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An Integral Transform-Weighted Residual Method for Solving Second Order Linear Boundary Value Differential Equations with Semi-Infinite Domain

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Abstract

Solving boundary value problem with semi-infinite domain in a conventional way or using some of the available approximate methods poses a lot of challenges or better still seems almost impossible over the years, and some of the alternative ways for crossing this hurdle is by fixing value for infinity that is in the domain. Here in this work, we present an Integral Transform (Aboodh Transform) - Weighted Residual Based Method (AT-WRM) to address the afore-mentioned challenge. Aboodh Transform was used to transform and at the same time used to find the inverse of the given differential equations while Weighted Residual via Collocation Method was used in order to avoid fixing value for infinity as usual by introducing e^{-ix} in trial function to decay the infinity that was part of the boundary condition. The accuracy of the presented method was authenticated by solving three different problems. The excellent results obtained from the three solved problems validate the accuracy and effectiveness of the method

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Keywords: Aboodh Transform, Weighted Residual, Partition Method, Boundary Value Problem, Semi-Infinite Domain.

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1. Introduction

Boundary conditions at infinity are commonly used to solve linear and nonlinear phenomena that emerge in a wide range of scientific and engineering sectors. For the solution of these types of problems, a variety of mathematical techniques and methodologies have been proposed. Oderinu and Aregbesola [1], for example, employed the weighted residual approach to address the challenge encountered in a semi-infinite domain by reducing the domain within (0,∞) utilizing the Gauss-Laguerre integration formula with Galerkin and Moment methods. Similarly, Odejide and Aregbesola [2] utilized the same strategy to tackle the problems in a semi-infinite domain, but the method of partition was employed. To handle problems in an unbounded domain, several researchers have employed a variety of strategies. Noor and Syed [3] used Variational Iteration Method that was modified. Adewumi et al. [4] used the combination of Laplace transformation method and weighted residual method to provide a series solution to problems in a semi-infinite domain, Peker et al. [5] employed differential transform method and Pade approximant

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to solve Blasius problem, which appears in many areas of science and engineering professions. Likewise, Sajid et al. [6] presented the hybrid variational iteration algorithm method based on the combination of variational iteration and shooting methods to obtain the result of the same Blasius problem, Akinola and Ogunlaran [7] presented solution to these types of boundary-value problems in a semi-finite domain by Sumudu transform decomposition method, several other methods have also been employed to solve these type of problem and they are: Adomian decomposition method [8], solution of the MHD falner-Skan flow by Hankel-Pade method [9], variational iteration approach [10] homotopy analysis decomposition method for the solution of viscous boundary layer flow Due to a moving sheet by Alao et al. [11], application of Laplace decomposition method to boundary value equation in a semi-infinite domain by Ogunsola et al. [12], a comparison study of numerical techniques for solving ordinary differential equations defined on a semi-infinite domain using rational Chebyshev functions by Ramadan [13], an order four continuous numerical method for solving general second order ordinary differential equations by Obarhua [14], numerical algorithms for direct solution of fourth order ordinary differential equations by Kuboye [15] to mention a few and many other analytical and numerical methods have also been employed to tackle problems of the nature considered.

Here, we would employ the combination of an integral transform proposed by Aboodh [16, 17] and Weighted Residual method by Crandall [18], Finlayson and Scriven [19], Vichnevetsky[20] for finding the solution of problems with semi infinite domain, inspired and motivated by the existing research in this direction, the researchers decided to couple weighted-residual method with Aboodh Transform (AT) in order to compliment the drawback of Aboodh Transform that cannot handle boundary value problems especially with semi-infinite domain. The suggested technique uses the Aboodh Transform method to generate a new form of trial function, which is then substituted into the original equation to obtain the residual function, then ultimately reduced by the proposed method.

