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Mathematical Modeling of Waves in a Porous Micropolar Fibre-reinforced Structure and Liquid Interface

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Abstract

The present investigation envisages on the Mathematical modeling of waves propagating in a porous micropolar fibre-reinforced structure in a half-space and liquid interface. The harmonic method of wave analysis is utilized, such that, the reflection and transmission of waves in the media were modelled and it's equations of motion analytically derived. It was deduced that incident longitudinal wave in the solid structure yielded four reflected waves given as; quasi–P wave (qLD), quasi–SV wave, quasi–transverse microrotational (qTM) wave and a wave due to voids and one transmitted wave known as the quasi-longitudinal transmitted (qLT) wave. The phase velocity in the liquid medium is independent of angle of propagation as observed. The corresponding amplitude ratios of propagations for both reflected and transmitted waves are analytically derived by employing Snell's law. The model would prove to be of relevance in the understanding of modeling of the behavior of propagation phenomena of waves in micropolar fibre-reinforecd machination systems resulting in solid/liquid interfaces especially in earth sciences and in particular seismology, amongst others.

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1. Introduction

Researchers in solid mechanics and in particular elastodynamics have always pursued for ways in trying to decipher the behaviors of certain disturbances in materials caused either naturally or artificially, especially in disciplines such as; engineering, structural designing, seismology, material sciences, and geophysics among others. Although of particular interest in this study is the porous micopolar fibre reinforced materials. Fibre reinforced materials play a collective host to the strength of materials used by construction engineers, physicists, material scientists etc., owing to their high tensile strength, low weight, and efficacy of fibre-reinforcement cum flexibility of usage [1]. Fibre reinforced media tends to be similar to another important type of material known as the Orthotropic material; which could be considered in the investigation of elastodynamic models. Some of these materials possess micro-rotation and translation of local points as given by Erigen [2], theory of micropolar elasticity. Thus, in describing the propagation of waves

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in such materials, a mathematical model is accompanied along with certain physical properties and parameters such as; voids in the material [3], pre-stress, as the case maybe. Voids are pores in a material which are taken as a volume fraction fields in equations describing such media. Mathematical models in terms of equations used for the study of reflection and transmission at a plane half-space of elastic media were initially introduced by Knott [4] and subsequently modified by Jeffreys [5], Gutenberg [6], etc.

In a similar vein, reflection and transmission coefficients in fluid-saturated porous media and boundary surfaces, have received great attention in literature [7-11]. Extensive works on fiber reinforced media and reflection of waves in an elastic media with some other physical properties of rotation, gravity, thermal effects etc., were also conducted [12-18]. Reflections of waves in a micropolar fibre reinforced medium with other physical properties of magneto-thermoelastic effects, rotation, etc., were also carried out [19-20].

Furthermore, boundary surfaces of some materials are plane, grooved or entirely of different shapes in nature. Grooved boundary surface could be visualized as a series of parallel furrows and ridges whose encounter in mechanical propagation of wave results to several effects especially across interfaces. Interestingly, some authors had worked on this concept of corrugated boundaries and other related wave propagation phenomena [21-29]. Also, certain discussions on materials [30-33] could aid understanding of material characterizations.

In spite of these contributions, the present study is also of particular interest, as it seeks to investigate the propagation of waves in plane boundary of a porous micropolar fibre reinforced solid structure and liquid interface. Also the motivation of the study stems from the fact that classical theories have short comings in modeling solid structure interactions and its understanding of behaviors for any given impact on it; hence micropolar theory of elasticity by Eringen [2], suggests the basic assumptions and subsequently, the consideration of 9x9 non-symmetric material matrix of Micropolar fibre-reinforcement. In studying this, we employed two concepts of wave propagation analyses called the normal mode analysis or harmonic analysis and Snell's law. Due to the composition of the media, when P-wave is incident on the solid structure, four waves are reflected in the solid structure while only quasi-longitudinal transmitted (qLT) wave is transmitted in the liquid medium. And by taken continuity conditions at the interface, the reflection and transmitted wave's propagation coefficients are derived.

