefficients.

Method	u_1	u_2	u_3	u_4	Ranking
tNS-set, score $s_1 = T - F$	0.530	0.343	0.530	0.371	$u_1 \sim u_3 \succ u_4 \succ u_2$
tNS-set, score s_2 (with I)	0.763	0.669	0.770	0.673	$u_3 \succ u_1 \succ u_4 \succ u_2$
N-PTEFSS, Stage B (time only, no Λ)	0.669	0.351	0.664	0.344	$u_1 \succ u_3 \succ u_2 \succ u_4$
N-PTEFSS, Stage C (with Λ)	0.695	0.164	0.894	0.502	$u_3 \succ u_1 \succ u_4 \succ u_2$

zation, time-weighting, and an effective-parameter adjustment are all defined natively on neutrosophic truth–indeterminacy–falsity triples, with each component carried separately through every operation and every proof. Its principal contribution is the Λ-adjustment of Definition 3.4: an operator that revises a $\langle T, I, F\rangle$ triple in the light of external, post-assessment confirmation, pushing truth upward while shrinking indeterminacy and falsity, and provably preserving the neutrosophic triple-sum bound $0 \le T + I + F \le 3$ (Proposition 4.12), a joint constraint with no counterpart in the single-valued fuzzy case, even though bound preservation by convexity is itself familiar there. We established the basic operations (union, intersection, complement, AND, OR) and proved complement involution, associativity, idempotency, distributivity, and De Morgan's laws for them, and we built a neutrosophic TOPSIS ranking procedure on top of the model.

Two results anchor the paper. First, a reduction theorem (Proposition 3.9) shows that when indeterminacy and falsity vanish the model specializes, term for term at the definitional layer, to the time-effective fuzzy soft set of [11], so N-PTEFSS is a proper generalization of that fuzzy predecessor rather than a relabelling. Second, the medical-diagnosis application (Section 6) demonstrated that the Λ-adjustment is decision-relevant, not merely descriptive: on a four-patient diagnostic ranking, time-weighting alone left the top of the order unchanged, but the effective-parameter adjustment reversed the top two alternatives, the patient with the strongest external confirmation overtook the one leading on the pre-confirmation readings, a decision that no purely pre-implementation assessment could recover. Sensitivity analyses, a scope ablation, and substitution of alternative distance and aggregation operators all left this reversal intact, indicating that it reflects the confirmatory evidence itself rather than any incidental modeling choice.

Several extensions remain open. The Λ-adjustment currently shrinks indeterminacy and falsity by the same factor (Remark 3.6), an asymmetric variant, in which confirmatory evidence resolves indeterminacy before it overturns an explicit negative, is well motivated in the diagnostic setting and would be worth developing with its own reduction check. A group-decision-making version, aggregating several experts' N-PTEFSSs before ranking, is a natural next step, as is the interaction of the Λ-adjustment with neutrosophic TOPSIS variants beyond the Biswas–Pramanik–Giri distance used here, for instance bipolar or interval-valued neutrosophic ranking procedures.

Data availability

The numerical data used in the worked medical-diagnosis example (Tables 1–3) and all sensitivity, ablation, and robustness results reported in Section 6 are illustrative values constructed for this paper and are reproduced in full within the manuscript. The complete Wolfram Language verification script used to compute and independently check every reported numerical value is reproduced in full in the Appendix.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this manuscript.