2. Basic Concept of the AT-WRM

2.1. Basic Idea of Aboodh Transform

Aboodh Transform is one out of many integral transforms defined for function of exponential order [16, 17] by:

$$A = \left\{ f(t) : \exists M, k_1, k_2 > 0, |f(t)| < Me^{-vt} \right\}, \tag{1}$$

where k_1, k_2 may be finite or infinite in a given set M. The Aboodh Transform of a function $f(t)$ is defined as:

$$A[f(t)] = K(v) = \frac{1}{v}\int_0^\infty f(t)e^{-vt}dt, t \geq 0, k_1 \leq v \leq k_2. \tag{2}$$

Aboodh Transform of some of the functions are:

$$A(c) = \frac{c}{v^2},$$

where c is a constant

$$A[t^n] = \frac{n!}{v^{n+2}}, A[e^{\pm at}] = \frac{1}{v^2 \pm av},$$

$$A[\sin(at)] = \frac{a}{v(v^2+a^2)}, A[\cos(at)] = \frac{1}{(v^2+a^2)}.$$

Aboodh transform of first, second and n^{th} derivatives are:

$$A[f'(t)] = vK(v) - \frac{f(0)}{v}$$

$$A[f''(0)] = v^2K(v) - \frac{f'(0)}{v} - f(0)$$

$$A[f^n(0)] = v^nK(v) - \sum_{k=0}^{n-1}\frac{f^k(0)}{v^{(2-n+k)}}$$

2.2. Basic Idea of Weighted Residual Method

Given a boundary value problem

$$M[u] = r(x), x \in D, \quad B_\mu[u] = \gamma_k, x \in \partial D. \tag{3}$$

$M[u]$ denotes a general differential operator, B_μ is the appropriate number of boundary conditions involved and D is the domain with boundary ∂D.

The solution of (3) is found by assuming an approximate solution called trial function to the dependent function $u(x)$ in the form:

$$\Omega(x,a) = w_0(x) + \sum_{i=1}^{n} a_i w_i(x), \tag{4}$$

where $\omega_i(x)$ are prescribed and satisfy the boundary conditions. Upon substitution of (4) into (3) the residual $R(x,a)$ in the differential equation is obtained.

The objective is to minimize the residual such that it becomes smaller and smaller or tend to zero as the number n of the function $\omega_i(x)$ increased in the successive approximations. Having obtained the residual, one of the methods, namely: Collocation, Partition, Moment, least square and Galerkin of minimizing the residual is employed.

3. Methodology- Aboodh Transform Weighted Residual Based Method (AT-WRM)

Suppose we have a differential equation:

$$L[u(x)] = f \tag{5}$$

in the domain ψ [1]

$$B_\gamma[u] = \psi \quad on \quad \partial\psi, \tag{6}$$

where $L[u(x)]$ denotes a general differential operator (linear or non-linear) involving spatial derivatives of dependent variable u, f is a known function of position, $B_\gamma[u]$ represents the appropriate number of boundary conditions, and ψ is the domain (semi-infinite in this case) with the boundary $\partial\psi$.

The itemized steps below were followed in solving this type of problem:

(i) We assumed a trial function that satisfies (6) in the form [10]

$$\sum_{i=0}^{n} d_i e^{-ikx}$$

, the assumption above is due to the condition at infinity, where d_i $i = 0, 1, 2, \cdot, n$ are constants to be determined and k is any constant value

(ii) Taking the Aboodh transform of (5) to have

$$A\Big[L[u(x)]\Big] = A[f] \tag{7}$$

(iii) Substituting the trial function obtained in step 1 into (7) to give

$$A\Big[L\Big[\sum_{i=0}^{n} d_i e^{-ikx} - f\Big]\Big] = 0. \tag{8}$$

(iv) Taking the Aboodh inverse transform of (8) to obtain a new trial function given as

$$A^{-1}\Big[A\Big[L\Big[\sum_{i=0}^{n} d_i e^{-ikx} - f\Big]\Big] = 0\Big] \tag{9}$$

(v) Imposing the boundary condition in finite domain on (9) to give a set of equations

(vi) Likewise, substituting (9) into (5) to obtain the Residual function which is then minimized using collocation method by making use of x_k which is the zeros of the Laguerre polynomial $L_n(x)$ given [2, 21]

$$L_n(x) = e^x \frac{d^n}{dx^n}(e^{-x}x^n)$$
$$A_k = \frac{(n!)^2}{x_k[L_n'(x_k)]^2} \tag{10}$$

Table 1: Laguerre Polynomial and the Corresponding Weight Functions

n	x_k	A_k
2	0.58578644	0.85355339
	3.41421356	0.14644661
4	0.32254769	0.60315410
	1.74576110	0.35741869
	4.53662030	0.03888791
	9.39507091	0.00053929
6	0.22284660	0.45896467
	1.18893210	0.41700083
	2.99273633	0.11337338
	5.77514357	0.01039920
	9.83746742	0.00026102
	15.98287398	0.00000090

(vii) The number of equations obtained equalled the number of constants to be determined and, so we solved the resulting algebraic equations through Maple 18 to obtain the constants d_i which are substituted back into the trial function in step 1 to find the solution.