2. Mathematical Formulation of the Problem

The constitutive relations for a micropolar fibre–reinforced elastic anisotropic solid with voids, considering some existing works [2-3], [34-35] follow as:

$$\sigma_{ij} = N_{ijmn}E_{mn} + S_{ijmn}\psi_{mn} + \zeta\phi\delta_{ij}, \tag{1}$$

$$m_{ij} = N_{jimn}E_{mn} + S_{mnji}\psi_{mn}, \tag{2}$$

we define the deformations and wryness tensors as:

$$E_{ij} = u_{j,i} + \varepsilon_{jim}\phi^*_m, \quad \psi_{mn} = \phi^*_{m,n}, \quad i = j = m = n = 1, 2, 3. \tag{3}$$

The balance laws in the absence of body forces are presented below as:

$$\sigma_{ij,i} = \rho\ddot{u}_j, \tag{4}$$

$$m_{ij,i} + \varepsilon_{jmn}\sigma_{mn} = \rho J\ddot{\phi}_j^*, \tag{5}$$

$$\xi_1(\phi_{,ii}) - \omega_o\phi - \varpi\dot{\phi} - \zeta(u_{i,i}) = \rho\kappa\ddot{\phi}. \tag{6}$$

where m_{ij}, σ_{ij}, ϕ^*_j, u_j and ϕ are the couple stress tensor, stress tensor, microrotation vector, displacement vector and volume fraction field respectively. ρ is the density of the solid medium; ζrepresents the voids parameter; J is the microinertia, N_{ijmn}, N_{jimn} are constants of material characterization such that non symmetric properties of N_{ijmn}, N_{jimn} and S_{ijmn} are observed. ε_{jim} is the Levi–Civita tensor and δ_{ij}, is the Kronecker–delta function. The given index after comma denotes partial derivative with respect to coordinate space and the superscript dot stipulates partial derivative with respect to time. We tried to consider the deformation in x_1x_3–plane. This is such that the displacements; $u_1 \neq u_3 \neq 0$, while $u_2 = 0$. Thus, this implies that the micro-rotation; $\phi^* = (0, \phi^*_2, 0)$. Repeated indexes of Einstein summation convention is used. Considering the fact the tensors are not symmetric in micropolar solid, in the sense that, a 9x9 matrix of the solid material characterization is utilized, Eqs. (4-6) in components form are given as:

$$N_1u_{1,11} + (N_2 + N_3)u_{3,13} + N_6u_{1,33} + N^*_1\phi^*_{2,3} + \zeta\phi_{,1} = \rho\ddot{u}_1, \tag{7}$$

$$N_4u_{3,11} + N_2u_{1,13} + N_5u_{3,33} + N_4\phi^*_{2,1} + \zeta\phi_{,3} = \rho\ddot{u}_3, \tag{8}$$

$$S_1\phi^*_{2,11} + S_2\phi^*_{2,33} + N^*_2\phi^*_2 + \chi_1u_{1,3} + \chi_2u_{3,1} = \rho J\ddot{\phi}^*_2, \tag{9}$$

$$\xi_1(\phi_{,ii}) - \omega_o\phi - \varpi\dot{\phi} - \zeta(u_{i,i}) = \rho\kappa\ddot{\phi}, \tag{10}$$

where

$$N_1 = (\lambda + 2\alpha + \beta + 4\mu_L - 2\mu_T),$$

$$N_2 = (\lambda + \alpha),\ N_3 = 2\mu_T,$$

$$N_4 = 2\mu_L, N_5 = (\lambda + 2\mu_T), N_6 = (2\mu_L - \mu_T),$$

$$N^*_1 = (3\mu_T - 2\mu_L), N^*_2 = (3\mu_T - 4\mu_L),$$

$$\chi_1 = N_4 - (N_3/2), \chi_2 = N_4 - N_3$$

3. Analytical Solution of the Problem

We consider a micropalar fibre–reinforced structure occupying the half–space $x_3 < 0$and a liquid medium occupying the half–space$x_3 > 0$. Thus, the two media constitutes an interface at the boundary. Hence, an assumption is made for the displacements in the media as taken below:

