Graph Theory: A Lost Component for Development in Nigeria

Olayiwola Babarinsa*

Department of Mathematics, Federal University Lokoja, Kogi State, Nigeria

Abstract

Graph theory is one of the neglected branches of mathematics in Nigeria but with the most applications in other fields of research. This article shows the paucity, importance, and necessity of graph theory in the development of Nigeria. The adjacency matrix and dual graph of the Nigeria map were presented. The graph spectrum and energies (graph energy and Laplacian energy) of the dual graph were computed. Then the chromatic number, maximum degree, minimum spanning tree, graph radius, and diameter, the Eulerian circuit and Hamiltonian paths from the dual graph were obtained and discussed.

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1. Introduction

The mathematical structures used to model pairwise relationships between things are studied in graph theory. Graph theory has applications in natural science, management and social sciences, arts and agricultural science, engineering and medicine, and information systems [1, 2]. It is the key subject essentially in mastering data science and is widely adopted by tech giants like Google, Amazon, Meta, Apple, and Microsoft.

The considerable origin of graph theory emanated from the problem of the Konigsberg bridges [3]. The city Konigsberg (now Kaliningrad in Western Russia) has two islands and mainlands with seven bridges connecting the lands. The challenge was to leave a point (home) and traverse each bridge exactly once and return to the same point [4, 5], see Figure 1.
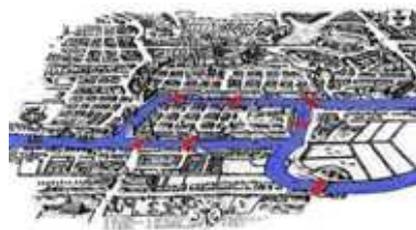
Figure 1. The bridges of Konigsberg.

In 1736, Swiss mathematician Leonhard Euler took up the problem of the seven bridges of Konigsberg and reconstructed it in a dual graph where the two mainlands and the two islands represent vertices and the seven bridges represent the edge (see Figure 2) and concluded that the task was not possible to do [6, 7]. His method of the proof of Konigsberg bridges leads to the concept of the Eulerian graph and to the introduction of a new branch of mathematics called graph theory.

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*Corresponding author tel. no: +2348060032554
Email address: olayiwola.babarinsa@fulokoja.edu.ng (Olayiwola Babarinsa)

Figure 2. A graph representation of the bridges of Konigsberg.

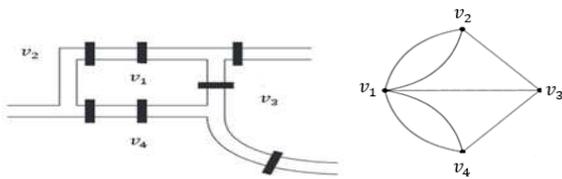In 1840, the development of graph theory started when German mathematician Ferdinand Mobius presented the idea of a complete graph and bipartite graph [8]. In 1852, a Scottish divine Francis Guthrie found the famous four-colour theorem [9]. Over two decades, the term "graph" was coined in 1878 by English mathematician Joseph Sylvester. He represented it as partitions of numbers by nodes placed in order at the points of a rectangular lattice [9].

This paper is organized into sections. In section 2, basic graph terminologies and illustrations are given, and in section 3, we discuss the impact of graph theory and its applications with examples emanating from Nigerian settings. Then the adjacency matrix and Laplacian matrix, and representation of the (Nigeria) map graph were computed and visualized, and the results were discussed.

2. Preliminary and Background in Graph Theory

Graphs harmonize vertices and edges that connect these vertices. This definition and its terminologies will be meaningless to researchers who want to pick up research interest in graph theory or to apply its usefulness in other research areas without understanding the rudimentary subject matter. It is important to note that an edge is an ambiguous word that may refer to both undirected edges and directed edges. Readers should be aware that in this paper whenever an "edge" and a "graph" are mentioned, they are referring to an undirected edge and a simple graph respectively, except otherwise stated.

2.1. Graph Models

To model a graph, there must exist an interaction. The interaction may be a mutually exclusive interaction (called an undirected edge) or just one-way interaction (called a directed edge or arc). Now, consider two or more objects (we say they are "vertices" or "nodes") and the interactions or connections between these objects are referred to as edges. We are much interested, in the diagrams, whether two given vertices are joined by a line (edge or arc) but the way they are joined is less important. For instance, Facebook users are vertices and their friendships are the edges, proteins are vertices and their interactions are edges, two or more elements are vertices and their bonds are the edges, particles are vertices and their collisions are the edges, footballers on the pitch are vertices and the passes made with the ball are the edges, dating apps/websites connecting people (vertices) by their interests (edges).