3.1. Numerical Illustration

Illustration 1:

$$8\frac{d^2u}{dx^2} + 2\frac{du}{dx} - u = 8e^{-\frac{x}{4}} \tag{11}$$

subject to the boundary conditions

$$u(0) = 1, u(\infty) = 0,$$

with exact solution given as:

$$u_{EXACT} = 9e^{-\frac{1}{2}x} - 8e^{-\frac{1}{4}x}.$$

Assuming the trial function

$$u = \sum_{i=0}^{n} d_i e^{-ikx}$$

with $n = 6$ and $k = \frac{1}{4}$ gives:

$$u = d_0 + d_1 e^{-\frac{1}{4}x} + d_2 e^{-\frac{1}{2}x} + d_3 e^{-\frac{3}{4}x} + d_4 e^{-x} + d_5 e^{-\frac{5}{4}x} + d_6 e^{-\frac{3}{2}x}. \tag{12}$$

Finding the Aboodh transform of (11) and substituting (12) into the resulting equation gives:

$$A[u] = K[v] = \frac{1}{v^2}\Big[\frac{u'(0)}{v} + u(0) + A\Big[e^{-\frac{x}{4}} + \frac{1}{8}u - \frac{1}{4}u'\Big]\Big]. \tag{13}$$

Upon application of inverse Aboodh Transform on (13), we have:

$$u = A^{-1}\Big[\frac{1}{v^2}\Big[\frac{u'(0)}{v} + u(0) + A\Big[e^{-\frac{x}{4}} + \frac{1}{8}u - \frac{1}{4}u'\Big]\Big]\Big]. \tag{14}$$

Substituting (14) into (11) gives:

$$\begin{aligned} R = {} & 3 + \frac{5}{16}d_3 e^{-\frac{3}{4}x} + \frac{3}{8}d_4 e^{-x} + \frac{7}{16}d_5 e^{-\frac{5}{4}x} + \frac{1}{2}d_6 e^{-\frac{3}{2}x} + \frac{1}{8}d_0 \\ & + \frac{1}{8}d_1 - e^{-\frac{1}{4}x} - \frac{1}{2}x + \frac{1}{32}d_0 - \frac{1}{128}d_0(x^2 + 16) \\ & - \frac{1}{72}d_3\Big(4 - 3x + 5e^{-\frac{3}{4}x}\Big) - \frac{1}{64}d_4\Big(5 - 5x + 3e^{-x}\Big) \\ & - \frac{1}{400}d_5\Big(36 - 45x + 14e^{-\frac{5}{4}}\Big) - \frac{1}{144}d_6\Big(14 - 21x + 4e^{-\frac{3}{2}x}\Big) \\ & - \frac{1}{8}e^{-\frac{1}{4}x}(16 + 3d_1) - \frac{1}{16}d_1(x - 4) \\ & + \frac{1}{36}d_3\Big(-3 - \frac{15}{4}e^{-\frac{3}{4}x}\Big) + \frac{1}{32}d_4(-5 - 3e^{-x}) + \frac{1}{200}d_5 \\ & \Big(-45 - \frac{35}{2}e^{-\frac{5}{4}x}\Big) + \frac{1}{72}d_6\Big(-21 - 6e^{-\frac{3}{2}x}\Big). \end{aligned} \tag{15}$$

Subjecting (14) to the initial condition $u(0) = 1$, and collocate (11) using the roots of Laguerre polynomial of six points at x = 0.22284660, 1.18893210, 2.99273633, 5.77514357, 9.83746742, 15.98287398 to take care of the boundary condition at infinity-$u(\infty) = 0$. This gives rise to seven algebraic equations with seven unknown altogether.