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Appendix A. Verification code (Wolfram Language)

The listing below is the complete Wolfram Language script used to compute and independently check the numerical claims of the paper. It reproduces, in the order in which the code is organised:

1. the symbolic bound-preservation proof of Proposition 4.12(b), obtained with `Reduce` rather than by numerical sampling;
2. the three-stage medical-diagnosis example of Section 6, namely the Stage B and Stage C matrices of Table 2 and the distances and closeness coefficients of Table 3, including the worked rows quoted in the text;
3. the sensitivity analysis in $\delta_\Lambda(u_3)$ behind Figure 2, together with the bisection that locates the crossover value $\delta_\Lambda(u_3^*) \approx 0.3075$;
4. the sensitivity analysis in the criterion weight ω_{e_1} behind Table 4 and Figure 3, with the remaining weights rescaled proportionally;
5. the ablation over the three Λ-adjustment scope variants reported in Table 5;
6. the cost-type criterion variant of Section 6.7, in which e_4 is re-read as a cost through the neutrosophic complement;
7. the robustness checks of Section 6.8: the normalized Hamming distance in place of the Euclidean distance, and SVNSWA aggregation in place of the componentwise weighted average at the time step;
8. the structural properties of the Λ-adjustment stated in Proposition 4.13;
9. the weight-then-ideal comparison of Section 6.9;
10. the degenerate-case check of Section 6.10, in which a uniform δ_Λ is applied to all four patients; and
11. the tNS-set baseline ranking of Section 6.11 (Table 7), computed without any Λ-adjustment.

It can be run directly in Mathematica (open the file, or evaluate the cells in order).

```
(* ::Package:: *)

(* ========================================
 ↪  ====================
N-PTEFSS VERIFICATION SCRIPT (Wolfram
 ↪  Language)
Reproduces every numerical claim in
 ↪  Sections 3 and 6 of the paper.
PART 1: symbolic proof of bound
 ↪  preservation (Proposition 4.12(b)) via
 ↪  Reduce.
PART 2: full numerical reproduction of the
 ↪  medical-diagnosis example
(4 patients, 4 parameters), the
 ↪  neutrosophic TOPSIS ranking,
and both sensitivity analyses (delta(u3)
 ↪  sweep, omega(e1) sweep).
========================================
 ↪  ================= *)

(* {; }{; }{; }- PART 1: symbolic
 ↪  bound-preservation proof {; }{; }{; }-
 ↪  *)
```

```
TLam[T_, d_] := T + (1 - T) d;
ILam[I_, d_] := I (1 - d);
FLam[F_, d_] := F (1 - d);

counterExampleSearch = Reduce[
0 <= T <= 1 && 0 <= I <= 1 && 0 <= F <= 1
 ↪  && 0 <= d <= 1 &&
!(0 <= TLam[T, d] <= 1 && 0 <= ILam[I, d]
 ↪  <= 1 && 0 <= FLam[F, d] <= 1),
{T, I, F, d}, Reals];
Print["Counterexample search (should be