Problems in almost every conceivable discipline can be solved using graph models. Graphs are widely used to describe the acquaintanceships between people, phone calls between two or more lines, links between websites, a collaboration between researchers, roadmaps, assignment of jobs, coloring (minimum color) the regions of a map, differentiating between isomers, food chain and food web, Sequencing of DNA, finding the shortest path between two cities, communication link between computers (mesh network), scheduling exams and assigning channels to television stations [2]. Some of the modeled graphs are acquaintanceship and friendship graphs, influence graphs, collaboration graphs, call graphs, molecular graphs, web graph and citation graphs, module dependency graphs, precedence graphs, concurrent processing, airline routes and road networks, tournaments, Niche overlap graphs in ecology and protein interaction graphs [2, 10]. These graph models can be created and visualized using graph software such as NetworkX, Gephi, GraphViz, Sage, or MATLAB.

2.2. Graphs Definition

To define a simple graph to a non-graph theorist one needs to "unconventionally" illustrate it. For instance, a tug of war between two people is a simple graph with two vertices (people) and an edge (rope), see Figure 3.

Figure 3. A simple graph: Tug of war between two people.

A simple graph is an undirected (non-directed) graph but if the edges have direction, then it is called a directed graph, mostly known as a digraph [11]. For example, see Figure 4, only the rag doll (v_1) exerts a greater force (arc) to pull a box (v_2).

Figure 4. A directed graph: A rag doll pulling a box.

The directed edges (arcs or arrows) of a digraph have head ends and tail ends, see Figure 5. An in-degree, $deg^-(v)$, is the number of arcs (head ends) towards a particular vertex, otherwise, it is an out-degree, $deg^+(v)$. A vertex that consists of zero $deg^-(v)$ is termed a source and a sink is a vertex with zero $deg^+(v)$ [12].

Figure 5. The head and tail end of an arc.

In addition, if there exist both undirected edges and directed edges in a graph then the graph is referred to as a mixed graph [13]. let us consider a weight lifter (v_3) with his hands exerting pressure (arcs) in balancing (edge) two ends (v_1 and v_2) of weight, see Figure 6.

Figure 6. A mixed graph: A man lifting weight.

Mathematically, a simple graph $G = (V, E)$ consists of a set of vertices V and a set of undirected edges E [14]. A directed graph (or a digraph) is a graph that contains only a set of directed edges with the set of vertices V [2]. A mixed graph $G = (V, E, A)$ is an ordered triple consisting of a set of vertices $V = \{v_1, v_2...v_n\}$, a set of undirected edges $E = \{e_1, e_2...e_n\}$, and a set of directed edges A [13, 15].

2.3. Graphs Classification

In considering the type of graph to use, one must know if the graph is directed or undirected, weighted or not weighted, and sometimes if it is cyclic or acyclic. Another thing to consider about a graph is whether loops are permitted. A loop (or buckle) is an edge that links a vertex to itself. A pendant is a vertex that joins exactly one other vertex.

An edge connects the vertices that define it. A graph with a countable number of vertices is termed a finite graph, and if the vertices are infinitely many is called an infinite graph [2]. A weighted graph or edge-labeled graph has edges (directed or undirected) having a non-negative value called weight. An unweighted graph can be considered a weighted graph if each of the edges has a numerical value of weight 1 [16]. The number of vertices (nodes or points) in G, written as $v(G)$, is the order of a graph while the number of edges in graph G, written as $e(G)$, is the size of a graph [17]. A vertex of degree zero is called isolated. With the terms and definitions, graphs can be classified based on the types, see Table 1 and Figure 7.
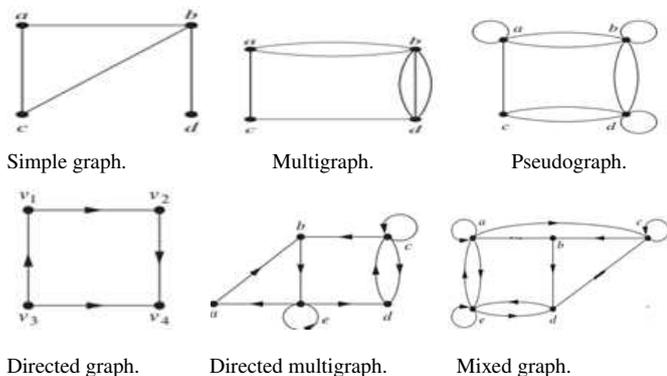

Simple graph.   Multigraph.   Pseudograph.