$$u_1 = Pe^{i\{k(x_jp_j)-\omega t\}}, \tag{11}$$

$$u_3 = Qe^{i\{k(x_j p_j)-\omega t\}}, \tag{12}$$

$$\phi_2^* = \phi^* e^{i\{k(x_j p_j)-\omega t\}}, \tag{13}$$

$$\phi = \phi_0 e^{i\{k(x_j p_j)-\omega t\}}, j = 1, 2. \tag{14}$$

where P, Q, ϕ^* and ϕ_0 are amplitudes of the wave displacements respectively. $c = \frac{\omega}{k}$ is the phase velocity of the wave, k is the wave number, and ω is the angular velocity of the wave. Introducing Eqs. (11-14) into Eqs. (7-10), yields the following equations below:

$$\begin{aligned}&\{k^2 D_1 - k^2 c^2 \rho\}P + \{k^2(N_2 + N_3)p_1 p_3\}Q\\ &-ikN_1^* p_3 \phi^* - ik\zeta p_1 \phi_0 = 0\end{aligned} \tag{15}$$

$$\begin{aligned}&\{N_2 k^2 p_1 p_3\}P + \{k^2 D_2 - c^2 k^2 \rho\}Q\\ &iN_4 k p_1 \phi^* - i\zeta k p_3 \phi_0 = 0,\end{aligned} \tag{16}$$

$$\begin{aligned}&-\{ik\chi_1 p_3\}P - \{ik\chi_2 p_1\}Q\\ &+\{k^2 D_3 - N_2 - \rho J k^2 c^2\}\phi^* = 0,\end{aligned} \tag{17}$$

$$\begin{aligned}&\{i\zeta k p_1\}P + \{i\zeta k p_3\}Q\\ &+(\xi_1 k^2 + \omega_0 - \varpi ikc - \rho\kappa k^2 c^2)\phi_0 = 0,\end{aligned} \tag{18}$$

where $D_1 = N_1 p_1^2 + N_6 p_2^2$, $D_2 = N_4 p_1^2 + N_5 p_2^2$, $D_3 = S_1 p_1^2 + S_2 p_3^2$,$p_1 = Sin\alpha$ and$p_2 = Cos\alpha$. For non–trivial solution, Eqs. (15-18) gives the quartic polynomial below:

$$\gamma^4 + E_1\gamma^3 + E_2\gamma^2 + E_3\gamma + E_4 = 0\,. \tag{19}$$

Here $\gamma = k^2$. This means that the characteristic Eq. (19) with complex coefficients E_1,E_2,E_3,and E_4 (See Appendix) gives four complex roots. Thus, the four waves propagate with complex phase velocities: c_1, c_2, c_3 and c_4 in the solid medium corresponding to the wave numbers $k_1, k_2, k_3$and k_4 respectively. Hence, the two dimensional model in the x_1x_3- plane of the micropolar fibre–reinforced solid half space, have four waves; quasi–P wave (qLD), quasi–SV wave, quasi–transverse microrotational (qTM) wave and wave due to voids travelling in the solid medium. Following Singh's work [36], for the liquid medium, consider $N_1 = N_2 = N_5 = \lambda_5$,$\rho = \rho_5$,$\phi_2^* = \chi_2 = \chi_1 = \zeta = N_4 = N_3 = \phi = 0$ into Eqs. (7-10), we obtain the equation below for a non–trivial solution:

$$c^{*4}\rho^2 - c^{*2}\lambda_5\rho\left(p_1^2 + p_3^2\right) = 0 \tag{20}$$

The roots of the characteristics Eq. (20) for the liquid medium can simply be represented as$c^* = \pm\sqrt{\lambda_5(p_1^2 + p_3^2)/\rho_5}$. This shows that one phase velocity; c_5,corresponding to the wave number$k_5$is taken. Thus, in the liquid medium, one of the roots of Eq. (20) will be negligible; this entails that only quasi–Longitudinal transmitted (qLT) wave can propagate. Also observe that $p_1^2 + p_3^2 = 1$. Hence, the phase velocity in the liquid medium will be independent of angle of propagation.