 ↪  False): ", counterExampleSearch];
Print["delta=0 identity: ",
 ↪  Simplify[TLam[T, 0] == T]];
Print["delta=1 saturates T: ",
 ↪  Simplify[TLam[T, 1] == 1]];
Print["I-update vanishes at I=0: ",
 ↪  Simplify[ILam[0, d] == 0]];
Print["F-update vanishes at F=0: ",
 ↪  Simplify[FLam[0, d] == 0]];

(* {; }- PART 2: medical-diagnosis example
 ↪  (4 patients, 4 params) {; }- *)

patients = {"u1", "u2", "u3", "u4"};
params = {"e1", "e2", "e3", "e4"};
w1 = 0.4; w2 = 0.6;

t1data = <|
"u1" -> <|
"e1" -> {0.6,0.3,0.1}, "e2" ->
 ↪  {0.5,0.4,0.1},
"e3" -> {0.7,0.2,0.1}, "e4" ->
 ↪  {0.5,0.3,0.2}|>,
"u2" -> <|
"e1" -> {0.5,0.4,0.1}, "e2" ->
 ↪  {0.7,0.2,0.1},
"e3" -> {0.4,0.4,0.2}, "e4" ->
 ↪  {0.4,0.4,0.2}|>,
"u3" -> <|
"e1" -> {0.7,0.2,0.1}, "e2" ->
 ↪  {0.6,0.3,0.1},
"e3" -> {0.5,0.3,0.2}, "e4" ->
 ↪  {0.5,0.3,0.2}|>,
"u4" -> <|
"e1" -> {0.5,0.3,0.2}, "e2" ->
 ↪  {0.5,0.3,0.2},
"e3" -> {0.3,0.6,0.1}, "e4" ->
 ↪  {0.7,0.2,0.1}|>
|>;
t2data = <|
"u1" -> <|
"e1" -> {0.7,0.2,0.1}, "e2" ->
 ↪  {0.6,0.3,0.1},
"e3" -> {0.8,0.1,0.1}, "e4" ->
 ↪  {0.6,0.2,0.2}|>,
"u2" -> <|
"e1" -> {0.4,0.4,0.2}, "e2" ->
 ↪  {0.8,0.1,0.1},
"e3" -> {0.4,0.4,0.2}, "e4" ->
 ↪  {0.4,0.4,0.2}|>,
"u3" -> <|
"e1" -> {0.8,0.1,0.1}, "e2" ->
 ↪  {0.7,0.2,0.1},
"e3" -> {0.6,0.3,0.1}, "e4" ->
 ↪  {0.6,0.2,0.2}|>,
"u4" -> <|
"e1" -> {0.6,0.3,0.1}, "e2" ->
 ↪  {0.5,0.3,0.2},
"e3" -> {0.3,0.6,0.1}, "e4" ->
 ↪  {0.8,0.1,0.1}|>
|>;

delta = <|"u1" -> 0.30, "u2" -> 0.10, "u3"
 ↪  -> 0.50, "u4" -> 0.35|>;
omegaBase = <|"e1" -> 0.35, "e2" -> 0.25,
 ↪  "e3" -> 0.25, "e4" -> 0.15|>;

WeightedAvg[a_List, b_List] := w1 a + w2 b;
LambdaAdjust[{t_, i_, f_}, d_] := If[0 < t
 ↪  < 1,
{t + (1 - t) d, i (1 - d), f (1 - d)},
{t, i, f}];

stageA = t2data;
stageB = Association@Table[
u -> Association@Table[
e -> WeightedAvg[t1data[u][e],
 ↪  t2data[u][e]],
{e, params}],
{u, patients}];
stageC = Association@Table[
u -> Association@Table[
e -> LambdaAdjust[stageB[u][e], delta[u]],
{e, params}],
{u, patients}];

NZDist[a_List, b_List] := Sqrt[Total[(a -
 ↪  b)^2]/3];

TopsisStage[stage_Association,
 ↪  omega_Association] :=
Module[{PIS, NIS, dPlus, dMinus, CC},
PIS = Association@Table[e -> {
        Max[Table[stage[u][e][[1]], {u,
         ↪  patients}]],
        Min[Table[stage[u][e][[2]], {u,
         ↪  patients}]],
        Min[Table[stage[u][e][[3]], {u,
         ↪  patients}]]}, {e, params}];
NIS = Association@Table[e -> {
        Min[Table[stage[u][e][[1]], {u,
         ↪  patients}]],
        Max[Table[stage[u][e][[2]], {u,
         ↪  patients}]],
        Max[Table[stage[u][e][[3]], {u,
         ↪  patients}]]}, {e, params}];
dPlus = Association@Table[
u -> Sqrt[Total[Table[
omega[e] NZDist[stage[u][e], PIS[e]]^2, {e,
 ↪  params}]]],
{u, patients}];