Directed graph.   Directed multigraph.   Mixed graph.

Figure 7. Graph classification.

2.4. Types of Graphs

Many complex graphs (such as network graph) have fundamental underlaying structure The following are the types of graphs in graph theory:

Complete graph.   Cyclic graph.   Wheel graph.   Planar graph.

Bipartite graph.   Regular graph.   Star graph.   Path graph.

Weighted graph.   Mixed hourglass.   Acyclic (Tree).   Acyclic (Forest).

Trivial graph.   Euler graph.   Halmitonian graph.   Null graph.

Hypercube graph.   Infinite graph.   Connected graph.   Friendship graph.
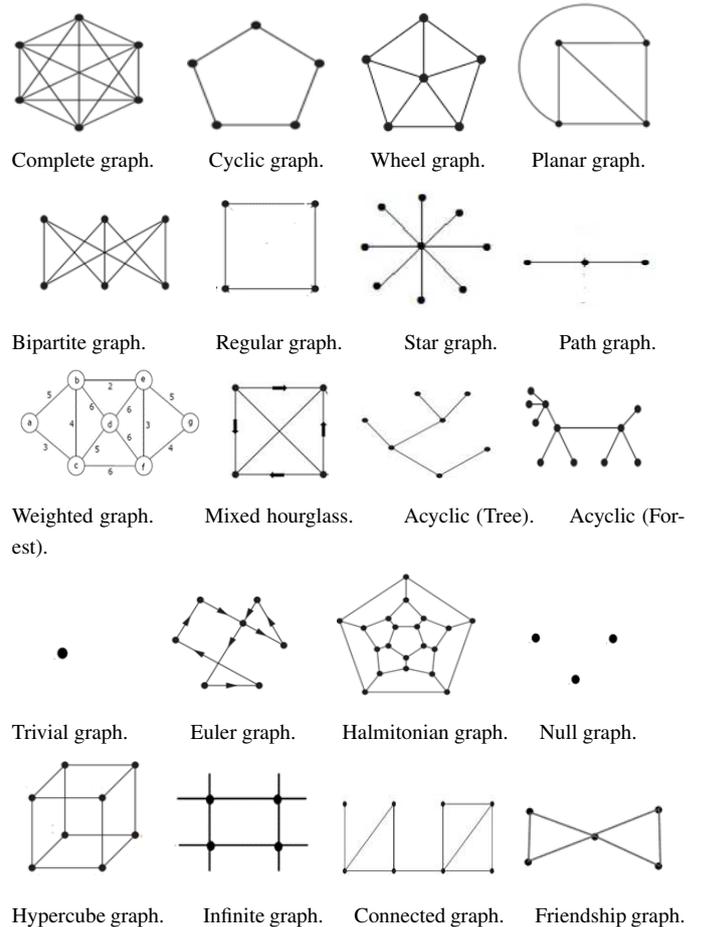
Figure 8. Basic graph types.

2.5. Matrices: Graph representations

Two types of matrices are mostly used to represent graphs; the adjacency matrix and incidence matrix. The adjacency matrix, $A(G) = [a_{i,j}]$, is an $n \times n$ matrix indexed by the vertices $\{v_1, v_2...v_n\}$ given as, see for instance

$$a_{i,j} = \begin{cases} 1 & \text{if } v_i v_j \text{ is an edge of } G \\ 0 & otherwise. \end{cases} \tag{1}$$

The adjacency matrix can determine if the graph is connected or not by checking the shortest path for every pair of vertices. The incidence matrix: Let $G = (V, E)$ such that $\{v_1, v_2...v_n\}$ are the set of vertices and $\{e_1, e_2...e_m\}$ the set of edges, then $M = [m_{i,j}]$ is an $n \times m$ matrix defined as [18]

$$m_{i,j} = \begin{cases} 1 & \text{when edge } e_j \text{ is incident with } v_i \\ 0 & otherwise. \end{cases} \tag{2}$$

Graph spectrum is the set of graph eigenvalues of $A(G)$ [8]. This spectrum can be used to obtain graph energy. Ivan Gutman introduced the concept of graph energy of a simple graph