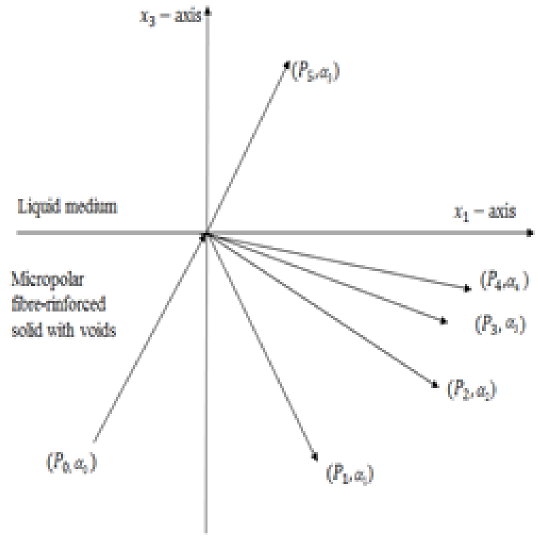

Figure 1. Schematic of the problem showing micropolar fibre–reinforced solid half-space with voids and liquid interface

4. Reflection/Transmission of Waves and the Geometry of the Problem

Let us consider the propagation of P-wave (Longitudinal wave) incident on a micopolar fibre-reinforced half-space in the x_1x_3-plane with voids such that the boundary is plane in nature and it's in an interface with a liquid medium. The geometry is demonstrated in Figure 1 below.

Also any one of the four waves can be chosen as incident wave. Figure 1 shows that when quasi–P wave(P_0) is incident at micropolar fibre–reinforced anisotropic solid and liquid interface, there exist four reflected waves as quasi–P (P_1) or qLD, quasi–SV (P_2) or qTD, quasi–TM (P_3) and wave due to voids (P_4)with their angles as $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ respectively. Also the transmitted wave exists as; quasi–longitudinal transmitted (qLT); (P_5)wave, with angleα_5.

5. Boundary Conditions and Results

The following are the boundary conditions taken at the interface of the micropolar fibre reinforced solid with voids and liquid:

1. Stresses at the common interface are continuous i.e. $\sigma_{33}^{\alpha} = \sigma_{33}^{l}$, $\sigma_{13}^{\alpha} = \sigma_{11}^{l}$,at$x_3 = 0$.
2. The conditions due to microrotation and voids takes the form:
   (a) $m_{32}^{\alpha} = 0$, observe that $m_{32} = 0 \Rightarrow \phi_{2,3}^* = 0$, and
   (b) $\phi_{,3}^{\alpha} = 0$, at$x_3 = 0$, respectively.
3. Normal displacements are continuous at the common interface:
   $u_3^{\alpha} = u_3^{l}$, at $x_3 = 0$.

We choose the displacement components, micro-rotation vectors and the volume fraction field as:

$$\left.\begin{array}{l} u_1^\alpha = A_\alpha d_1^\alpha e^{i\mu_\alpha},\ u_3^\alpha = F^\alpha A_\alpha d_1^\alpha e^{i\mu_\alpha}, \\ \phi_2^* = ik_\alpha G^\alpha A_\alpha d_1^\alpha e^{i\mu_\alpha}, \\ \phi^\alpha = H^\alpha A_\alpha d_1^\alpha e^{i\mu_\alpha},\ u_1^l = A_l d_1^l e^{i\mu_l}, \\ u_3^l = I^l A_l d_1^l e^{i\mu_l}. \end{array}\right\}, \tag{21}$$