Solving the seven equations for the unknown constants through Maple 18, we have:

$$\begin{aligned} & d_0 = 1.88748 \times 10^{-7}, d_1 = -8.000002108199, d_2 = 9.000009965150, \\ & d_3 = -0.000023454290, d_4 = 0.000028981727, \\ & d_5 = -0.000018037567, d_6 = 0.000004464431. \end{aligned} \tag{16}$$

Substituting (16) into the assumed trial function (12) gives the desired solution to (11) as:

$$\begin{aligned} u_{AT-WRM} &= 1.88748\times10^{-7} - 8.000002108199e^{\frac{-1}{4}x} \\ &+ 9.000009965150e^{\frac{-1}{2}x} - 0.000023454290e^{\frac{-3}{4}x} \\ &+ 0.000028981727e^{-x} - 0.000018037567e^{\frac{-5}{4}x} \\ &+ 0.000004464431e^{\frac{-3}{2}x} \end{aligned} \tag{17}$$

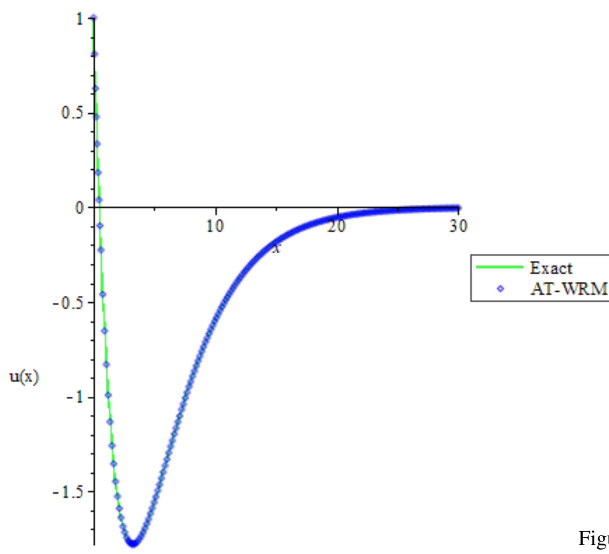

Figure 1: Graphical Validation of the Initial Boundary Condition of Illustration 1

Illustration 2:

$$8\frac{d^2u}{dx^2} + 2\frac{du}{dx} - u = 8e^{-\frac{3x}{4}}, \quad 0 \le x \le \infty, \tag{18}$$

with the exact solution

$$u_{EXACT} = -3e^{-\frac{1}{2}x} + 4e^{-\frac{3}{4}x}.$$

To obtain the solution to (18), salient points or procedures itemized in section 3 were carried out one after the other with the same trial function as (12), the same number of points ($n = 6$) and $k = \frac{1}{4}$. Hence, the constant terms of (12) and the approximate result of (18) are obtained as:

$$d_0 = -2.2517\times10^{-9}, d_1 = 2.5194\times10^{-8}, d_2 = -3.000000119291,$$
$$d_3 = 4.000000281182, d_4 = -3.47939\times10^{-7}, d_5 = 2.16854\times10^{-7},$$
$$d_6 = -5.3749\times10^{-8} \tag{19}$$

and

$$\begin{aligned} u_{AT-WRM} &= -2.2517\times10^{-9} + 2.5194\times10^{-8}e^{-\frac{1}{4}x} \\ &- 3.000000119291e^{-\frac{1}{2}x} + 4.000000281182e^{-\frac{3}{4}x} \\ &- 3.47939\times10^{-7}e^{-x} + 2.16854\times10^{-7}e^{-\frac{5}{4}x} \\ &- 5.3749\times10^{-8}e^{-\frac{3}{2}x} \end{aligned} \tag{20}$$

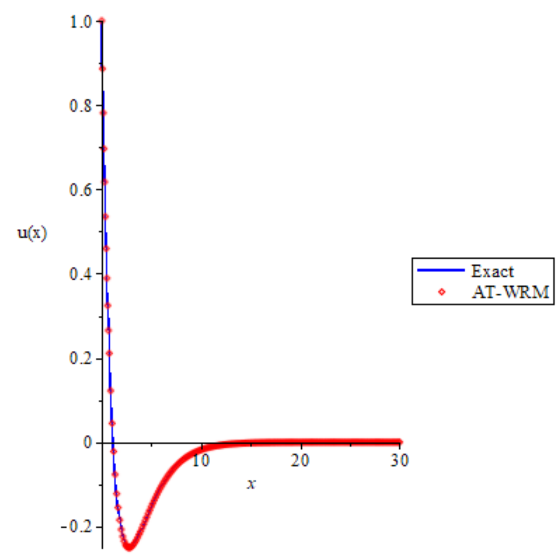

Figure 2: Graphical Validation of the Initial Boundary Condition of Illustration 2