Table 1. Summary of graph classification

Graph classification	Graph type	Edges	Multiple Edges?	Loops?
Weighted or unweighted	Simple graph	Undirected	No	No
	Multigraph	Undirected	Yes	No
Directed or undirected	Pseudograph	Undirected	Yes	Yes
	Directed graph	Directed	No	No
Cyclic or acyclic	Directed multigraph	Directed	Yes	Yes
	Mixed graph	Directed and undirected	Yes	Yes

from theoretical chemistry [19]. Nikiforov [20] gave the extended concept of graph energy to any matrix. Therefore, energy of a graph $E(G)$ is the sum of absolute values of eigenvalues, $\lambda_i(G); i = 1, 2, ..., n$, of $A(G)$ [21]. That is,

$$E(G) = \sum_{i=1}^{n} |\lambda_i(G)|. \tag{3}$$

Some of the forms of matrices used in graph theory are the degree matrix and (Signless) Laplacian matrix [22]. A degree matrix, $D(G) = [d_{i,j}]$, is an $n \times n$ diagonal matrix defined as

$$d_{i,j} = \begin{cases} \deg(v_i) \text{ if } i = j \\ 0 \quad otherwise. \end{cases} \tag{4}$$

Furthermore, the Laplacian matrix, $L(G) = D(G) - A(G)$, has spectrum that consist of $\mu_i; i = 1, 2, ..., n$ such that $\mu_1(L(G)) \geq \mu_2(L(G)) \geq ... \geq \mu_n(L(G)) \geq 0$, where $L(G)$ is symmetric and positive semidefinite [23]. The Laplacian energy is defined as

$$LE(G) = \sum_{i=1}^{n} \left| \mu_i - \left(\frac{2 \times edges}{vertices} \right) \right|. \tag{5}$$

A graph G with n vertices is hyperenergetic if $E(G) > 2n - 2$, hypoenergetic if $E(G) < n$ and non-hypoenergetic if $E(G) \geq n$ [24, 10]. Applications of graph energies are pattern recognition, object identification, face recognition, image analysis and processing which are widely adopted by military and police [25, 26]. In the area of medicine, attempts has been made to use it in neuronal network and epidemics see [27] and the references therein.

3. Graph Theory and Nigeria

In this section, the paucity of graph theory and representation of the Nigeria map (all 36 states and F.C.T) are presented, and the results are discussed.

3.1. The paucity of Graph Theory Applications in Nigeria

A graph theorist, a boundless researcher, has the most edge for collaboration. Thus, it will be difficult, if not impossible, to mention fields where graph theory is not applicable. In Nigeria, graph theory is one of the aspects of discrete mathematics that has not been considered beyond undergraduate and postgraduate studies, as an elective course, in mathematics, computer science, and engineering. The few published articles from

Nigerian researchers that mentioned graph theory only made references to the terms that apply to the research interest of the authors. Presently, no Nigerian mathematician has taken graph theory, especially graph energy, as his/her research interest, except perhaps the author of this paper whose Erdos number and Dijkstra number are 4 and 5 respectively, see reference in [28].

In the Nigeria setting, for instance, linking BVN to many bank accounts or NIN to SIM registration is achievable by graph theory. In today's Nigerian politics, graph theory is an essential tool for political analysis. We unconsciously apply graph theory in our daily activities such as the shortest distance we take to get to our destinations. The shortest distance usually has weights, and the weighted graph has huge applications in transportation systems. Almost every ministry in Nigeria such as NCC, NCDC, NIPOST, WAEC, NHIS, CBN, FAAN, FHA, INEC, NBS, NPA, and NPC can deploy graph theory for the effective performance of the agency in terms of data collection, analysis, and prediction. Besides, one of the greatest uses of graph theory in Nigeria is in town planning but it is not fully utilized. The paucity of graph usage can be seen in almost all sectors and places in Nigeria. For example, let us consider the chaotic urbanization of Lokoja (with an area of $3,180 km^2$) where two major equipped government hospitals (Kogi State Specialist Hospital and Federal Medical Centre Lokoja) are $2km$ apart of 3 minute's drive. It is obvious that in the planning of the establishment of these hospitals, a sudden cardiac arrest of a patient was not considered for the outskirt residents of Lokoja nor of other bordering local government areas. Another chaotic planning in Lokoja is the two tertiary institutions (Kogi State polytechnic and Federal University Lokoja), where the two institutions are about $8.3km$ apart on 6 minute's drive. Furthermore, emergency exits during natural disasters (such as a flood) or terrorist attacks are not considered in planning the state capital with three entrance/exit roads that consist of only two roads (Lokoja-okenne/Abuja-Lokoja road and Ganaja/Lokoja-Ajaokuta road) which lead to Lokoja. Thus, the cons of the urbanization of Lokoja outweigh its pros.