where $\mu_\alpha = k_\alpha(x_1 p_1^\alpha + x_3 p_3^\alpha - c_\alpha t)$, $\alpha = 0$ correspond to incident wave, $\alpha = 1,2,3,4$ corresponds to reflected waves in the solid medium and$\mu_\ell = k_\ell(x_1 p_1^\ell + x_3 p_3^\ell - c_\ell t)$, where $\ell = 5$ corresponds to quasi-longitudinal transmitted (qLT) wave in the inviscid liquid medium. Also, the coupled relations $F^\alpha, G^\alpha$and $H^\alpha$are obtained from Eqs (15-18) for the solid medium, i.e.

$$F^\alpha = -F_1^\alpha / F_2^\alpha,$$
$$G^\alpha = ik_\alpha(\chi_2 p_1^\alpha F^\alpha + \chi_1 p_3^\alpha)/(k_\alpha^2(D_3^\alpha - \rho J c_\alpha^2) - N_2),$$
$$H^\alpha = -(k_\alpha \zeta i(p_3^\alpha F^\alpha + p_1^\alpha))/(k_\alpha^2(\xi_1 - \rho\kappa c_\alpha^2) + \omega_0 - \varpi ikc).$$

where

$$F_1^\alpha = p_3^\alpha\{(D_1^\alpha - \rho c_\alpha^2 - N_2(p_1^\alpha)^2)(k_\alpha^2(D_3^\alpha - \rho J c_\alpha^2) - N_2) + \chi_1((p_3^\alpha)^2 N_1^* - (p_1^\alpha)^2 N_4)\},$$
$$F_2^\alpha = p_1^\alpha\{(\chi_2(p_3^\alpha)^2 N_1^* - (p_1^\alpha)^2 N_4) + ((p_3^\alpha)^2(N_2 + N_3) - D_2^\alpha + \rho c_\alpha^2)(k_\alpha^2(D_3^\alpha - \rho J c_\alpha^2) - N_2)\},$$
$$D_1^\alpha = N_1(p_1^\alpha)^2 + N_6(p_3^\alpha)^2,\ D_2^\alpha = N_4(p_1^\alpha)^2 + N_5(p_3^\alpha)^2,$$
$$\text{and}\ D_3^\alpha = S_1(p_1^\alpha)^2 + S_2(p_3^\alpha)^2,$$

Similarly, the relation I^ℓ for the liquid medium assumes the form:

$$I^\ell = \{\lambda_\ell((p_1^\ell)^2 - p_1^\ell p_3^\ell) - \rho_\ell c_\ell^2\}/\lambda_\ell((p_3^\ell)^2 - p_1^\ell p_3^\ell) - \rho_\ell c_\ell^2,\ \ell = 5.$$

Using Eq. (21) into the boundary conditions, a system of equation is obtained after using Snell's law i.e., the coefficients;a_{ij} of the system are made possible by using Snell's law such that:$k_0 p_1^{(0)} = k_1 p_1^1 = k_2 p_1^2 = k_3 p_1^3 = k_4 p_1^4 = k_5 p_1^5 = k$, and $k_0 c_0 = k_1 c_1 = k_2 c_2 = k_3 c_3 = k_4 c_4 = k_5 c_5 = \omega$.

Thus, the system obtained takes the form:

$$a_{ij} Z_j = b_i, i = j = 1,2,3,4,5. \tag{22}$$

where

$$a_{1r} = k_r d_1^r\{(N_2 p_1^r + N_5 p_3^r F^r)\}/k_0 d_1^0\{(N_2 p_1^0 + N_5 p_3^0 F^0)\},$$

$$a_{1\ell} = -k_\ell d_1^\ell \lambda_\ell\{(p_1^\ell + p_3^\ell I^\ell)\}/k_0 d_1^0\{(N_2 p_1^0 + N_5 p_3^0 F^0)\},$$

$$a_{2r} = k_r d_1^r\{(N_4 F^r p_1^r + G^r)\}/k_0 d_1^0\{(N_4 F^0 p_1^0 + G^0)\},$$