dMinus = Association@Table[
u -> Sqrt[Total[Table[
omega[e] NZDist[stage[u][e], NIS[e]]^2, {e,
 ↪  params}]]],
{u, patients}];
CC = Association@Table[
u -> If[dPlus[u] + dMinus[u] > 0,
dMinus[u]/(dPlus[u] + dMinus[u]), 0.],
{u, patients}];
<|"PIS" -> PIS, "NIS" -> NIS, "D+" ->
 ↪  dPlus, "D-" -> dMinus, "CC" -> CC|>];

resA = TopsisStage[stageA, omegaBase];
resB = TopsisStage[stageB, omegaBase];
resC = TopsisStage[stageC, omegaBase];
RankOf[res_] := Reverse@SortBy[patients,
 ↪  res["CC"][#] &];

Do[
Module[{res = pair[[1]], nm = pair[[2]]},
Print["Stage ", nm, ":  ",
StringRiffle[Table[
```

```
u <> ": CC=" <>
 ↪  ToString[Round[res["CC"][u], 0.0001]],
{u, patients}], "   "]];
Print["  Ranking: ",
 ↪  StringRiffle[RankOf[res], " > "]]],
{pair, {{resA, "A"}, {resB, "B"}, {resC,
 ↪  "C"}}}];

(* {; }- Sensitivity 1: delta(u3) sweep,
 ↪  exact flip threshold {; }- *)
CCGapAtDelta3[d3v_?NumericQ] := Module[{sc,
 ↪  res},
sc = Association@Table[u ->
 ↪  Association@Table[
e -> LambdaAdjust[stageB[u][e],
If[u == "u3", d3v, delta[u]]],
{e, params}],
{u, patients}];
res = TopsisStage[sc, omegaBase];
res["CC"]["u1"] - res["CC"]["u3"]];
Do[
Print["delta(u3)=", d3,
"  gap=", Round[CCGapAtDelta3[d3],
 ↪  0.0001]],
{d3, {0.10, 0.20, 0.30, 0.32, 0.40, 0.50,
 ↪  0.60}}];
threshold =
 ↪  Quiet@FindRoot[CCGapAtDelta3[d3v],
 ↪  {d3v, 0.32}];
Print["Exact flip threshold: ", threshold];

(* {; }- Sensitivity 2: omega(e1) sweep,
 ↪  proportional rescaling {; }- *)
RescaledOmega[we1_] := Module[
{baseRest = <|"e2" -> 0.25, "e3" -> 0.25,
 ↪  "e4" -> 0.15|>,
        restSum, remain},
restSum = Total[baseRest]; remain = 1 -
 ↪  we1;
Association[Prepend[
KeyValueMap[#1 -> remain (#2/restSum) &,
 ↪  baseRest],
"e1" -> we1]]];
Do[
Module[{om = RescaledOmega[we1], res},
res = TopsisStage[stageC, om];
Print["omega(e1)=", we1, "  CC=",
 ↪  Normal[Round[#, 0.0001] & /@
 ↪  res["CC"]],
"  Ranking: ", StringRiffle[RankOf[res], "
 ↪  > "]]],
{we1, {0.05, 0.15, 0.25, 0.35, 0.45, 0.55,
 ↪  0.65}}];

(* {; }- Ablation: scope of the
 ↪  Lambda-adjustment (A / B / C) {; }- *)
Etilde = {"e1", "e3"};
deltaPerParam = <|
"u1" -> <|"e1" -> 0.35, "e3" -> 0.25|>,
"u2" -> <|"e1" -> 0.05, "e3" -> 0.15|>,
"u3" -> <|"e1" -> 0.60, "e3" -> 0.40|>,
"u4" -> <|"e1" -> 0.30, "e3" -> 0.40|>|>;
Do[
Print["avg check ", u, ": ",
Mean[Values[deltaPerParam[u]]], " vs ",
 ↪  delta[u]],
{u, patients}];

stageCA = stageC; (* current definition,
 ↪  already computed above *)

stageCB = Association@Table[
u -> Association@Table[
e -> If[MemberQ[Etilde, e],
LambdaAdjust[stageB[u][e], delta[u]],
 ↪  stageB[u][e]],
{e, params}],
{u, patients}];

stageCC = Association@Table[
u -> Association@Table[
e -> If[MemberQ[Etilde, e],
LambdaAdjust[stageB[u][e],