This catastrophic planning can be found in each state of the country. Another deficiency of graph theory in Nigeria can be noticed in the examination timetable scheduling or lecture timetable in the Nigerian tertiary institutions. Students often complain of having two or three examinations in a day and sometimes two examinations at the same time. To solve the problem of scheduling examination timetables, graph coloring should be applied.

Table 2. Adjacency matrix of states (and F.C.T) in Nigeria.

	v_1^*	v_2^*	v_3^*	v_4^*	v_5^*	v_6^*	v_7^*	v_8^*	v_9^*	v_{10}^*	v_{11}^*	v_{12}^*	v_{13}^*	v_{14}^*	v_{15}^*	v_{16}^*	v_{17}^*	v_{18}^*	v_{19}^*	v_{20}^*	v_{21}^*	v_{22}^*	v_{23}^*	v_{24}^*	v_{25}^*	v_{26}^*	v_{27}^*	v_{28}^*	v_{29}^*	v_{30}^*	v_{31}^*	v_{32}^*	v_{33}^*	v_{34}^*	v_{35}^*	v_{36}^*	v_{37}^*
v_1^*	0	0	1	1	0	0	0	0	0	1	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
v_2^*	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
v_3^*	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
v_4^*	1	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
v_5^*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0
v_6^*	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
v_7^*	0	0	0	0	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0
v_8^*	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
v_9^*	0	0	1	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
v_{10}^*	0	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0
v_{11}^*	1	0	0	0	0	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
v_{12}^*	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
v_{13}^*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0
v_{14}^*	1	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
v_{15}^*	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
v_{16}^*	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
v_{17}^*	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
v_{18}^*	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	1
v_{19}^*	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
v_{20}^*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
v_{21}^*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0
v_{22}^*	0	0	0	1	0	0	1	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	1	0	0	0	0	0	0	1
v_{23}^*	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0
v_{24}^*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
v_{25}^*	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1
v_{26}^*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	1	1	1	0	0	0	0	0	0	1	1
v_{27}^*	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0
v_{28}^*	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0
v_{29}^*	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	0	1	1	0	1	0	0	0	0	0	0	0
v_{30}^*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	1	0	0	0	0	0	0	0	0
v_{31}^*	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0
v_{32}^*	1	0	1	1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
v_{33}^*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
v_{34}^*	0	1	0	0	1	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0
v_{35}^*	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
v_{36}^*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
v_{37}^*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0

3.2. Representation of Nigeria Map in Graph

The Nigeria map, including all the states and Federal Territory Capital (F.C.T), can be represented in a graph, see Figure 9. First, let the border between states be represented by edges and the states be vertices. The connection between the borders (edges) and the states (vertices) is the graph obtained from the Nigeria map.

Figure 9. The Map of Nigeria with states (and F.C.T).

Now, let Abia $-v_1^*$, Adamawa $-v_2^*$, Akwa Ibom $-v_3^*$, Anambra $-v_4^*$, Bauchi $-v_5^*$, Bayelsa $-v_6^*$, Benue $-v_7^*$, Borno $-v_8^*$, Cross River $-v_9^*$, Delta $-v_{10}^*$, Ebonyi $-v_{11}^*$, Edo $-v_{12}^*$, Ekiti $-v_{13}^*$, Enugu $-v_{14}^*$, Gombe $-v_{15}^*$, Imo $-v_{16}^*$, Jigawa $-v_{17}^*$, Kaduna $-v_{18}^*$, Kano $-v_{19}^*$, Katsina $-v_{20}^*$, Kebbi $-v_{21}^*$, Kogi $-v_{22}^*$, Kwara $-v_{23}^*$, Lagos $-v_{24}^*$, Nasarawa $-v_{25}^*$, Niger $-v_{26}^*$, Ogun $-v_{27}^*$, Ondo $-v_{28}^*$, Osun $-v_{29}^*$, Oyo $-v_{30}^*$, Plateau $-v_{31}^*$, Rivers $-v_{32}^*$, Sokoto $-v_{33}^*$, Taraba $-v_{34}^*$, Yobe $-v_{35}^*$, Zamafara $-v_{36}^*$ and F.C.T $-v_{37}^*$.