$$a_{2\ell} = -k_\ell d_1^\ell \lambda_\ell\{(I^\ell p_1^\ell)\}/k_0 d_1^0\{(N_4 F^0 p_1^0 + G^0)\},$$

$$a_{3r} = k_r^2 d_1^r p_3^r G^r / k_0^2 d_1^0 p_3^0 G^0,\quad a_{3\ell} = 0,$$

$$a_{4r} = k_r p_3^r H^r d_1^r / k_0 p_3^0 H^0 d_1^0,\quad a_{4\ell} = 0$$

$$a_{5r} = F^r d_1^r / F^0 d_1^0,\ a_{5\ell} = -I^\ell d_1^\ell / F^0 d_1^0.$$

Hence, Z_j is the reflection and transmission coefficients, and the amplitude ratios of the reflected and transmitted waves are$Z_i = |A_i/A_0|$, $b_i = -1$,$i = j = 1,2,3,4,5$, $r = 1,2,3,4$, and$\ell = 5$. Also observe that k_0=k_1, c_0=$c_1$and the speed of the waves; c_0, c_1, c_2, c_3, c_4, and c_5 depends upon the material parameters. The components of propagation and unit displacement vectors are as follows:

$$\begin{array}{l} p_1^{(0)} = Sin\alpha_0,\quad p_3^{(0)} = Cos\alpha_0,\quad d_1^{(0)} = Sin\alpha_0, \\ d_3^{(0)} = Cos\alpha_0, \\ p_1^{(1)} = Sin\alpha_1,\ p_3^{(1)} = -Cos\alpha_1,\ d_1^{(1)} = Sin\alpha_1, \\ d_3^{(1)} = -Cos\alpha_1, \\ p_1^{(2)} = Sin\alpha_2,\ p_3^{(2)} = -Cos\alpha_2,\ d_1^{(2)} = Cos\alpha_2, \\ d_3^{(2)} = Sin\alpha_2, \\ p_1^{(3)} = Sin\alpha_3,\ p_3^{(3)} = -Cos\alpha_3,\ d_1^{(3)} = Cos\alpha_3, \\ d_3^{(3)} = Sin\alpha_3, \\ p_1^{(4)} = Sin\alpha_3,\ p_3^{(4)} = -Cos\alpha_4,\ d_1^{(4)} = Cos\alpha_4, \\ d_3^{(4)} = Sin\alpha_4, \\ p_1^{(5)} = Sin\alpha_5,,\ p_3^{(5)} = Cos\alpha_5,\quad d_1^{(5)} = Sin\alpha_5, \\ d_3^{(5)} = Cos\alpha_5. \end{array} \tag{23}$$

6. Conclusion

This article dealt wholly on the formulation and the analytical solution of waves in a micropolar fibre-reinforced medium having pores and interfaced with liquid medium. Four reflected waves namely; quasi–P or qLD, quasi–SV or qTD, quasi–TM and wave due to voids traveling in the solid medium were found while quasi–longitudinal transmitted (qLT) was found traveling in the liquid medium due to the negligible viscosity of the liquid. The analytical derivation of the amplitude ratios of both reflected and transmitted waves respectively, were derived and presented. Also, we observed that the phase velocity in the liquid medium is independent of the angle of propagation and the reverse is the case in the solid medium such that the phase velocity of propagation were found to be dependent on the angle of propagation. Thus, it is worthy of note to state that the significance of the model should prove useful to new researchers, scientists, material scientists working to ascertain models that could be imperative in predicting some seismological analyses in some solid/liquid structures in terms of propagating phenomena. Future works to this study-"mathematical modeling of waves in a porous micropolar fibre reinforced structure and liquid interface", could incorporate rotating viscoelasticity of the solid/liquid media, grooved boundary conditions and non-local effects to the model.