Illustration 3: Consider the problem solved by Odejide [2]

$$\frac{d^2u}{dx^2} + 2\frac{du}{dx} - 2u = -e^{-2x}, \quad 0 \le x \le \infty \tag{21}$$

subject to the boundary conditions

$$u(0) = 1, \quad u(\infty) = 0.$$

The exact solution to (21) is:

$$u_{EXACT} = \frac{1}{2}\left(e^{(-1+\sqrt{i})x} + e^{-}2x\right)$$

This time around the new trial function

$$u = \sum_{i=0}^{n} d_i e^{(-1+\sqrt{i})x}$$

Table 2: Comparison between AT-WRM and the Exact for Illustration 1

x	u_{EXACT}	u_{AT-WRM}	$\lvert u_{EXACT} - u_{AT-WRM}\rvert$
0.0	1.000000000000	0.9999999999995	5.2791×10^{-13}
0.1	0.758585524280	0.7585855246842	$4.04668539 \times 10^{-10}$
0.2	0.533701366318	0.5337013671228	$8.04471306 \times 10^{-10}$
0.3	0.324423897197	0.3244238983415	$1.144766413 \times 10^{-9}$
0.4	0.129877433414	0.1298774348212	$1.406555289 \times 10^{-9}$
0.5	-0.050768173034	-0.05076817144136	$1.593182583 \times 10^{-9}$
0.6	-0.218299825266	-0.2182998235440	$1.721015535 \times 10^{-9}$
0.7	-0.373463358686	-0.3734633568723	$1.812797763 \times 10^{-9}$
0.8	-0.516965610304	-0.5169656084104	$1.893055570 \times 10^{-9}$
0.9	-0.649476385479	-0.6494763834940	$1.985061000 \times 10^{-9}$
1.0	-0.771630327158	-0.7716303250490	$2.1089594485 \times 10^{-9}$

Table 3: Comparison between AT-WRM and the Exact for Illustration 2

x	u_{EXACT}	u_{AT-WRM}	u_{MWR}[2]	$\lvert u_{EXACT} - u_{AT-WRM}\rvert$	$\lvert u_{EXACT} - u_{MWR}\rvert$
0.0	1.000000000000	0.999999999999	0.9999999995	7.0×10^{-13}	5.0×10^{-10}
0.1	0.857285671812	0.8572856718066	0.8572856728	$5.51848747 \times 10^{-12}$	1.8×10^{-9}
0.2	0.728319651592	0.7283196515825	0.7283196512	$1.032911322 \times 10^{-11}$	8.0×10^{-10}
0.3	0.611940945763	0.6119409457479	0.6119409463	$1.445554232 \times 10^{-11}$	3.0×10^{-10}
0.4	0.507080623493	0.5070806234760	0.5070806239	$1.765087242 \times 10^{-11}$	1.0×10^{-10}
0.5	0.412754765950	0.4127547659297	0.4127547648	$1.994249358 \times 10^{-11}$	1.2×10^{-9}
0.6	0.328057944442	0.3280579444209	0.3280579430	$2.151874020 \times 10^{-11}$	1.0×10^{-9}
0.7	0.252157188311	0.2521571882890	0.2521571879	$2.264794352 \times 10^{-11}$	1.1×10^{-9}
0.8	0.184286406269	0.1842864062451	0.1842864050	$2.362236467 \times 10^{-11}$	1.0×10^{-9}
0.9	0.123741227565	0.1237412275399	0.1237412271	$2.472101384 \times 10^{-11}$	1.0×10^{-10}
1.0	0.069874231826	0.06987423180009	0.06987423173	$2.618660748 \times 10^{-11}$	2.7×10^{-10}

is assumed for (21) with $n = 6$. Following the same process as done for the illustrations 1 and 2 we have:

$$\begin{aligned} d_0 &= -3.464061362 \times 10^{-11}, d_1 = 0.5000000099703, \\ d_2 &= -9.79486026690 \times 10^{-8}, d_3 = 0.5000003319506, \\ d_4 &= -5.104366705162 \times 10^{-7}, d_5 = 3.679792467715^{-7}, \\ d_6 &= -1.014800109354 \times 10^{-8} \end{aligned} \tag{22}$$

and

$$\begin{aligned} u_{AT-WRM} &= -3.464061362 \times 10^{-11} e^{-x} + 0.5000000099703 e^{-2x} \\ &- 9.79486026690 \times 10^{-8} e^{-2414213562x} \\ &+ 0.5000003319506 e^{-2732050808x} \\ &- 5.104366705162 \times 10^{-7} e^{-3.000000x} \\ &+ 3.679792467715^{-7} e^{-3.236067977x} \\ &- 1.014800109354 \times 10^{-8} e^{-3.449489743x} \end{aligned} \tag{23}$$