Thus, the representation of vertices and edges gives the adjacency matrix of the states (and F.C.T) in Nigeria, see Table 2. Using the Sage and Graphviz packages, the representation of Table 2 in Voronoi diagram and dual graphs are given in Figure 10, 11 and 12 respectively.

3.3. Results and Discussions

The followings are the results and useful analysis obtained from Figure 12. There is no presence of a loop since no state can share a border with herself. Lagos (v_{24}^*) is a pendant because it borders almost entirely the ocean and is suitable for a seaport. The colour number indicated on the vertices is the chromatic number. Thus, the chromatic number $\chi_{(G)}$ (the minimum number of colours required for colouring the regions of a graph) is 4. Readers are advised to colour the Nigeria map with 4 colours such that no two states (vertices) with the same colour share a border (edge).

The largest number of neighbors of a vertex in the graph, the maximum degree, is 11 which corresponds to v_{22}^*. That means Kogi state (v_{22}^*) shares a border with 11 other states, which makes it a commercial and geographical state of Nigeria. For the commercial purpose, Amazon (Cadabra Inc) can establish a warehouse in Kogi state to minimize the cost of transportation.
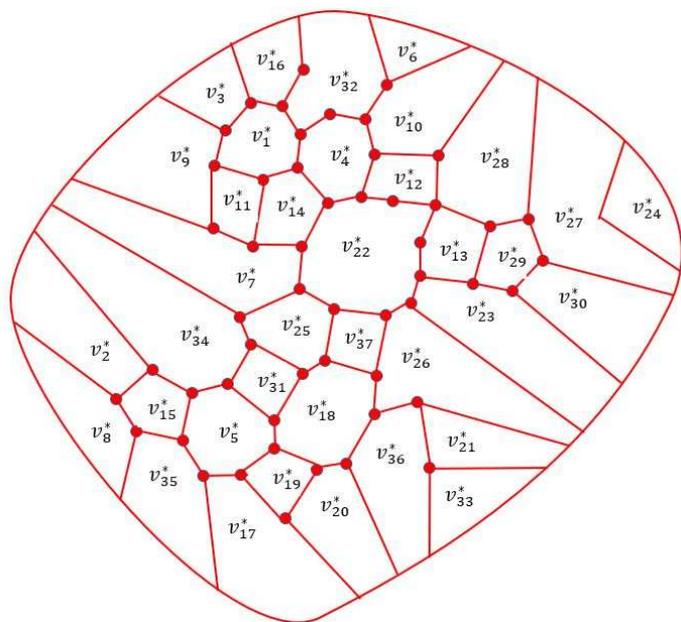
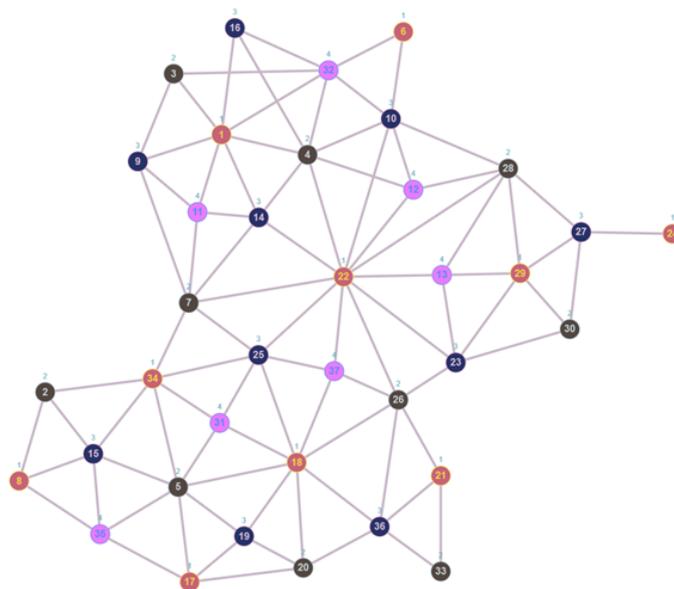Figure 10. The Voronoi diagram of Nigeria map.
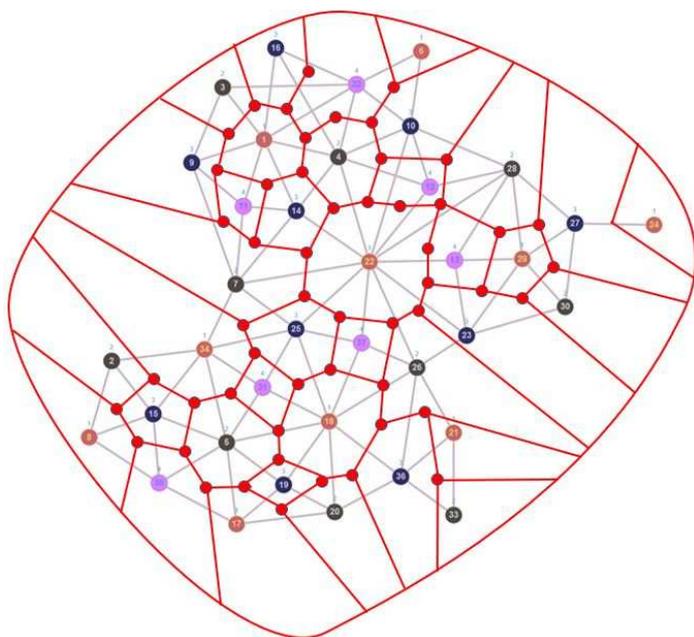