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Appendix

$$
\begin{aligned}
E_1 = & (D_1(D_3\left(i^2\zeta^2p_3^2-\omega^2\rho\xi_1\right)-i^2N_4p_1^2\xi_1\chi_2 \\
& -D_2(\left(J\omega^2\rho+N_2\right)\xi_1+D_3\left(i\omega\varpi+\omega^2\kappa\rho-\omega_0\right))) \\
& +D_2\left(D_3\left(i^2\zeta^2p_1p_3-\omega^2\rho\xi_1\right)-i^2p_3^2\xi_1\chi_1\left(N_1\right)^*\right) \\
& +p_1p_3^2(-D_3(i^2\zeta^2N_3p_1-N_2^2p_1\left(\omega\left(i\varpi+\omega\kappa\rho\right)-\omega_0\right) \\
& \qquad +N_2(i^2\zeta^2p_3+p_1(i^2\zeta^2+N_3\left(-\omega\left(i\varpi+\omega\kappa\rho\right)+\omega_0\right)))) \\
& +p_1\xi_1(N_2^3+N_2^2\left(J\omega^2\rho+N_3\right)\ +i^2N_3N_4\chi_1+ \\
& N_2\begin{pmatrix} J\omega^2\rho N_3 \\ +i^2\begin{pmatrix} N_4\chi_1+ \\ \chi_2\left(N_1\right)^* \end{pmatrix} \end{pmatrix}))))/D_3\left(D_1D_2-N_2\left(N_2+N_3\right)p_1^2p_3^2\right)\xi_1,
\end{aligned}
$$

$$
\begin{aligned}
E_2 = & (-i^2\omega^2\zeta^2\rho D_3p_1p_3-i^2\omega^2\zeta^2\rho D_3p_3^2+i^2J\omega^2\zeta^2 \\
& \rho N_2p_1^2p_3^2+i^2\zeta^2N_2^2p_1^2p_3^2-iJ\omega^3\varpi\rho N_2^2p_1^2p_3^2 \\
& \qquad -J\omega^4\kappa\rho^2N_2^2p_1^2p_3^2-i\omega\varpi N_2^3p_1^2p_3^2-\omega^2\kappa\rho N_2^3p_1^2p_3^2 \\
& +i^2J\omega^2\zeta^2\rho N_3p_1^2p_3^2+i^2\zeta^2N_2N_3p_1^2p_3^2 \\
& \qquad -iJ\omega^3\varpi\rho N_2N_3p_1^2p_3^2-J\omega^4\kappa\rho^2N_2N_3p_1^2p_3^2 \\
& -i\omega\varpi N_2^2N_3p_1^2p_3^2-\omega^2\kappa\rho N_2^2N_3p_1^2p_3^2 \\
& \qquad +i^2J\omega^2\zeta^2\rho N_2p_1p_3^3+i^2\zeta^2N_2^2p_1p_3^3+\omega^4\rho^2D_3\xi_1 \\
& -i^3\omega\varpi N_2N_4p_1^2p_3^2\chi_1-i^2\omega^2\kappa\rho N_2N_4p_1^2p_3^2\chi_1 \\
& \qquad -i^3\omega\varpi N_3N_4p_1^2p_3^2\chi_1-i^2\omega^2\kappa\rho N_3N_4p_1^2p_3^2\chi_1 \\
& +i^4\zeta^2N_4p_1p_3^3\chi_1-i^4\zeta^2N_4p_1^3p_3\chi_2+i^2\omega^2\rho N_4p_1^2\xi_1\chi_2 \\
& \qquad +J\omega^2\rho N_2^2p_1^2p_3^2\omega_0+N_2^3p_1^2p_3^2\omega_0+J\omega^2\rho N_2N_3p_1^2p_3^2\omega_0 \\
& +N_2^2N_3p_1^2p_3^2\omega_0+i^2N_2N_4p_1^2p_3^2\chi_1\omega_0 \\
& \qquad +i^2N_3N_4p_1^2p_3^2\chi_1\omega_0+D_1(-i^2J\omega^2\zeta^2\rho p_3^2-i^2\zeta^2N_2p_3^2 \\
& +J\omega^4\rho^2\xi_1+\omega^2\rho N_2\xi_1+i^3\omega\varpi N_4p_1^2\chi_2 \\
& \qquad +i^2\omega^2\kappa\rho N_4p_1^2\chi_2+\omega^2\rho D_3\left(i\omega\varpi+\omega^2\kappa\rho-\omega_0\right) \\
& +D_2\left(J\omega^2\rho+N_2\right)\left(\omega\left(i\varpi+\omega\kappa\rho\right)-\omega_0\right) \\
& \qquad -i^2N_4p_1^2\chi_2\omega_0)-i^4\zeta^2p_3^4\chi_1\left(N_1\right)^*+i^2\omega^2\rho p_3^2\xi_1\chi_1\left(N_1\right)^*
\end{aligned}
$$