 ↪  deltaPerParam[u][e]],
stageB[u][e]],
{e, params}],
{u, patients}];

Do[
Module[{variant = pair[[1]], nm =
 ↪  pair[[2]], res},
res = TopsisStage[variant, omegaBase];
Print["Variant ", nm, ":  CC=",
 ↪  Normal[Round[#, 0.0001] & /@
 ↪  res["CC"]],
"  Ranking: ", StringRiffle[RankOf[res], "
 ↪  > "]]],
{pair, {{stageCA, "A (current)"}, {stageCB,
 ↪  "B (Etilde-restricted)"},
                {stageCC, "C
                 ↪  (per-parameter)"}}}];

(* {; }- Cost-type criterion: e4 as cost
 ↪  via neutrosophic complement {; }- *)
Complement3[{t_, i_, f_}] := {f, 1 - i, t};
stageCcost = Association@Table[
u -> Association@Table[
e -> If[e === "e4",
 ↪  Complement3[stageC[u][e]],
 ↪  stageC[u][e]],
{e, params}],
{u, patients}];
Module[{res = TopsisStage[stageCcost,
 ↪  omegaBase]},
Print["e4 as COST:  CC=", Normal[Round[#,
 ↪  0.0001] & /@ res["CC"]],
"  Ranking: ", StringRiffle[RankOf[res], "
 ↪  > "]]];

(* {; }- Alternative distance: normalized
 ↪  Hamming {; }- *)
NZHamming[a_List, b_List] := Total[Abs[a -
 ↪  b]]/3;
TopsisStageDist[stage_, omega_, dist_] :=
 ↪  Module[{PIS, NIS, dP, dM},
PIS = Association@Table[e -> {
        Max[Table[stage[u][e][[1]], {u,
         ↪  patients}]],
        Min[Table[stage[u][e][[2]], {u,
         ↪  patients}]],
        Min[Table[stage[u][e][[3]], {u,
         ↪  patients}]]}, {e, params}];
NIS = Association@Table[e -> {
        Min[Table[stage[u][e][[1]], {u,
         ↪  patients}]],
        Max[Table[stage[u][e][[2]], {u,
         ↪  patients}]],
        Max[Table[stage[u][e][[3]], {u,
         ↪  patients}]]}, {e, params}];
dP = Association@Table[
u -> Sqrt[Total[Table[
omega[e] dist[stage[u][e], PIS[e]]^2, {e,
 ↪  params}]]], {u, patients}];
dM = Association@Table[
u -> Sqrt[Total[Table[
```

```
omega[e] dist[stage[u][e], NIS[e]]^2, {e,
↪  params}]]], {u, patients}];
Association@Table[
u -> If[dP[u] + dM[u] > 0, dM[u]/(dP[u] +
↪  dM[u]), 0.], {u, patients}]];
Module[{cc = TopsisStageDist[stageC,
↪  omegaBase, NZHamming]},
Print["Hamming distance:  CC=",
↪  Normal[Round[#, 0.0001] & /@ cc]]];

(* {; }- Alternative aggregation: SVNSWA at
↪  the time step {; }- *)
SVNSWA[a_List, b_List, wa_, wb_] := {
        1 - (1 - a[[1]])^wa (1 -
        ↪  b[[1]])^wb,
        a[[2]]^wa b[[2]]^wb,
        a[[3]]^wa b[[3]]^wb};
stageBsv = Association@Table[
u -> Association@Table[
e -> SVNSWA[t1data[u][e], t2data[u][e], w1,
↪  w2], {e, params}],
{u, patients}];
stageCsv = Association@Table[
u -> Association@Table[
e -> LambdaAdjust[stageBsv[u][e],
↪  delta[u]], {e, params}],
{u, patients}];
Module[{res = TopsisStage[stageCsv,
↪  omegaBase]},
Print["SVNSWA aggregation:  CC=",
↪  Normal[Round[#, 0.0001] & /@
↪  res["CC"]],
"  Ranking: ", StringRiffle[RankOf[res], "
↪  > "]]];