Figure 12. The graph of Nigeria map.

Figure 11. The dual graph of Nigeria map.

Using Kogi state as a reference, the other warehouses can be situated in Kaduna state (v_{18}^*), Ogun state (v_{27}^*), Abia state (v_1^*) and Gombe state (v_{15}^*) which will not only cover five geopolitical zone of the country but also saves the company the cost to build minimum number of warehouses require to serve Nigeria and neighbouring countries. These strategic states will serve other neigbouring countries, except Kogi state, if the needs arise. For instance, Ogun state warehouse can directly serve Benin Republic, Kaduna state can serve part of Niger Republic, Abia state serves part of Cameroon, and Gombe state can serve both Chad Republic and part of Cameroon.

The connected component of the graph is 1. The weight of the Minimum Spanning Tree (MST) is 36. The graph radius (minimum eccentricity of any vertex in the graph) and graph diameter (maximum eccentricity) from the figure are 4 ($v_7^* \Rightarrow v_{22}^* \Rightarrow v_{28}^* \Rightarrow v_{27}^* \Rightarrow v_{24}^*$) and 7 ($v_8^* \Rightarrow v_2^* \Rightarrow v_{34}^* \Rightarrow v_7^* \Rightarrow v_{22}^* \Rightarrow v_{28}^* \Rightarrow v_{27}^* \Rightarrow v_{24}^*$) respectively.

Figure 12 has no Euler circuit (simple circuit containing every edge) because there are more than two states $\left(v_2^*, v_3^*, v_8^*, v_{16}^*, v_{21}^*\right.$ and $\left.v_{30}^*\right)$ bordering exactly three states, or two states ($v_{14}^*, v_{23}^*, v_{29}^*$ and v_{36}^*) bordering exactly five states. This implies Euler circuit cannot exist since the number of odd degrees is higher than two. The Hamiltonian path from the dual graph indicates that a Nigerian who is willing to visit all states without traversing any state twice can do so by following these paths: $v_1^* \Rightarrow v_3^* \Rightarrow v_9^* \Rightarrow v_7^* \Rightarrow v_{11}^* \Rightarrow v_{14}^* \Rightarrow v_4^* \Rightarrow v_{16}^* \Rightarrow v_{32}^* \Rightarrow v_6^* \Rightarrow v_{10}^* \Rightarrow v_{12}^* \Rightarrow v_{22}^* \Rightarrow v_{25}^* \Rightarrow v_{37}^* \Rightarrow v_{18}^* \Rightarrow v_5^* \Rightarrow v_{31}^* \Rightarrow v_{34}^* \Rightarrow v_2^* \Rightarrow v_8^* \Rightarrow v_{15}^* \Rightarrow v_{35}^* \Rightarrow v_{17}^* \Rightarrow v_{19}^* \Rightarrow v_{20}^* \Rightarrow v_{36}^* \Rightarrow v_{33}^* \Rightarrow v_{21}^* \Rightarrow v_{26}^* \Rightarrow v_{23}^* \Rightarrow v_{13}^* \Rightarrow v_{28}^* \Rightarrow v_{29}^* \Rightarrow v_{30}^* \Rightarrow v_{27}^* \Rightarrow v_{24}^*$. From Table 2, the determinant of the adjacency matrix is -6822 (clockwise orientation) and the spectrum of the adjacency matrix are