$$+i^4\zeta^2 p_1^2 p_3^2\chi_2\,(N_1)^* - i^3\omega\varpi N_2 p_1^2 p_3^2\chi_2$$
$$(N_1)^* - i^2\omega^2\kappa\rho N_2 p_1^2 p_3^2\chi_2\,(N_1)^* + i^2 N_2 p_1^2 p_3^2\chi_2\omega_0\,(N_1)^*$$
$$+D_2(-i^2\zeta^2\left(J\omega^2\rho + N_2\right)p_1 p_3$$
$$+J\omega^4\rho^2\xi_1 + \omega^2\rho N_2\xi_1 + \omega^2\rho D_3(\omega\,(i\varpi + \omega\kappa\rho) - \omega_0)$$
$$+i^3\omega\varpi p_3^2\chi_1\,(N_1)^* + i^2\omega^2\kappa\rho p_3^2\chi_1$$
$$(N_1)^* - i^2 p_3^2\chi_1\omega_0\,(N_1)^*))/D_3\left(D_1 D_2 - N_2\begin{pmatrix} N_2+ \\ N_3\end{pmatrix} p_1^2 p_3^2\right)\xi_1,$$

$$E_3 = -\omega^2\rho(i\omega^3\varpi\rho D_3 + \omega^4\kappa\rho^2 D_3 - i^2 J\omega^2\zeta^2\rho p_1 p_3 -$$
$$i^2\zeta^2 N_2 p_1 p_3 - i^2 J\omega^2\zeta^2\rho p_3^2 - i^2\zeta^2 N_2 p_3^2$$
$$+J\omega^4\rho^2\xi_1 + \omega^2\rho N_2\xi_1 + i^3\omega\varpi N_4 p_1^2\chi_2$$
$$+i^2\omega^2\kappa\rho N_4 p_1^2\chi_2 + D_2\left(J\omega^2\rho + N_2\right)\left(i\omega\varpi + \omega^2\kappa\rho - \omega_0\right)$$
$$+D_1\left(J\omega^2\rho + N_2\right)(\omega\,(i\varpi + \omega\kappa\rho) - \omega_0) - \omega^2\rho D_3\omega_0$$
$$-i^2 N_4 p_1^2\chi_2\omega_0 + i^3\omega\varpi p_3^2\chi_1\ \ (N_1)^* + i^2\omega^2\kappa\rho p_3^2\chi_1\,(N_1)^*$$
$$-i^2 p_3^2\chi_1\omega_0\,(N_1)^*)/D_3\begin{pmatrix} D_1 D_2 - \\ N_2\,(N_2 + N_3)\,p_1^2 p_3^2\end{pmatrix}\xi_1,$$

$$E_4 = \omega^4\rho^2\left(J\omega^2\rho + N_2\right)$$
$$\left(\omega\begin{pmatrix} i\varpi+ \\ \omega\kappa\rho\end{pmatrix} - \omega_0\right)/D_3\begin{pmatrix} D_1 D_2 - \\ N_2\,(N_2 + N_3)\,p_1^2 p_3^2\end{pmatrix}\xi_1.$$