(* {; }- Structural properties of the
↪  Lambda-adjustment (Prop 4.13) {; }- *)
Clear[Tt, Ii, Ff, dd];
TL[Tt_, dd_] := Tt + (1 - Tt) dd;
IL[Ii_, dd_] := Ii (1 - dd);
FL[Ff_, dd_] := Ff (1 - dd);
Print["dT_L/dd = ", Simplify[D[TL[Tt, dd],
↪  dd]], " (>=0)"];
Print["dI_L/dd = ", Simplify[D[IL[Ii, dd],
↪  dd]], " (<=0)"];
Print["dF_L/dd = ", Simplify[D[FL[Ff, dd],
↪  dd]], " (<=0)"];
Print["dT_L/dT = ", Simplify[D[TL[Tt, dd],
↪  Tt]], " (>=0: monotone in input T)"];

(* {; }- Weight-then-ideal vs
↪  ideal-then-weight (Stage C) {; }- *)
TopsisWeightThenIdeal[stage_, omega_] :=
↪  Module[{W, PIS, NIS, dP, dM},
W = Association@Table[
u -> Association@Table[e -> omega[e]
↪  stage[u][e], {e, params}],
{u, patients}];
PIS = Association@Table[e -> {
        Max[Table[W[u][e][[1]], {u,
        ↪  patients}]],
        Min[Table[W[u][e][[2]], {u,
        ↪  patients}]],
        Min[Table[W[u][e][[3]], {u,
        ↪  patients}]]}, {e, params}];
NIS = Association@Table[e -> {
        Min[Table[W[u][e][[1]], {u,
        ↪  patients}]],
        Max[Table[W[u][e][[2]], {u,
        ↪  patients}]],
        Max[Table[W[u][e][[3]], {u,
        ↪  patients}]]}, {e, params}];
dP = Association@Table[
u -> Sqrt[Total[Table[NZDist[W[u][e],
↪  PIS[e]]^2, {e, params}]]],
{u, patients}];
dM = Association@Table[
u -> Sqrt[Total[Table[NZDist[W[u][e],
↪  NIS[e]]^2, {e, params}]]],
{u, patients}];
Association@Table[
u -> If[dP[u] + dM[u] > 0, dM[u]/(dP[u] +
↪  dM[u]), 0.], {u, patients}]];
Module[{cc = TopsisWeightThenIdeal[stageC,
↪  omegaBase]},
Print["weight-then-ideal:  CC=",
↪  Normal[Round[#, 0.0001] & /@ cc],
"  Ranking: ", StringRiffle[
Reverse@SortBy[patients, cc[#] &], " >
↪  "]]];

(* {; }- Degenerate check: uniform delta
↪  preserves the Stage-B ranking {; }- *)
Do[
Module[{sc, res},
sc = Association@Table[
u -> Association@Table[
e -> LambdaAdjust[stageB[u][e], dU], {e,
↪  params}], {u, patients}];
res = TopsisStage[sc, omegaBase];
Print["uniform delta=", dU, ":  Ranking ",
StringRiffle[RankOf[res], " > "]]],
{dU, {0.0, 0.2, 0.35, 0.5, 0.8}}];

(* {; }- Baseline: tNS-set score-based
↪  ranking (no Lambda) on Stage B {; }- *)
score1[{Tt_, Ii_, Ff_}] := Tt - Ff;
score2[{Tt_, Ii_, Ff_}] := (Tt + (1 - Ii) +
↪  (1 - Ff))/3;
AggScore[stage_, sfn_] :=
↪  Association@Table[
u -> Total[Table[omegaBase[e]
↪  sfn[stage[u][e]], {e, params}]],
{u, patients}];
Module[{r1 = AggScore[stageB, score1], r2 =
↪  AggScore[stageB, score2]},
Print["tNS-set score s1=T-F:  ",
↪  Normal[Round[#, 0.001] & /@ r1],
"  Ranking: ",
↪  StringRiffle[Reverse@SortBy[patients,
↪  r1[#] &], " > "]];
Print["tNS-set score s2(with I):  ",
↪  Normal[Round[#, 0.001] & /@ r2],
"  Ranking: ",
↪  StringRiffle[Reverse@SortBy[patients,
↪  r2[#] &], " > "]]];
```