$\lambda_1^* = -3.0149, \ \lambda_2^* = -2.5616, \ \lambda_3^* = -2.3765, \ \lambda_4^* = -2.3457,$
$\lambda_5^* = -2.1655, \ \lambda_6^* = -2.1054, \ \lambda_7^* = -2.0253, \ \lambda_8^* = -1.9140,$
$\lambda_9^* = -1.7695, \ \lambda_{10}^* = -1.7151, \ \lambda_{11}^* = -1.5978, \ \lambda_{12}^* = -1.4870,$
$\lambda_{13}^* = -1.3401, \ \lambda_{14}^* = -1.3257, \ \lambda_{15}^* = -1.2377, \ \lambda_{16}^* = -1.1903,$
$\lambda_{17}^* = -0.7940, \ \lambda_{18}^* = -0.6964, \ \lambda_{19}^* = -0.5332, \ \lambda_{20}^* = -0.3504,$
$\lambda_{21}^* = -0.2638, \ \lambda_{22}^* = 0.0641, \ \lambda_{23}^* = 0.1458, \ \lambda_{24}^* = 0.2270,$
$\lambda_{25}^* = 0.6558, \ \lambda_{26}^* = 0.6833, \ \lambda_{27}^* = 0.9538, \ \lambda_{28}^* = 1.0645,$
$\lambda_{29}^* = 1.4782, \ \lambda_{30}^* = 1.8905, \ \lambda_{31}^* = 2.2471, \ \lambda_{32}^* = 2.6553,$
$\lambda_{33}^* = 3.0397, \ \lambda_{34}^* = 3.3508, \ \lambda_{35}^* = 4.0026, \ \lambda_{36}^* = 4.7088,$
$\lambda_{37}^* = 5.6430.$ Thus, the energy of the graph is $E(G) = 65.6201,$

which is non-hypoenergetic.

To compute the Laplacian energy (see Equation 5) from Table 2, we evaluate the degree matrix via Equation 4. Then its spectrum are

$\mu_1^* = 0.0000$, $\mu_2^* = 0.2841$, $\mu_3^* = 0.4619$, $\mu_4^* = 0.6405$,
$\mu_5^* = 1.0892$, $\mu_6^* = 1.3870$, $\mu_7^* = 1.4319$, $\mu_8^* = 1.9546$,
$\mu_9^* = 2.3059$, $\mu_{10}^* = 2.5968$, $\mu_{11}^* = 2.8290$, $\mu_{12}^* = 2.9777$,
$\mu_{13}^* = 3.2076$, $\mu_{14}^* = 3.3923$, $\mu_{15}^* = 3.5828$, $\mu_{16}^* = 4.0672$,
$\mu_{17}^* = 4.1943$, $\mu_{18}^* = 4.5491$, $\mu_{19}^* = 4.5880$, $\mu_{20}^* = 5.0092$,
$\mu_{21}^* = 5.1446$, $\mu_{22}^* = 5.4446$, $\mu_{23}^* = 5.6693$, $\mu_{24}^* = 5.8715$,
$\mu_{25}^* = 5.9542$, $\mu_{26}^* = 5.9889$, $\mu_{27}^* = 6.1590$, $\mu_{28}^* = 6.7064$,
$\mu_{29}^* = 7.2008$, $\mu_{30}^* = 7.4258$, $\mu_{31}^* = 7.4721$, $\mu_{32}^* = 7.6544$,
$\mu_{33}^* = 8.0111$, $\mu_{34}^* = 8.3096$, $\mu_{35}^* = 8.6034$, $\mu_{36}^* = 9.5428$,
$\mu_{37}^* = 12.2923$ and its Laplacian energy is $LE(G) = 87.6227$. The energies of this graph could be used by the security agencies (such as NIA, DIA and DSS) to counter terrorism in Nigeria, since the country has (land and coastline) border of about $19,382 km$ with 1400 illegal entry points.

4. Conclusion

The retrogressive development in Nigeria is shown by the paucity of application of graph theory. The dual graph of the Nigeria map shows the connectivity between states which can be used as a transportation heuristic for road, water, and rail systems. Further research implementations and applications of graph theory to the ministries, agencies, or sectors in Nigeria will be important for the country to be the giant of Africa.

Acknowledgments

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