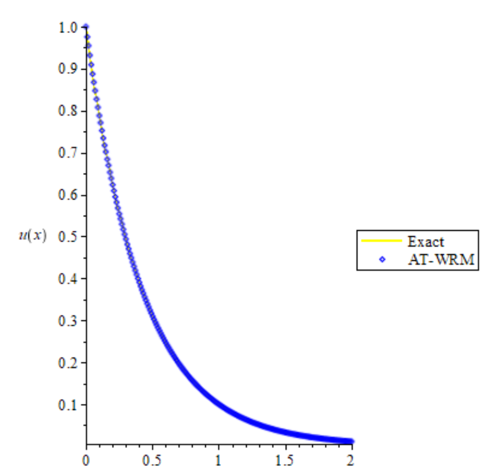

Figure 3: Graphical Validation of the Initial Boundary Condition of Illustration 3

3.2. Discussion of Results

An Integral Transform -Weighted Residual based technique has been successfully used in solving second order linear boundary value problems with semi- infinite domain. In order to examine the reliability of the proposed method ,three different illustrations were considered. Tables 2, 3 and 4 show the results of the three illustrations respectively and in all the three tables it was observed that absolute difference between exact

Table 4: Comparison between AT-WRM and the Exact for Illustration 3

x	u_{EXACT}	u_{AT-WRM}	u_{MWR}[2]	$\lvert u_{EXACT} - u_{AT-WRM}\rvert$	$\lvert u_{EXACT} - u_{MWR}\rvert$
0.0	1.000000000000	1.000000000001	1.00000000	1.346801×10^{-13}	0.0000000
0.1	0.7898337356130	0.7898337356130	0.7898312080	8.472091×10^{-14}	2.5275×10^{-6}
0.2	0.6246723675307	0.6246723675306	0.6246709988	9.183796×10^{-14}	1.3692×10^{-6}
0.3	0.4947063913440	0.4947063913439	0.4947069736	6.180149×10^{-14}	5.826×10^{-6}
0.4	0.3922992773092	0.3922992773093	0.3923013693	$1.0272004 \times 10^{-13}$	2.0923×10^{-6}
0.5	0.3114991915313	0.3114991915314	0.3115024402	9.230426×10^{-14}	3.2486×10^{-6}
0.6	0.2476617911460	0.2476617911461	0.2476660586	$1.4779032 \times 10^{-13}$	4.2673×10^{-6}
0.7	0.1971585649673	0.1971585649672	0.1971637757	$9.0332704 \times 10^{-14}$	5.2108×10^{-6}
0.8	0.1571511081549	0.1571511081545	0.1571571826	$3.30057946 \times 10^{-13}$	6.0744×10^{-6}
0.9	0.1254162556993	0.1254162556989	0.1254231315	$4.38164242 \times 10^{-13}$	6.8759×10^{-6}
1.0	0.1002104788741	0.1002104788736	0.1002181358	$5.38651959 \times 10^{-13}$	$7.6569 \times 1^{0-6}$

solution and Aboodh Transform-Weighted Residual Method is very negligible. Figures 1, 2 and 3 also show graphical comparison between the exact solution and Aboodh Transform-Weighted Residual Method for the three illustrations and it was observed that there are no significant difference between the two sets of solutions.

4. Conclusion

This research work has presented Aboodh Transform-Weighted Residual Method (AT-WRM) as a means of providing solution to three different second order linear boundary value problems in a semi-infinite domain. The results obtained so far as presented in the tables and figures showed that the method is superb, provides brilliant result and at the same time proves a vital tool to surmount the challenges of seeking how to handle the condition at infinity. Moreover, it has been shown that the combination of an integral transform-Adoodh Transform and Weighted Residual Method demostrate an excellent mathematical tool in obtaining near exact result if not the real exact solution of any boundary value problem and most especially the one with semi-infinite domain.

Motivation

The reason for compling the two methods is to be able to solve boundary value problems within semi-infinite domain by the process of Adoodh Transform method as Adoodh Transform cannot independently deal with boundary condition at infinity naturally. In addition,to improve the efficiency of the method of Weighted Residual Method.

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